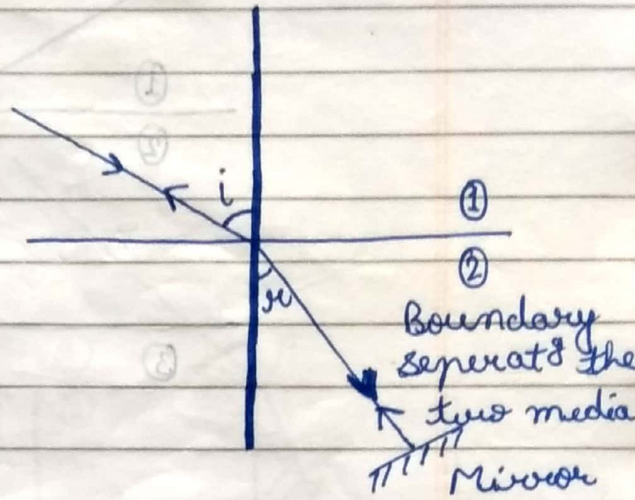
Ch-9

Unit-6 Optics

Refraction of light :-

The phenomena of change in path of light as it passes obliquely from one transparent medium to another is called refraction of light.



Snell's Law :-

First Law

(i) $\frac{\sin i}{\sin r}$ = Constant $({}^{1}\mu_2)$ { The incident ray, the refracted ray and the normal to the interface at the point of incidence, all lie in the same plane.

Second Law

(ii) The ratio of the sine of the angle of incidence and the sine of the angle of refraction is constant for a given pair of media

Principle of reversibility :

$\frac{\sin i}{\sin r} = {}^{1}\mu_2$

$\frac{\sin r}{\sin i} = {}^{2}\mu_1$

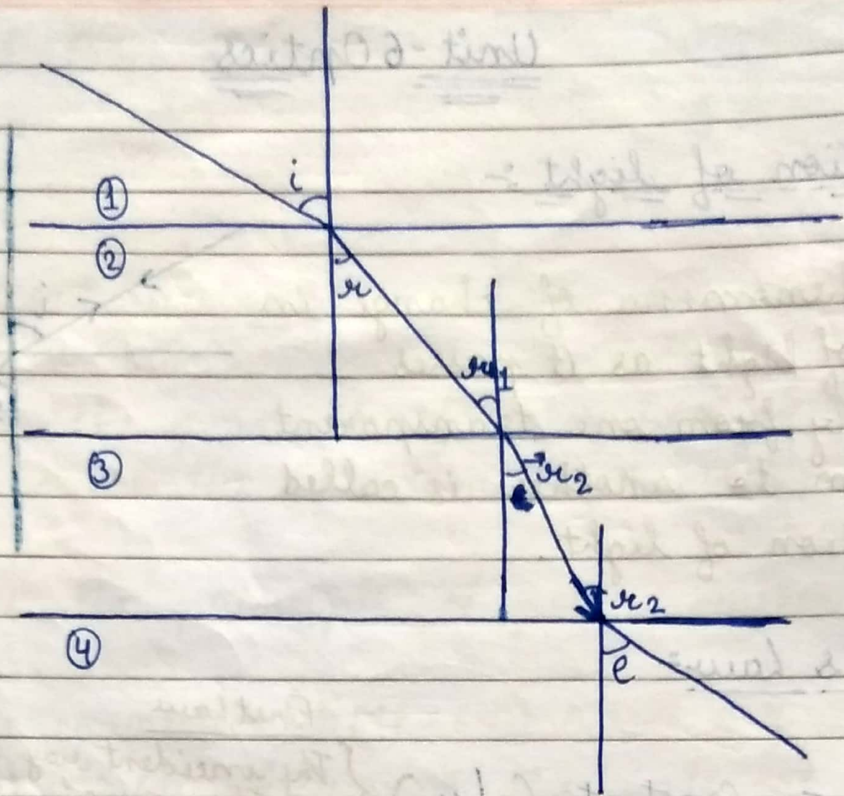
In general

${}^{a}\mu_b \times {}^{b}\mu_a = 1$

$\boxed{{}^{a}\mu_b = \frac{1}{{}^{b}\mu_a}}$

Multiplying, $1 = {}^{1}\mu_2 \times {}^{2}\mu_1$

Refraction through multiple media :



For first pair of Medium

$$ {}^{1}\mu_2 = \frac{\sin i}{\sin r_1} \longrightarrow (1) $$

For second pair of Medium

$$ {}^{2}\mu_3 = \frac{\sin r_1}{\sin r_2} \longrightarrow (2) $$

For third pair of Medium

$$ {}^{3}\mu_1 = \frac{\sin r_2}{\sin e} \longrightarrow (3) $$

Multiplying ---- $\quad {}^{1}\mu_2 \times {}^{2}\mu_3 \times {}^{3}\mu_1 = 1.$

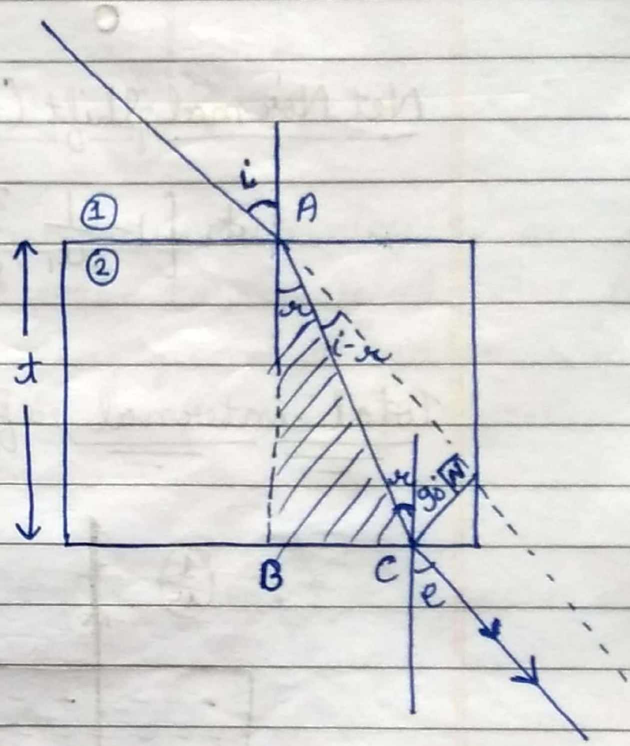
$$ {}^{2}\mu_3 = \frac{1}{{}^{1}\mu_2 \times {}^{3}\mu_1} $$

