1.1 Magnetic Flux

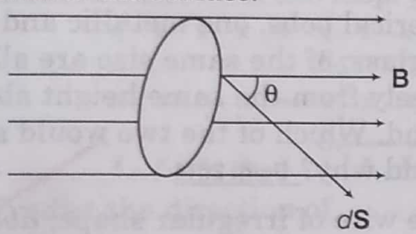
The magnetic flux represents total magnetic lines of force passing normally through a given area placed in a magnetic field.

Magnetic flux ϕ_B through an area dS in a magnetic field $\mathbf{B}$ is defined as, $\phi_B = \int \mathbf{B} \cdot d\mathbf{S}$.

where, $\mathbf{B}$ is the magnetic field and $d\mathbf{S}$ is the area element.

Magnetic flux, $\phi = \mathbf{B} \cdot \mathbf{S} = BS\cos\theta$,

where, S is the area of surface and θ is the angle between the direction of magnetic field and normal to the surface.



SI unit of magnetic flux is Weber (Wb).

CGS unit of magnetic flux is Maxwell (Mx).

$$1\ \text{Wb} = 10^8\ \text{Mx} = 1\ \text{Tm}^2$$

Magnetic flux is a scalar quantity and its dimensional formula is $[ML^2T^{-2}A^{-1}]$.

1.2 Electromagnetic Induction

The phenomenon to generate induced current or induced emf by changing the magnetic flux linked with a closed circuit is known as Electromagnetic Induction (EMI).

Faraday's Law of Electromagnetic Induction

Faraday's First Law

Whenever the amount of magnetic flux linked with the closed loop or circuit changes, an emf induces in the loop or circuit which lasts so long as change in flux continues.

Faraday's Second Law

The induced emf in a closed loop or circuit is directly proportional to the rate of change of magnetic flux linked with the closed loop or circuit.

i.e. $e \propto \frac{(-)\,\Delta\phi}{\Delta t} \Rightarrow e = -N\frac{\Delta\phi}{\Delta t}$

where, N = number of turns in loop.

1.3 Lenz's Law

According to this law, the direction of induced emf or induced current is such that it always opposes the cause that produces it. i.e. it opposes the change in magnetic flux.

Induced Current

- If N is the number of turns and R is the resistance of a coil, then the magnetic flux linked with its each turn changes by $d\phi$ in short time interval dt, the induced current flowing through the coil is

$$I = \frac{|e|}{R} = -\frac{1}{R}\left(N\frac{\Delta\phi}{\Delta t}\right)$$

- If induced current is produced in a coil rotated in a uniform magnetic field, then the flux through the coil $\phi = BA\cos\omega t$

  because the coil rotates with an angular velocity ω and turns through an angle θ in time t, $\theta = \omega t$,

  $\therefore \quad \phi = B\cos\omega t$

  Hence, $\varepsilon = -\frac{d\phi}{dt} = -\frac{d}{dt}(BA\cos\omega t) \Rightarrow \varepsilon = BA\sin\omega t$

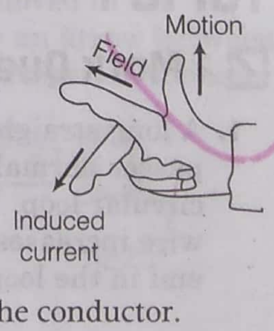
  If coil has N turns, then $\varepsilon = NBA\omega\sin\omega t$

  Hence, $I = \frac{NBA\omega\sin\omega t}{R} = I_0\sin\omega t$

  where, $I_0 = \frac{NBA\omega}{R}$ = Peak value of induced current.

Fleming's Right Hand Rule

If the thumb, forefinger and central finger of right hand are stretched mutually perpendicular to each other such that the forefinger points the direction of magnetic field, thumb points towards the direction of motion of conductor, then central finger points towards the direction of induced current in the conductor.



