Ch-2

Electrostatic Potential And Capicitance

Electric Potential :-

Electric Potential at a point is defined as the amount of work done per unit charge in bringing a unit (+)ve test charge from infinity to that point.

Mathematically, $V = \frac{W}{q_0}$ $\begin{cases} \text{It is a scalar quantity} \\ \text{SI unit is Volt.} \end{cases}$

Potential at a point due to charge :-

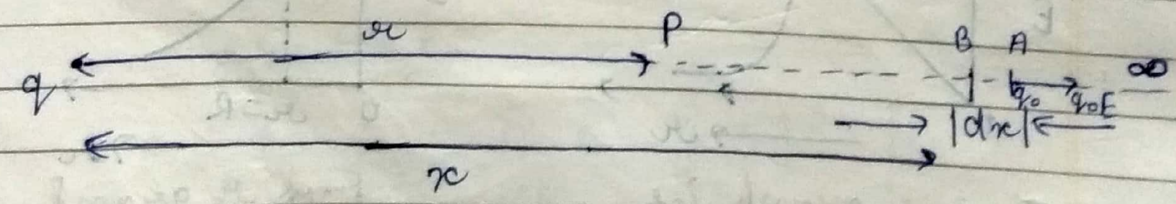
$$dW = -F\,dx$$
$$= q_0 E\,dx$$
$$= -q_0 E\,dx$$
$$= -q_0 \frac{1}{4\pi\epsilon_0}\frac{q}{x^2}\,dx$$

$$\therefore W_{\infty P} = \int dW = -\int_{\infty}^{r} \frac{1}{4\pi\epsilon_0}\frac{q q_0}{x^2}\,dx$$

$$W_{\infty P} = \frac{q q_0}{4\pi\epsilon_0}\int_{\infty}^{r}\frac{dx}{x^2}$$

$$W_{\infty P} = -\frac{q q_0}{4\pi\epsilon_0}\left[-\frac{1}{x}\right]_{\infty}^{r} = \frac{q q_0}{4\pi\epsilon_0}\left[\frac{1}{r} - \frac{1}{\infty}\right] = \frac{1}{4\pi\epsilon_0}\frac{q q_0}{r}$$

$$\therefore V_P = \frac{W_{\infty P}}{q_0} = \frac{\frac{1}{4\pi\epsilon_0}\frac{q \cancel{q_0}}{r}}{\cancel{-q_0}} = \boxed{\frac{1}{4\pi\epsilon_0}\frac{q}{r}}$$



Electric field as a gradient of Potential :-

- $dW = -q_0 E\, dx$

$\frac{dW}{q_0} = -E \cdot dx$

$\therefore dV = -E dx$

$\therefore E = -\frac{dx}{dx}$

$\therefore$ In General $\boxed{E = \frac{-dv}{dr}}$

$\frac{dv}{dr}$ is called as Potential gradient

$\cancel{q_0 E dx}$ $\quad \frac{dv}{dr} = 0$, $dv = 0$; $dv = \frac{W}{q_0}$

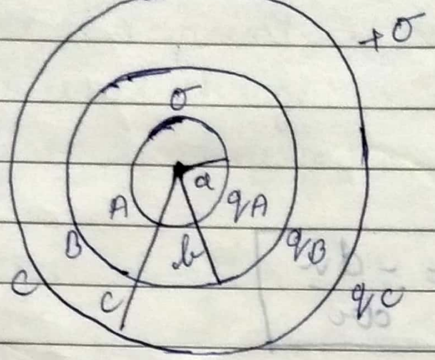
$\cancel{= -q_0 \frac{1}{4\pi\epsilon_0} \frac{q}{x^2} dx}$

$\cancel{\therefore W_{\infty P} = \int dW = -\int_\infty^r \frac{1}{4\pi\epsilon_0} \frac{q q_0}{x^2} dx}$

Show diagramatically that electric field at a point is non-zero & potential is zero.

Q. Find the potential of 3 shells a, b, c.
If the shell a & c are at the same potential the obtain the relation b/w the radii a, b, c.