$$ \boxed{{}^{2}\mu_3 = \frac{{}^{1}\mu_3}{{}^{1}\mu_2}} $$

Absolute refractive index:-

$$^2\mu_3 = \frac{\mu_3}{\mu_2} \quad ,$$

Lateral Shift:-



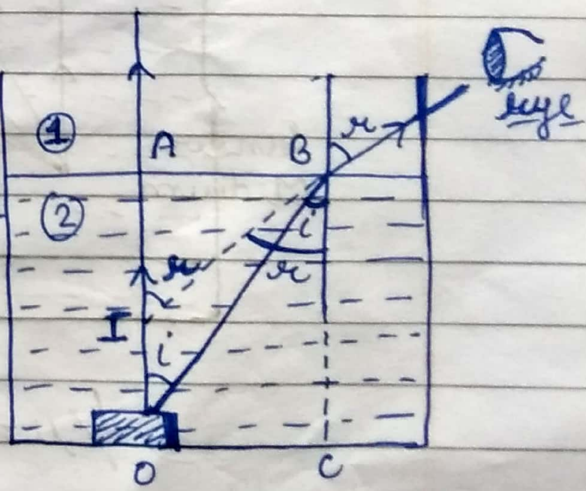
$$CN = AN \sin(i-r)$$

$$= \frac{AB}{\cos r}\sin(i-r)$$

$$\boxed{CN = \frac{t}{\cos r}(\sin i - r)}$$

Required Lateral Shift

Normal Shift:-



Let $^1\mu_2 = \mu$

$$^2\mu_1 = \frac{\sin i}{\sin r} = \frac{AB}{OB} \Big/ \frac{AB}{BI} = \frac{AB}{OB}\times\frac{BI}{AB}$$

$$= \frac{BI}{OB} \approx \frac{AI}{OA}$$

$$\frac{1}{\mu} = \frac{AI}{OA}$$

$$\mu = \frac{OA}{AI}$$

$$\mu = \frac{\text{Real depth}}{\text{Apparent depth}}$$

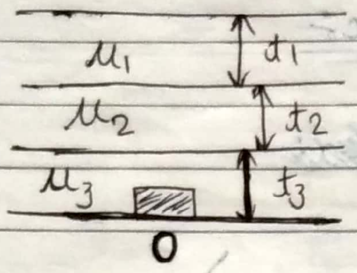
$$N.S = OI = OA - AI$$

$$= OA\left[1 - \frac{AI}{OA}\right]$$

$$= \boxed{t\left[1 - \frac{1}{\mu}\right]}$$

Lateral Shift

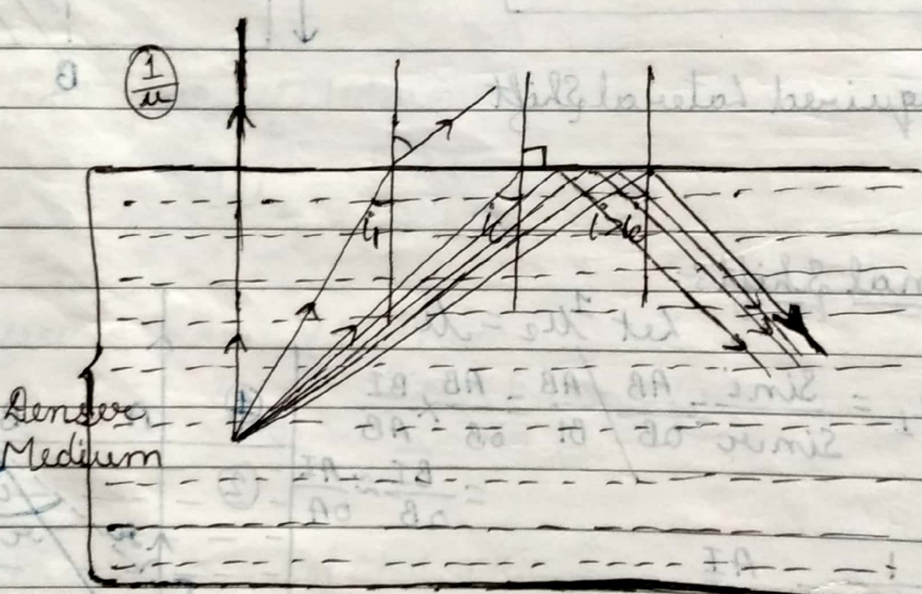
Normal Shift



Net Normal Shift (a case multiple media)

$$t_1\left[1-\frac{1}{\mu_1}\right]+t_2\left[1-\frac{1}{\mu_2}\right]+t_3\left[1-\frac{1}{\mu_3}\right]+---$$