Motional Emf due to Translatory Motion

Let a conducting rod of length l be moving with a uniform velocity v perpendicular to a uniform magnetic field B, an induced emf is set up. The magnitude of the induced emf will be

$$e = El, e = Blv$$

$$[\text{In equilibrium } Fl = Fm, lE = lvB, E = vB]$$

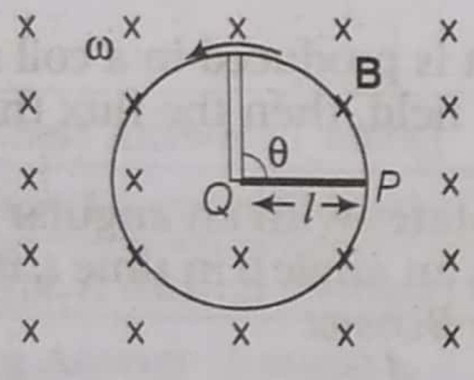
If the rod is moving such that it makes an angle θ with the direction of the magnetic field, then $e = Blv \sin\theta$. Hence, for the motion parallel to B, the induced emf is zero.

Motional Emf due to Rotational Motion

The induced emf developed between two ends of conductor of length l rotating about one end with angular velocity ω in a direction perpendicular to magnetic field is given by,

$$\varepsilon = \frac{B\omega l^2}{2} = Bl^2\pi\nu = \frac{Bl^2\pi}{T},$$

where, ν = frequency (cycle/s) and T = time period.



2.1 Eddy Currents

The current induced in bulk piece of conductor when magnetic flux linked with the conductor changes is known as eddy currents. The magnitude of eddy current is given by

$$i = \frac{\text{Induced emf}}{\text{Resistance}} = \frac{e}{R}$$

Direction of eddy currents can be given by Lenz's law or by Fleming's right hand rule.

- Eddy current causes undesirable heating and wastage of power in transformer. The heat produced by eddy currents may even damage the insulation of coils.
- Eddy current can be minimised by taking laminated core which consists of thin metallic sheets insulated from each other by varnish and placed normal to the direction of magnetic field.

Applications of Eddy Currents

Inspite of the undesirable effects, eddy currents are used in many ways. Some of them are given below:

(i) Speedometer (ii) Induction meter
(iii) Induction furnace
(iv) Electromagnetic shielding
(v) Electromagnetic damping
(vi) Energy meter
(vii) Induction motor

2.2 Inductance

Flux linkage of a closely wound coil is directly proportional to the current I i.e. $\phi_B \propto I$

For a closed wound coil of N-turns, the same magnetic flux is linked with all turns. The flux ϕ_B through the coil changes, each turn contributes to the induced emf. Therefore, flux linked with the coil (flux linkage) is equal to $N\phi_B$.

Then, total flux, $N\phi_B \propto I$.

The constant of proportionality in this relation is called inductance.

Self-Induction

The phenomenon of production of induced emf in a coil, when a current passes through it, undergoes a change.

$\because$ Total flux linked with coil, $N\phi \propto I$

$$N\phi = LI$$

where, ϕ = flux linked with each turn and
L = coefficient of self-induction or self-inductance.

Also, induced emf, $e = -\frac{d\phi}{dt} = -L\frac{dI}{dt}$

SI unit of self-induction is Henry (H),

where $L = \frac{\varepsilon}{dI/dt}$

1 Henry (H) = 1 V-s/A or 1 Tm^2/A or ohm-s.

Self-Inductance of a Long Solenoid

Self-induction of a long solenoid is one whose length is very large as compared to its area of cross-section. The magnetic field B at any point inside such a solenoid is practically constant and given by $L = \frac{\mu N^2 A}{l}$

where, N = number of turns,
A = area of solenoid, and l = length of solenoid.

2.2 Mutual Induction

The phenomenon of generation of induced emf in secondary coil when current linked with primary coil changes is known as mutual induction.

$$N_2\phi_2 = MI_1$$

where, $N_2\phi_2$ = flux linked with secondary coil
I_1 = current in primary coil

According to Faraday's Law, $e_2 = \frac{-MdI_1}{dt}$

SI unit of mutual inductance is Henry (H).

