Chapter- 12, 13

Nuclei

Nucleus :

1 Fermi $= 10^{-15}$ m

1 amu $= \frac{1}{12}$ of mass C^{12} atom

$$= \frac{1}{12} \times 1.992678 \times 10^{-26} \text{ Kg}$$

$$= 1.660565 \times 10^{-27} \text{ Kg}$$

$$E = mc^2$$

$$= 1.660565 \times 10^{-27} \times (3 \times 10^8)^2 \text{ J}$$

$$\boxed{E = 93.5 \text{ MeV}}$$

$$\boxed{1 \text{ amu} \equiv 931.5 \text{ MeV}}$$

Nuclear Size :

$$4\pi R^3 \propto A$$

$$R \propto A^{1/3}$$

$$R = R_0 A^{1/3}$$

where $R_0 = 1.2 \times 10^{-15}$ m

$= 1.5$ Fermi

Nuclear Density :

$$\text{Nuclear Density} = \frac{\text{Mass of nucleous}}{\text{Volume of nucleus}} = \frac{m \times A}{\frac{4}{3}\pi R^3}$$

$$= \frac{mA}{\frac{4}{3}\pi (R_0 A^{1/3})^3}$$

$$= \frac{3m\cancel{A}}{4\pi R_0^3 \cancel{A}} = \frac{3m}{4\pi R_0^3}$$

$$= \frac{3 \times 1.67 \times 10^{-27}}{4 \times 3.14 \times (1.2 \times 10^{-15})^3}$$

$$= 2.3 \times 10^{17} \ Kg \ m^{-3}$$

Nuclear force :-

Mass Defect :- (Δm)

$$\Delta m = [Z m_p + (A-Z) m_n] - m({}_Z X^A)$$

Packing fraction :

$$P.F \text{ of a nucleus} = \frac{\text{Mass defect}}{\text{Mass number}} = \frac{\Delta m}{A}$$

Binding Energy :-

$$B.E = \Delta m . c^2 \text{ Joules}$$

or

$$B.E = \Delta m \times 931.5 \text{ MeV}$$

It may be defined as the energy required to extract one nucleons into its constituent protons and neutrons and to seperate them to such a large distance that they may not interact with each other.

$$\Delta E_b = [Zm_H + (A-Z)\,m_n - m]\,c^2.$$

Binding energy per nucleon:-

$$\Delta E_{bn} \text{ or } \bar{B} = \frac{B.E.}{A} = \frac{[Zm_H + (A-Z)m_n - m]\,c^2}{A}$$

$\therefore A = 30 - 170.$

Expression for binding energy.

$$\Delta m = Zm_p + (A-Z)\,m_n - m_N \rightarrow (1)$$

$$\Delta E_b = \Delta m \times c^2 = [Zm_p + (A-Z)\,m_n - m_N]\,c^2 \rightarrow (2)$$

$$(\Delta E_b)_e = [X(m_N + Zm_e) - m(^A_Z X)]\,c^2 \rightarrow (3)$$

$$(\Delta E_b)_e = 0$$

$$\therefore m_N + Zm_e - m(^A_Z X) = 0$$

$$m_N = m(^A_Z X) - Zm_e$$

$$\Delta E_b = [Zm_p + (A-Z)\,m_n - m(^A_Z X) + Zm_e]\,c^2$$

$$\Delta E_b = [Z(m_p + m_e) + (A-Z)\,m_n - m(^A_Z X)]\,c^2 \rightarrow (4)$$

But $m_p + m_e = m_H \equiv$ mass of hydrogen atom

Eq^n (4) can be written in terms of m_H as,

$$\boxed{\Delta E_b = [Zm_H + (A-Z)\,m_n - m(^A_Z X)]\,c^2} \rightarrow (5).$$

Q11. Calculate the binding energy per nucleon of ${}^{40}_{20}Ca$ nucleus. Given,

$$m({}^{40}_{20}Ca) = 39.962589 \text{ amu}$$
$$m_n = 1.008665 \text{ amu}$$
$$m_p = 1.007825 \text{ amu}$$