Ans.



$$V_C = \frac{1}{4\pi\epsilon_0}\frac{q_c}{c} - \frac{1}{4\pi\epsilon_0}\frac{q_B}{c} + \frac{1}{4\pi\epsilon_0}\frac{q_A}{c} = \frac{1}{4\pi\epsilon_0}\left[\frac{q_c}{c} - \right.$$

$$V_B = \frac{1}{4\pi\epsilon_0}\frac{q_c}{B} - \frac{1}{4\pi\epsilon_0}\frac{q_B}{B} + \frac{1}{4\pi\epsilon_0}\frac{q_A}{B} = \frac{1}{4\pi\epsilon_0}\left[\frac{q_c}{B} - \frac{q_B}{B} + \right.$$

$$V_A = \frac{1}{4\pi\epsilon_0}\frac{q_c}{A} - \frac{1}{4\pi\epsilon_0}\frac{q_B}{A} + \frac{1}{4\pi\epsilon_0}\frac{q_A}{A} = \frac{1}{4\pi\epsilon_0}\left[\frac{q_c}{A} - \frac{q_B}{A} + \right.$$

$$q_c = \sigma 4\pi \epsilon_0^2$$

(ii) $V_A = V_c$

$\cancel{\frac{q_0 - q_B + q_A}{c}}$

$$\frac{1}{4\pi\epsilon_0}\left[\frac{q_A}{a} - \frac{q_B}{b} + \frac{q_c}{c}\right] = \frac{1}{4\pi\epsilon_0}\left[\frac{q_0}{c} - \frac{q_B}{c} + \frac{q_A}{c}\right]$$

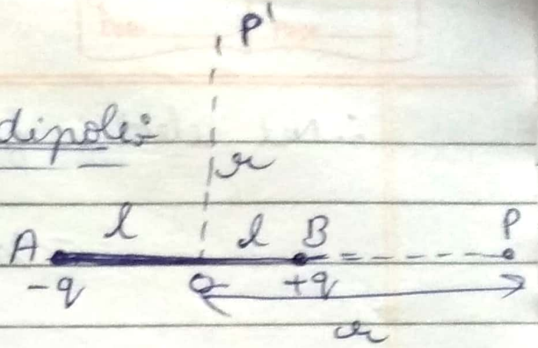
$$\frac{4\pi a^2\sigma}{a} - \frac{4\pi b^2\sigma}{b} + \frac{4\pi c^2\sigma}{c} = \frac{4\pi c^2\sigma}{c} - \frac{4\pi b^2\sigma}{c} + \frac{4\pi a^2\sigma}{c}$$

$$a - b + c = c - \frac{b^2}{c} + \frac{a^2}{c}$$

$$\cancel{(a-b)} = \frac{(a+b)\cancel{(a-b)}}{c}$$

$$\boxed{c = a+b}$$

Potential at axial point due to dipole:-



$$V_{PA} = \frac{1}{4\pi\epsilon_0} \frac{-q}{(r+l)}$$

$$V_{PB} = \frac{1}{4\pi\epsilon_0} \frac{q}{(r-l)}$$

$$V_P = V_{PA} + V_{PB}$$

$$V_P = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r-l} - \frac{1}{r+l}\right]$$