[Where $B_2 = \mu_0 n_2 I_2$, $B_1 = \mu_0 n_1 I_1$
$\phi_1 = B_2 AN_1 = \mu_0 n_2 I_2 \cdot AN_1$,
$\phi_2 = B_1 AN_2 = \mu_0 n_1 I_1 \cdot AN_2$]

1 Henry (H) = 1 V-s/A or 1 Tm^2/A.

Mutual inductance (M) of closely wound solenoids, $M = \frac{\mu_0 N_1 N_2 A}{l}$

where, N_1 and N_2 = number of turns in primary and secondary solenoids, A = area of solenoid and l = length of solenoid

Some Important Points Related to Inductance

- Two inductors are in parallel combination, then equivalent inductance is given by

$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$$

where, L_1, L_2 = coefficient of self-inductances of both coils.

Two inductors are in series combination

$$L = L_1 + L_2$$

- Magnetic energy stored in an inductor

$$U = \frac{1}{2} LI^2$$

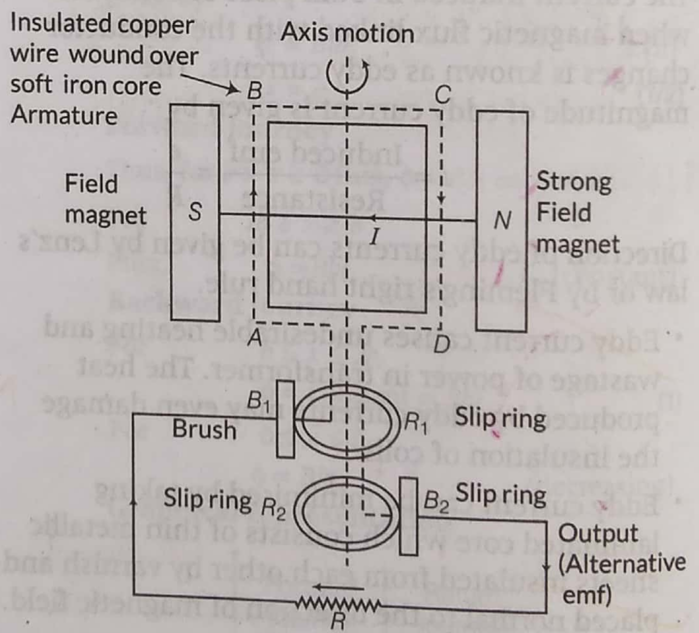
where, I is the current in the inductor.

2.3 AC Generator

A generator is an electrical machine that produces electrical energy from mechanical work, just the opposite of what a motor does. In a generator, the shaft is rotated by some mechanical means, such as an engine or a turbine starts working and an emf is induced in the coil.

Principle

An AC generator is based on the phenomenon of electromagnetic induction, which states that when a coil is rotated in uniform magnetic field magnetic flux linked with a conductor (or coil) changes, an emf is induced in the coil.



AC generator

Theory and Working

As, the armature of coil is rotated is uniform magnetic field, angle θ between the field and the normal to the coil changes continuously. Therefore, magnetic flux linked with the coil changes and an emf is induced in the coil. According to Fleming's right hand rule, current induced in AB from A to B and it is from C to D in CD. In the external circuit, current flows from B_2 to B_1.

If e is the emf induced in the coil, then

$$e = \frac{-Nd\phi}{dt} \Rightarrow e = NBA\omega \sin\omega t$$

[By Faraday's flux rule

$$\varepsilon = -\frac{d\phi}{dt} = -\frac{d}{dt}(NBA\cos t)$$

$$= NBA\omega \sin\omega t]$$

where , N = number of turns in the coil,

B = strength of magnetic field,

A = area of each turn of the coil,

ω = angular velocity of rotation of the coil

and $$I = \frac{e}{R} = \frac{NBA\omega}{R}\sin\omega t,$$

where R = resistance of the coil.

By Faraday's flux rule

$$\varepsilon = -\frac{d\phi}{dt} = -\frac{d}{dt}[NBA\cos\omega t]$$

$$= NBA\omega \sin\omega t$$