Total internal reflection:-



$$1/\mu=\frac{\sin I_c}{\sin 90'}$$

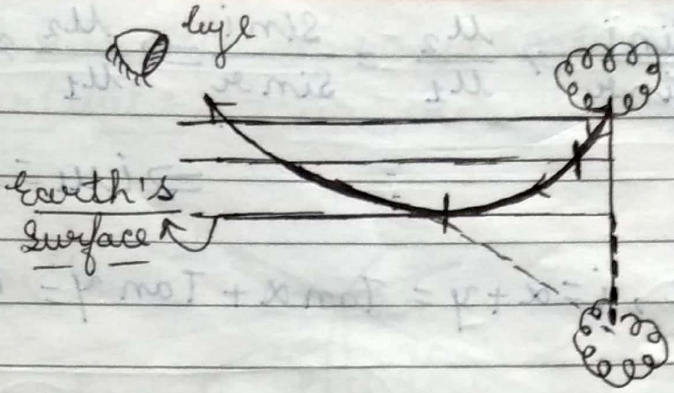
$\boxed{\mu=\frac{1}{\sin i}}$ or $\boxed{i=\sin^{-1}\frac{1}{\mu}}$

Applications of T.I.R :-

① ~~Mirage~~

Optical illusion observed in deserts or distant places is known as Mirage effect. In this observation hot surface seems as shimmering pond of water.

(1) Mirage :-



Def^n :- It is an optical illusion observed in deserts or over hot extended surface like coal-tarred road, due to which a traveller sees a shimmering pond of water some distance ahead of him & in which surrounding objects like trees etc, appear inverted.

Total Reflecting Prisms :- {Right angled Prisms}

These prism may be used in three ways:

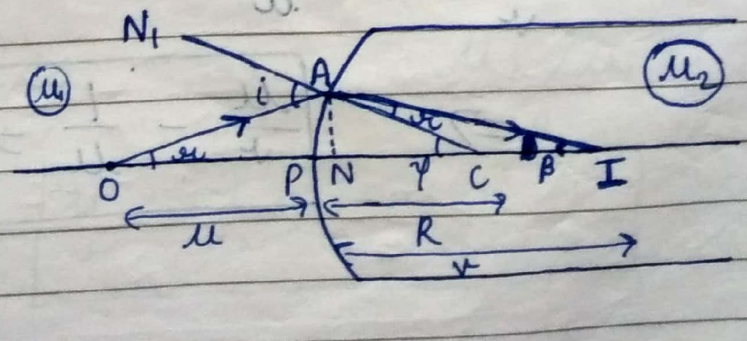
(i) To deviate a ray through 90°.
(ii) To invert an image with deviation of rays through 180.
(iii) To invert an image without deviation of rays (Erecting Prism).

Refraction through spherical surface :-

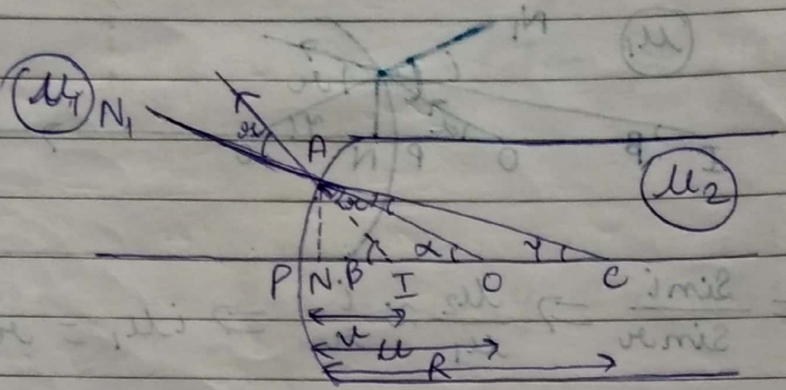
Imp

* Through Convex spherical surfaces:

When object O is left in rarer medium then & image formed is Real :-



(III) When object O is kept in a denser medium and image formed is virtual :



$$ {}^{2}\mu_1 = \frac{\sin i}{\sin r} \Rightarrow \frac{\mu_1}{\mu_2} = \frac{i}{r} \Rightarrow i\mu_2 = r\mu_1 \longrightarrow (1) $$

In $\triangle AOC$, $\alpha = i + \gamma \Rightarrow i = \alpha - \gamma = \tan\alpha - \tan\gamma$

$$ i = \frac{AN}{NO} - \frac{AN}{NC} = \frac{AN}{PO} - \frac{AN}{PC} = \frac{AN}{-u} - \frac{AN}{-R} = \frac{AN}{R} - \frac{AN}{u} \rightarrow (2) $$

In $\triangle AIC$, $\beta = r + \gamma \Rightarrow r = \beta - \gamma = \tan\beta - \tan\gamma =$

$$ \frac{AN}{NI} - \frac{AN}{NC} = \frac{AN}{PI} - \frac{AN}{PC} $$

$$ r = \frac{AN}{-v} - \frac{AN}{-R} = \frac{AN}{R} - \frac{AN}{v} \rightarrow (3) $$

Putting for i & r in eq^n (1)

$$ \mu_2\left[\frac{AN}{R} - \frac{AN}{u}\right] = \mu_1\left[\frac{AN}{R} - \frac{AN}{v}\right] $$

$$ \frac{\mu_2}{R} - \frac{\mu_2}{u} = \frac{\mu_1}{R} - \frac{\mu_1}{v} $$

$$ \boxed{\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R}} $$

$$\frac{1}{v} - \frac{\mu_2/\mu_1}{u} = \frac{1 - \frac{\mu_2}{\mu_1}}{R}$$

$$\boxed{\frac{1}{v} - \frac{\mu}{u} = \frac{1-\mu}{R}}$$

~~from a point~~

~~Light through in air falls on a convex spherical glass~~

Q. Ex 36 (SL Arora) Page no. 9·36.

Ans. $\mu_1 = 1$, $\mu_2 = 1.5$, $u = -100$ cm., $R = 20$ cm.