$\frac{q_B}{c} + \frac{q_A}{c}]$

$$V_P = \frac{q}{4\pi\epsilon_0}\left[\frac{r+l-r+l}{r^2-l^2}\right]$$

$+\frac{q_A}{B}]$

$$V_P = \frac{q\,2l}{4\pi\epsilon_0 (r^2-l^2)}$$

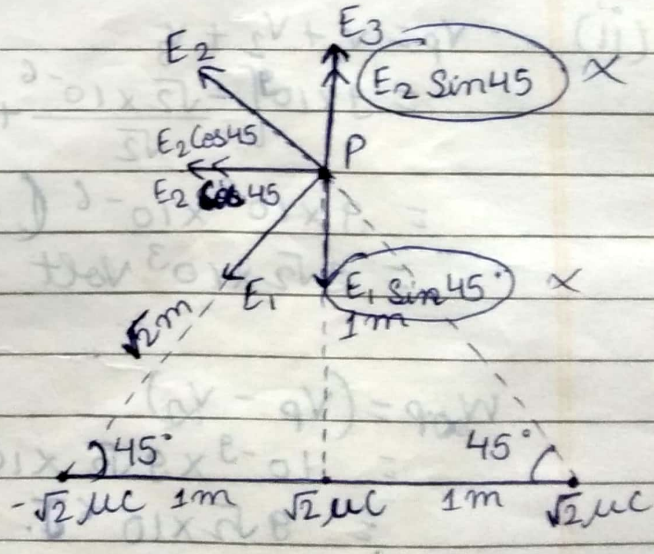
$\frac{q_A}{A}]$

$$\boxed{V_P = \frac{1}{4\pi\epsilon_0} \frac{P}{r^2-l^2}}$$

Special Case If $r >> l$ then

$$\boxed{V = \frac{1}{4\pi\epsilon_0} \frac{P}{r^2}}$$

Q. In the given fig. find
(i) Electric field (E) & Potential (V) at P.



(ii) Findout the work done in bringing a test charg of 1 nano coulomb from ∞ to point P.

Ans.
$$E' = E_1 \cos 45 + E_2 \cos 45$$
$$= 2E_1 \cos 45$$
$$= 2 \times 9 \times 10^9 \times \frac{\sqrt{2} \times 10^{-6}}{(\sqrt{2})^2} \times \frac{1}{\cancel{\sqrt{2}}}$$

$$E' = 9 \times 10^3 \ NC^{-1}$$

(ii) $W_{\infty P} = q_0 (V_P - V_\infty)$

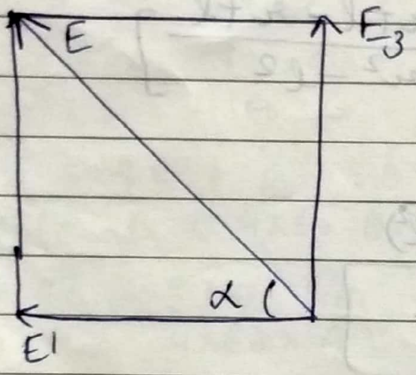
$\therefore$ Net electric field $= \sqrt{E'^2 + E_3^2}$

~~$= 9\times10^3$~~

$= \sqrt{(9\times10^3)^2 + (9\sqrt{2}\times10^3)^2}$

$= 9\times10^3\sqrt{1+2} = \boxed{9\sqrt{3}\times10^3\ NC^{-1}}$

$\because \left[\dfrac{9\times10^9 \times \sqrt{2}\times10^{-6}}{(1)^2} = 9\sqrt{2}\times10^3\ NC^{-1} \right]$



$\tan\alpha = \dfrac{E_3}{E'} = \dfrac{9\sqrt{2}\times10^3}{9\times10^3}$

$= \sqrt{2}$

$\boxed{\alpha = \tan^{-1}(\sqrt{2})}$

(ii) $V_P = V_1 + V_2 + V_3$

$= 9\times10^9\left[\dfrac{-\sqrt{2}\times10^{-6}}{\sqrt{2}} + \dfrac{\sqrt{2}\times10^{-6}}{1} + \dfrac{\sqrt{2}\times10^{-6}}{\sqrt{2}} \right]$

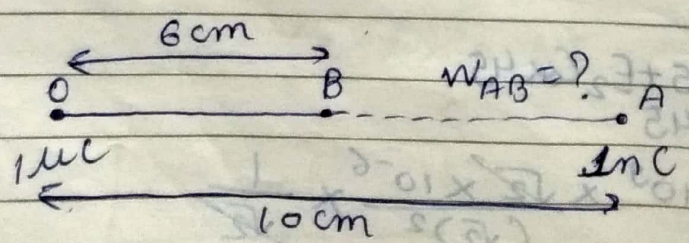
$= 9\times10^9\times10^{-6}(\sqrt{2})$

$= 9\sqrt{2}\times10^3$ Volt

$W_{\infty P} = (V_P - V_\infty)$

$= 10^{-9}\times9\sqrt{2}\times10^3$

$= 9\sqrt{2}\times10^{-6}$ J.