There are 20 protons
There are 20 neutrons

Mass of 20 protons $= 20 \times 1.008665$ amu $= 20.1565$ amu
Mass of 20 neutrons $= 20 \times \cancel{39}\, 1.007825$ amu
$= \cancel{40.3298 \text{ amu}}$
$= 20.1733$ amu

Total mass $= 20.1565 + 20.1733$
$= 40.3298$ amu.

$$\text{B.E for nucleon} = \frac{[Z m_p + (A-Z) m_n - m({}_Z X^A)] \times 931.5}{A}$$

$$\Delta m = 20 \times 1.007825 + (40-20) \times 1.008665 - 39.962589$$

$$\text{B.E} = 0.367211 \times 931 = 341.87 \text{ MeV}$$

$$\text{B.E per nucleon} = \frac{341.87}{40} = 8.547 \text{ MeV}$$

Radioactive decay Laws:

at $t = 0$ $\quad N_0$, $\quad t = 0$, $\quad N_0$
at $t = t$ $\quad N$, $\quad t = t$ $\quad N$

$$\frac{dN}{dt} \propto -N \quad , \quad \frac{dN}{dt} \propto -N$$

$$\frac{dN}{dt} = -\lambda N$$

$$\frac{1}{N}\frac{dN}{dt} = -\lambda dt$$

On integrating $\log N = -\lambda t + C$.

As at $t = 0$, $N = N_0$

$\therefore \log N_0 = 0 + C$

$\log N = -\lambda t + \log N_0$

$\log N - \log N_0 = -\lambda t$

$\log \left[\frac{N}{N_0}\right] = -\lambda t$

$\log \frac{N}{N_0} = -\lambda t$

$\boxed{N = N_0 e^{-\lambda t}}$

$N = N_0 e^{-t}$ where $\lambda = 1/t$ or $t = 1/\lambda$

$N = \frac{N_0}{2.718}$

$\frac{N}{N_0} = e^{-\lambda t}$

$N = 0.368$

$N = 36.8\% N_0$

$N = N_0 e^{-\lambda t}$

~~Disintegration constant is reciprocal of that time in which total no. of decayed atoms becomes 36.8% of initial atoms in the radioactive sample~~

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Half-Life:- $[T_{1/2}$ or $T]$

As per as defination

$N = \frac{N_0}{2}$

From ① eq^n we have,

$\frac{\cancel{N_0}}{2} = \cancel{N_0} e^{-\lambda t}$

$\frac{1}{2} = e^{-\lambda t}$

$2 = e^{\lambda T}$

$\frac{1}{2} = \frac{1}{e^{\lambda t}}$

$$\log_e 2 = \log_e e^{\lambda T} = \lambda T \log_e e$$

$$2.303 \log_{10} 2 = \lambda T \times 1$$
$$2.303 \times 0.3010 = \lambda T$$
$$0.693 = \lambda T$$
$$\boxed{T = \frac{0.693}{\lambda}}$$

$$\log_e 2 = \lambda t$$
$$2.303 \log_{10} 2 = \lambda t$$
$$2.303 \times 0.3010 = \lambda t$$
$$\lambda t = 0.693$$
$$t = \frac{0.693}{\lambda}$$

Mean Life or Average Life:

$$\tau_{av} \text{ or } \tau = \frac{1}{\lambda}$$
$$= \frac{1}{0.693/T}$$
$$= \frac{T}{0.693}$$
$$\boxed{\tau = 1.44 T}$$

Activity of a radioactive substance

$$R = \frac{dN}{dt}$$
$$= -\left[N_0 e^{-\lambda t} \times (-\lambda)\right]$$
$$= N_0 \lambda e^{-\lambda}$$
$$\boxed{R = \lambda N}$$