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1.5}{v} + \frac{1}{100} = \frac{1.5-1}{20} = \frac{1}{40}$$

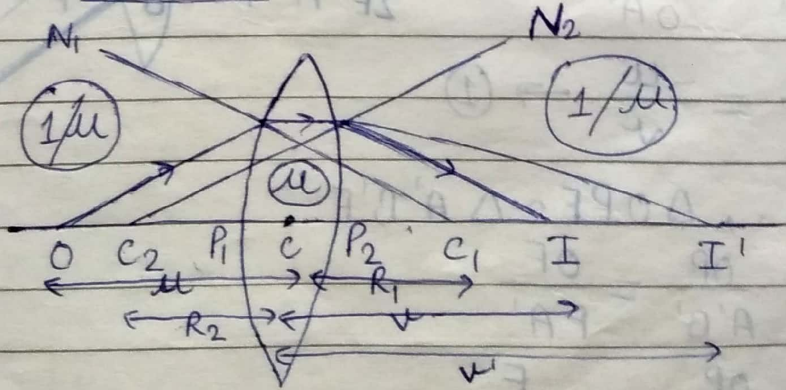
$$\frac{3}{2v} = \frac{1}{40} - \frac{1}{100} = \frac{5-2}{200} = \frac{3}{200}$$

$$v = \pm 100 \text{ cm.}$$

$\cancel{\frac{0.5}{200}} = \frac{1}{40}$

$\cancel{40}$

Lens Maker Formula :



For the refraction at surface XP_1Y we have,

$$\frac{\mu}{v'} - \frac{1}{u} = \frac{\mu - 1}{R_1} \longrightarrow (1)$$

For the refraction at surface XP_2Y

$$\frac{1/\mu}{v} - \frac{1}{v'} = \frac{\frac{1}{\mu}-1}{R_2}$$

$$\frac{1}{\mu v} - \frac{1}{v'} = \frac{1-\mu}{\mu R_2}$$

$$\frac{1}{v} - \frac{\cancel{\mu}}{\cancel{v'}} = \frac{1-\mu}{R_2} \rightarrow (2)$$

Adding (1) & (2), we have

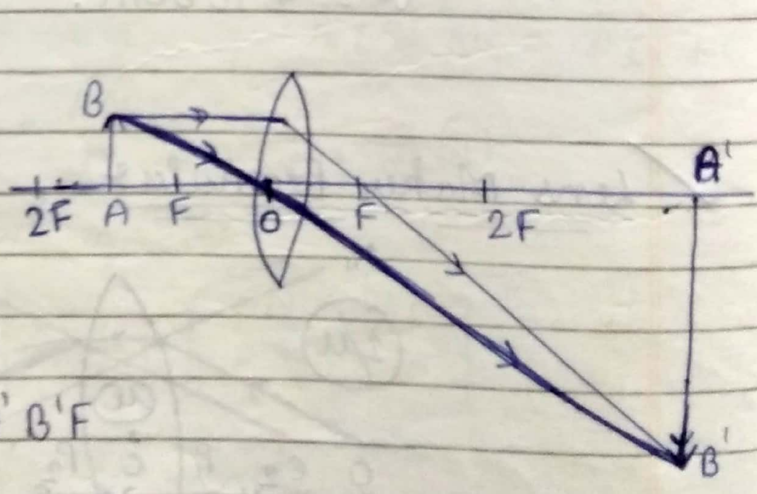
$$\frac{1}{v} - \frac{1}{u} = \frac{\mu-1}{R_1} - \frac{\mu-1}{R} = (\mu-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

2f $u = \infty$ then $v = f$.

$$\frac{1}{f} - \frac{1}{\infty} = (\mu-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\boxed{\frac{1}{f} = (\mu-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

Lens Formula :



$\Delta AOB \sim \Delta OA'B'$

$$\frac{AB}{A'B'} = \frac{OA}{OA'}$$

$$\frac{AB}{A'B'} = \frac{-u}{v} \rightarrow (1)$$

Again, $\Delta OPF \sim \Delta A'B'F$

$$\frac{OP}{A'B'} = \frac{OF}{FA'}$$

$$\frac{OP}{A'B'} = \frac{F}{OA'-OF}$$

$$\frac{OP}{A'B'} = \frac{F}{v-F} \rightarrow (2)$$

Power of lens, $P = \frac{1}{f}$

Equating ① & ②

$$\frac{-u}{v} = \frac{f}{v-f}$$

$$-uv + uf = vF$$
$$uv = uf - vF$$

Dividing by uv on both side

$$\boxed{\frac{1}{f} = \frac{1}{v} - \frac{1}{u}}$$

Magnification:-

The ratio of size of image to the size of ~~image~~ object

$$m = \frac{\text{Size of image}}{\text{Size of object}}$$

$\Delta OAB \sim \Delta OA'B'$

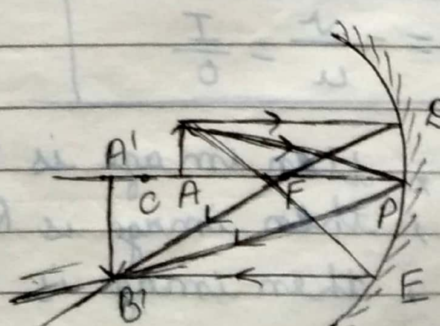
$$\frac{AB}{A'B'} = \frac{-u}{v} \rightarrow ①$$

$$\therefore \left[\frac{AB}{A'B'} = \frac{OA}{OA'}\right]$$

$$\frac{O}{I} = \frac{-u}{v}$$

$$\boxed{m = \frac{I}{O} = \frac{-v}{u}}$$ H.P.