Q. Find the amount of work done in rotating an electric dipole, of dipole moment 3.2×10^{-8} Cm, from its position of stable equilibrium, to the position of unstable equilibrium, in a uniform electric field of intensity 10^4 N/C

Ans.

CAPACITANCE :

Capacity of a conductor to store charge is known as capacitance of conductor.

$$q \propto V$$

$$q = CV$$

$$C = \frac{q}{V}$$

It is a scalar quantity.

It's SI unit is FARAD

Capacity of a spherical Conductor

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

$$\frac{q}{V} = 4\pi\epsilon_0 r$$

$$\boxed{C = 4\pi\epsilon_0 r}$$

For ex: In case of an earth

$R = 6400 Km$

$C = 4\pi\epsilon_0 R$

$C = \cancel{\frac{1}{4\pi\epsilon_0}} 1/9\times10^{-9} \times 6400\times1000$

$= \frac{64}{9} \times 10^{-4}$

$= 7.1 \times 10^{-4}$

$= 710 \times 10^{-6}$ F

$= 710 \mu F$

Find radius of that sphere whose cap. is 1F.

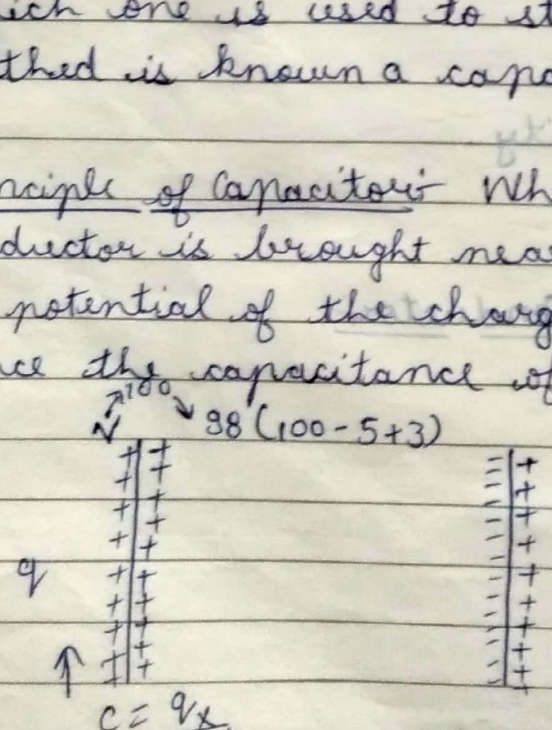
$R = \frac{C}{4\pi\epsilon_0}$

$= 9\times10^{9}$ m

Principle of ~~Capacitance~~ Capacitor:

Capacitor: An arrangement of two conductors out of which one is used to store charge & other remains earthed is known a capacitor

Principle of Capacitor: When a identical uncharged conductor is brought near the charged conductor then the potential of the charged conductor decreases & hence the capacitance of charged conductor increases.



(Fig1(a))