Mirror formula:-



In $\Delta ABF \sim \Delta FPE$

$$\frac{AB}{PE} = \frac{AF}{FP} \text{ or,}$$

$$\frac{AB}{A'B'} = \frac{-u-(-f)}{-f}$$

$$\frac{AB}{A'B'} = \frac{-u-(-f)}{-f} = \frac{u-f}{f}.$$

Again $\Delta FPQ \sim \Delta FA'B'$

$$\frac{PQ}{A'B'} = \frac{PF}{A'F} \Rightarrow \frac{AB}{A'B'} = \frac{-f}{A'P-PF} = \frac{-f}{-v-(-f)}$$

$$\frac{AB}{A'B} = \frac{f}{v-f} \rightarrow (2)$$

Equating (1) & (2)

$$\frac{u-f}{f} = \frac{f}{v-f}$$

$$uv - uf - vf + f^2 = f^2$$

$$uv = uf + vf$$

Dividing of uvf

$$\frac{\cancel{uv}}{\cancel{uvf}} = \frac{\cancel{uf}}{\cancel{uvf}} + \frac{\cancel{vf}}{\cancel{uvf}}$$

$$\boxed{\frac{1}{f} = \frac{1}{v} + \frac{1}{u}}$$

<u>Magnification formula</u>:

$\Delta ABP \sim \Delta A'B'P$

$$\frac{AB}{A'B'} = \frac{AP}{A'P}$$

But $m = \frac{-I}{O}$

$$\therefore m = \frac{A'B'}{AB} = \frac{A'P}{AP} = \frac{-I}{O} = \frac{-v}{-u}$$

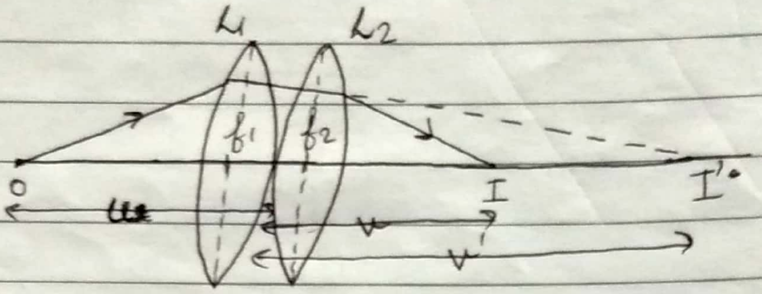
$$\therefore \boxed{m = -\frac{v}{u} = \frac{I}{O}}$$

If $m>1$, then image is VIRTUAL

If $m=1$, then image is REAL & of some size

If $m<1$, then image is REAL

Lens Combination :-



For refraction at L_1, we can write from lens formula

$$\frac{1}{v'} - \frac{1}{u} = \frac{1}{f_1} \longrightarrow (1)$$

For refraction at L_2

$$\frac{1}{v} - \frac{1}{v} = \frac{1}{f_2} \longrightarrow (2)$$

For an equivalent lens we can write

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \longrightarrow (3)$$

where f = focal length of equivalent lens

Adding (1) & (2)

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2} \longrightarrow (4)$$

Compairing (1) & (2)

$$\boxed{\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2}}$$ and for power $\boxed{P = P_1 + P_2}$

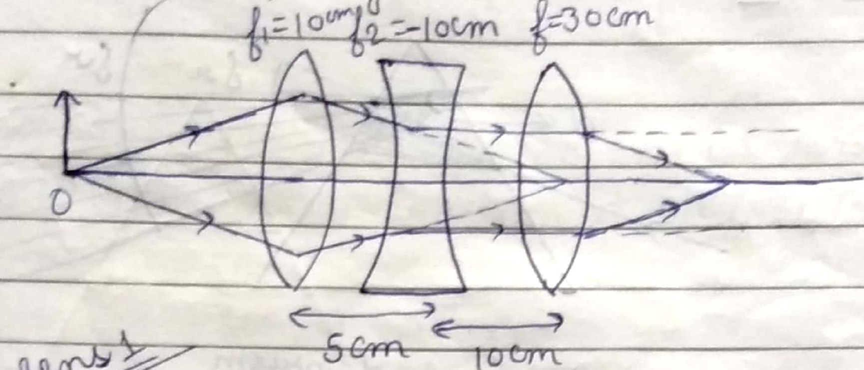
For n lens combination

$$\boxed{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \text{-----} + \frac{1}{f_n}}$$ & for power $\boxed{P = P_1 + P_2 ---}$

Note:- If the two lens are seperated by a distance x the formula becomes $\boxed{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{x}{f_1 f_2}}$

Q. Ex 81, (SL Arora) Page no. 9.61.

Ans.



For lens 1

$$\frac{1}{v_1} - \frac{1}{-30} = \frac{1}{10}$$

$$\frac{1}{v} = \frac{1}{10} - \frac{1}{30} = \frac{3-1}{30} = \frac{\cancel{2}}{\cancel{30}\,15}$$

$v_1 = 15cm$

$u_2 = 15 - 5 = 10cm$

For lens 2

$$\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f}$$

$$\frac{1}{v_2} - \frac{1}{10} = \frac{-1}{10}$$

$v_2 = \infty$

$u_3 = v_2$

For lens 3

$$\frac{1}{v_3} - \frac{1}{u_3} = \frac{1}{f_3}$$

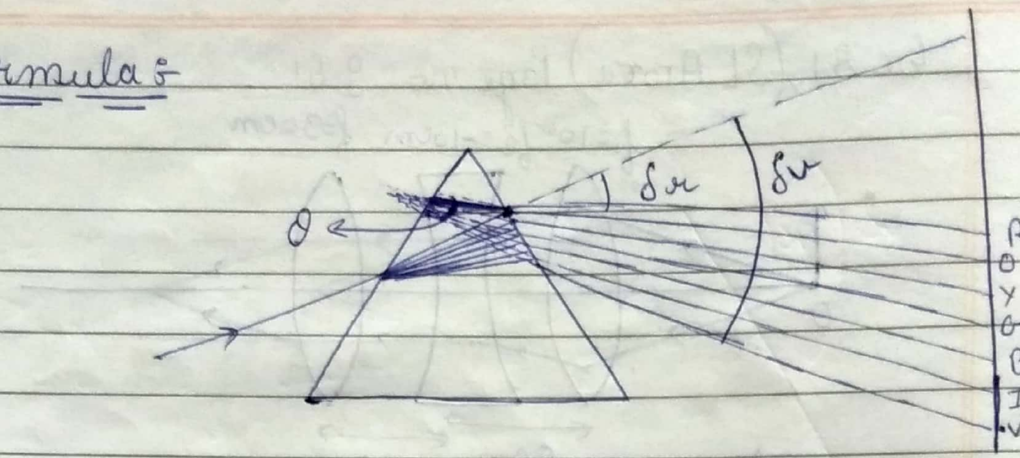
$$\frac{1}{v_3} - \frac{1}{\infty} = \frac{1}{30}$$

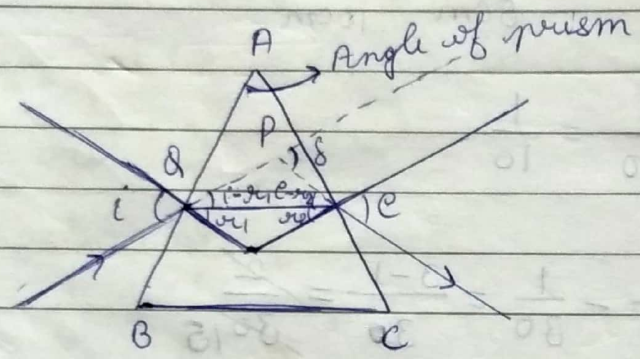
$v_3 = 30 cm$

The final image is formed 30cm right of third lens.

+Pn

Prism Formula :





In ΔPQS, $\delta = (i - r_1) + (e - r_2)$

$\delta = i + e - (r_1 + r_2) \rightarrow (1)$

In ΔQRS, $r_1 + r_2 + \angle QRS = 180^\circ \rightarrow (2)$

In $\Delta AQRS$, $\angle A + \angle QRS = 180^\circ \rightarrow (2)$

Comparing (2) & (3) we have

$\angle A = r_1 + r_2$

$\delta = i + e - \angle A$

$\delta + A = i + e.$

When $\delta = \delta_{min.}$, then $i = e$ & $r_1 = r_2 = r$ (say)

From (4) we have, $A = r + r$

$\therefore r = \frac{A}{2}$

From (5) we have, $\delta_m + A = i + i$

$\therefore i = \frac{A + \delta_m}{2}$

From snell's law

~~$\int \frac{\tan\theta}{1+\sin\theta}$~~

$$\boxed{\mu = \frac{\sin i}{\sin r} = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin A/2}}$$ This is known as prism formula.

Special Case:- For thin prism, As i & r will be very small then for refraction at surface AB we have.

$$\mu = \frac{\sin i}{\sin r_1} \approx \frac{i}{r_1}$$

$$i = \mu r_1 \rightarrow (6)$$

& for reflection at AC, we have,

$$\mu = \frac{\sin e}{\sin r_2} \approx \frac{i}{r_2}$$

$$e = \mu r_2 \rightarrow (7)$$

Putting for i & r in eq^n (5)

$$A + \delta = \mu r_1 + \mu r_2$$
$$= \mu(r_1 + r_2)$$

$$A + \delta = \mu A$$
$$\delta = \mu A - A$$
$$\boxed{\delta = A(\mu - 1)}$$

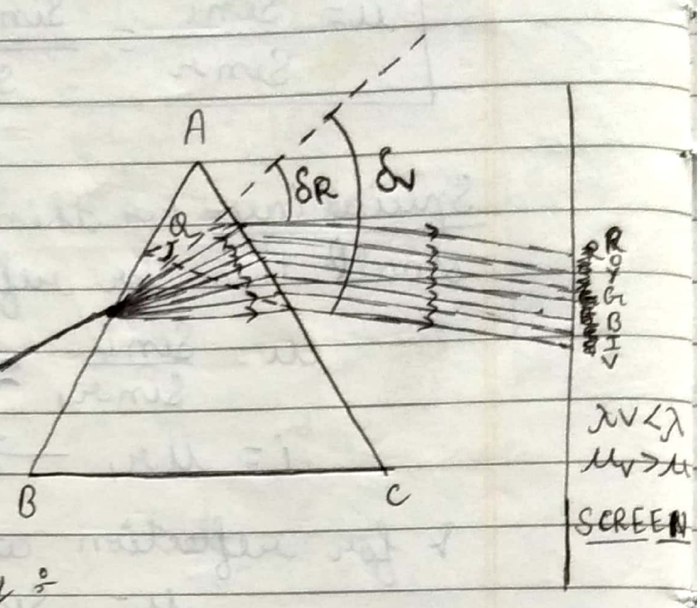
Rayliegh law of scattering:- According to it,

$$\text{Amount of scattering} \propto \frac{1}{\lambda^4}$$

Cauchy's formula :-

$$\boxed{\mu = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4}}$$

Angle of Dispersion (θ) :- and dispersive power



$$\delta = A(\mu - 1)$$

$\delta_V > \delta_R$, $\theta = \delta_V > \delta_R$.