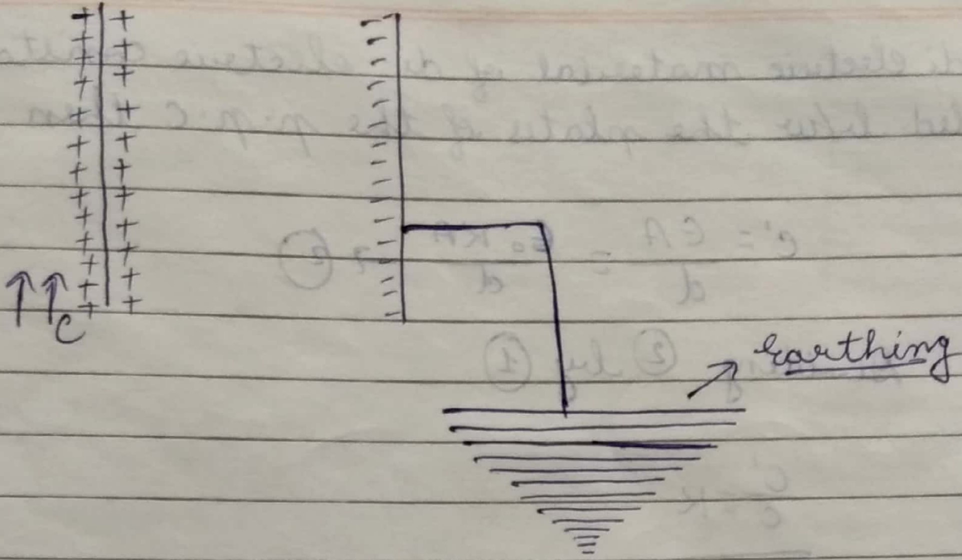
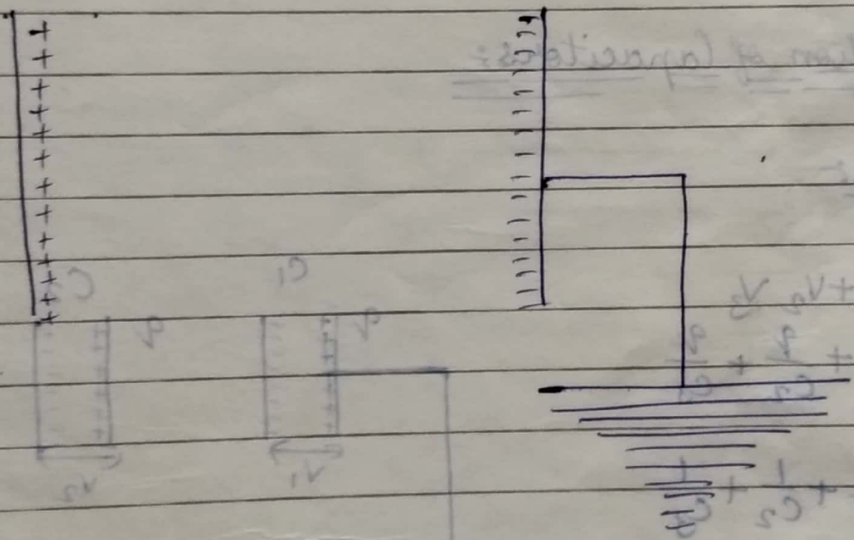
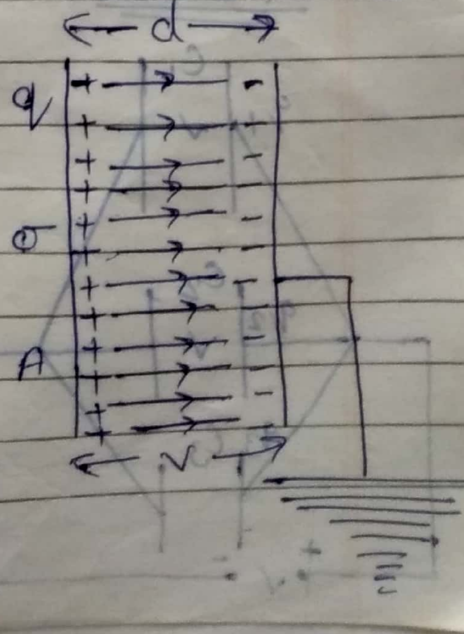


Fig (b) (i)



Parallel Plate Capacitor :-



We know that, $E = \frac{V}{d}$

OR, $\frac{\sigma}{\epsilon_0} = \frac{V}{d}$

OR, $\frac{q/A}{\epsilon_0} = \frac{V}{d}$

$\therefore \frac{q}{V} = \frac{\epsilon_0 A}{d}$

$$\boxed{C = \frac{\epsilon_0 A}{d}}$$

If di electric material of di-electric constant K is filled b/w the plates of the p.p.c then