Dispersion power $\omega = \dfrac{\theta}{\delta_Y} = \dfrac{\delta_V - \delta_R}{\delta_Y}$

$$= \frac{A(\mu_V - 1) - A(\mu_R - 1)}{A(\mu_Y - 1)} \Rightarrow \frac{\mu_Y - \mu_R}{\mu_Y - 1}$$

Rayleigh's law of scattering :-

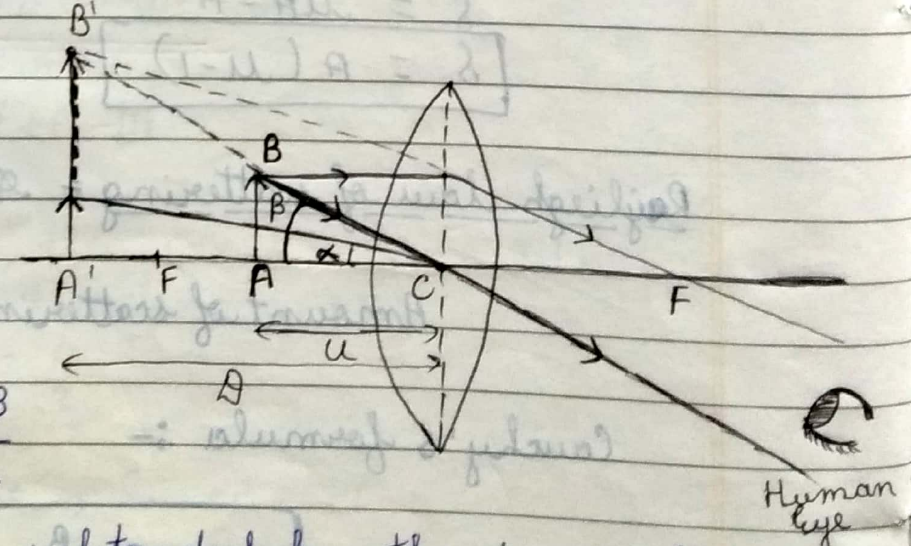
$$\text{Amount of scattering} \propto \frac{1}{\lambda^4}$$

Cauchy's formula :-

$$\mu = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + \ldots$$

Optical Instrument :-

Simple Microscope :-



$$M = \frac{\beta}{\alpha} \approx \frac{\tan\beta}{\tan\alpha}$$

Where, β = Angle subtended by the image formed formed at lddv over the eye.

α = Angle subtended by the object (assumed to be kept at lddv) over the eye.

$$M = \frac{AB/AC}{A'P/A'C} = \frac{A'C}{AC} \times \frac{\cancel{AB}}{\cancel{AP}} = \frac{A'C}{AC} = \frac{\cancel{+}D}{\cancel{+}u} = \frac{D}{u}$$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$1 - \frac{v}{u} = \frac{v}{f}$$

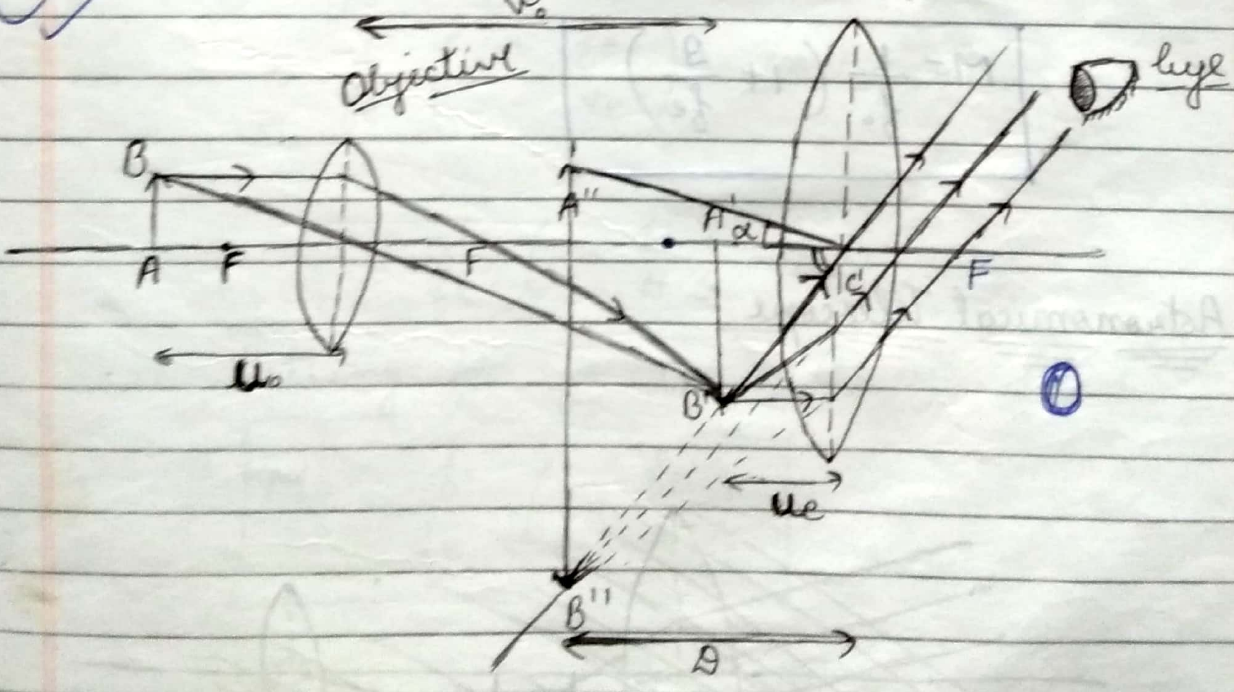
$$\frac{v}{u} = 1 - \frac{v}{f} = 1 - \frac{(-D)}{f} = 1 + \frac{D}{f}$$

$$\frac{A'C}{AC} = \frac{v}{u} = 1 + \frac{D}{f}$$

Special Case :-
If final img. is formed at infinity then

$$\boxed{M = \frac{D}{f}}$$

Compound Microscope :-



①

$$M = \frac{\beta}{\alpha} \quad \frac{\tan\beta}{\tan\alpha} = \frac{A'B''/A''C'}{A''P/A''C'}$$

$$= \frac{A''B''}{A''C'} \times \frac{A''C'}{A''P} = \frac{A''B}{AB} = \frac{A''B''}{A'B'} \times \frac{A'B'}{AB}$$

$$M = m_e \times m_o \longrightarrow (1)$$

find Me From lens formula $\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$ for eyepiece.

OR,

$$1 - \frac{v_e}{u} = \frac{v_e}{f_e} = \frac{v_e}{u_e} = 1 - \frac{v_e}{f_e} = 1 - \frac{(-D)}{f_e}$$

$$= 1 + \frac{D}{f_e}$$

$$\therefore m_e = 1 + \frac{D}{f_e} \longrightarrow (2)$$

Also, $m_o = \frac{v_o}{u_o} \longrightarrow (3)$

From (1), (2) & (3)

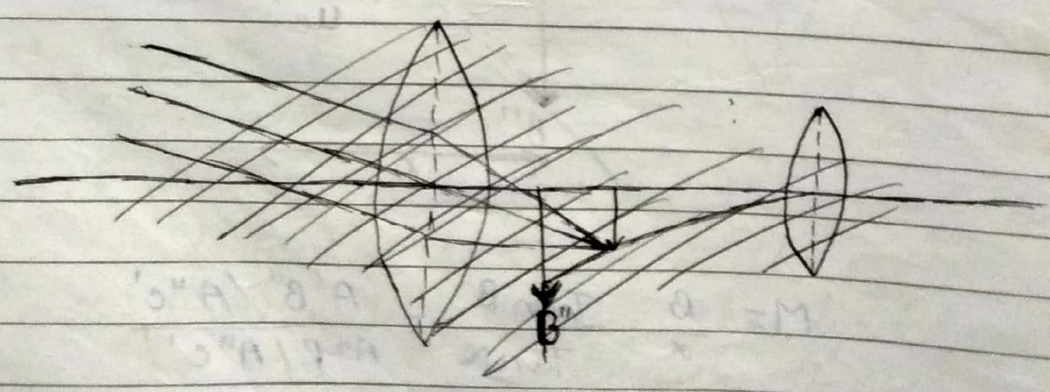
$$M = \frac{v_o}{u_o}\left(1 + \frac{D}{f_e}\right)$$

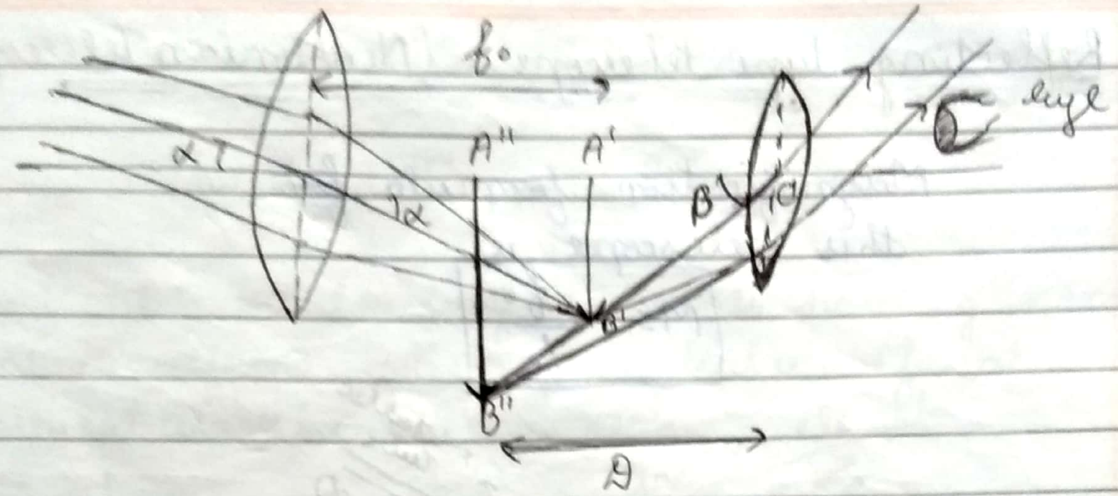
Now $v_o \approx L$ as f_e is small

$u_o \approx f_o$ as f_o is small

$$\boxed{M = \frac{1}{f_o}\left(1 + \frac{D}{f_e}\right)}$$

Astronomical Telescope :-





$$M = \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha} = \frac{A'B' / A'C'}{A'B' / A'C} = \frac{A'C}{A'C'}$$

$$= \frac{f_o}{u_e} \rightarrow (1)$$

$$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$$

$$\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e}$$

$$= \frac{1}{f_e}\left(\frac{f_e}{v_e} - 1\right)$$

$$= \frac{-1}{f_e}\left(1 - \frac{f_e}{v_e}\right) \qquad v_e = -D$$

$$\frac{1}{u_e} = \frac{-1}{f_e}\left(1 + \frac{f_o}{D}\right) \rightarrow (2)$$

Putting for $\frac{1}{v_e}$ in eq. (1) we have

$$\boxed{M = -\frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right)}$$

M =

SBG STUDY

Reflecting type telescope :- (Newtonian Telescope) :-

Magnification formula for this telescope is

$$\boxed{M = -\frac{f_o}{f_e}}$$