$$C' = \frac{\epsilon A}{d} = \frac{\epsilon_0 K A}{d} \rightarrow (2)$$

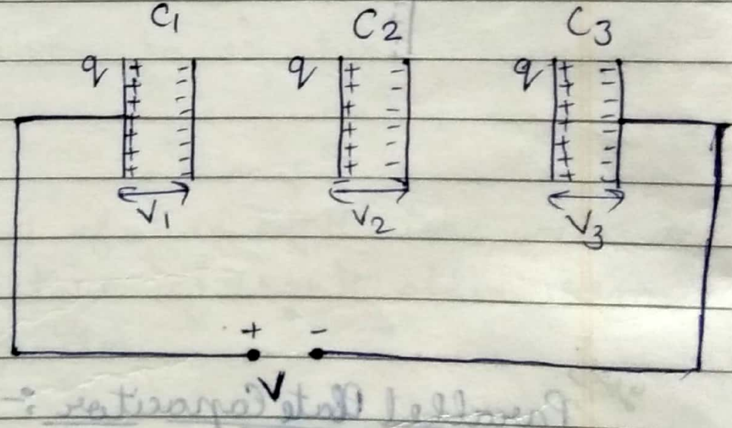
Dividing (2) by (1)

$$\frac{C'}{C} = K$$

$$\boxed{K = \frac{Cm}{Cv}}$$

Combination of Capacitors:

(i) In Series:

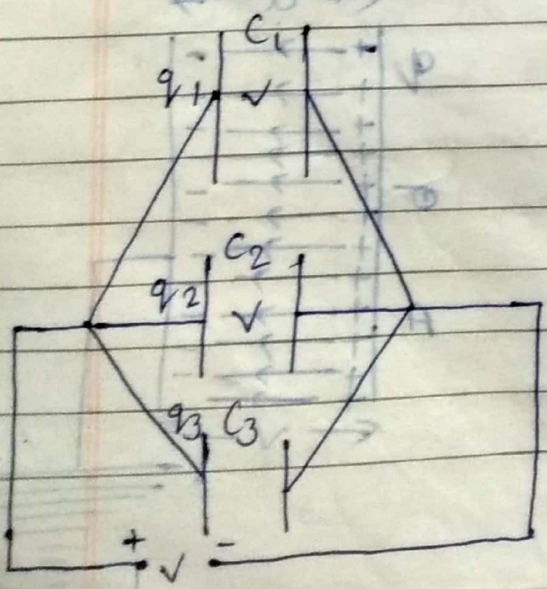


$$V = V_1 + V_2 + V_3$$

$$\frac{q}{C} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

(ii) In Parallel:

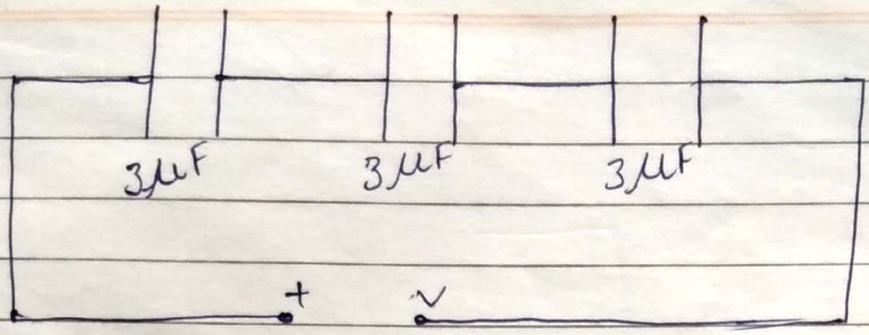


$$q = q_1 + q_2 + q_3$$

$$CV = C_1V + C_2V + C_3V$$

$$\boxed{C = C_1 + C_2 + C_3}$$

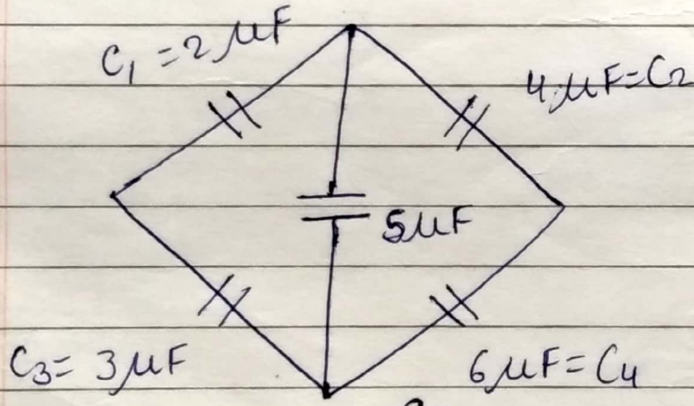
Q.

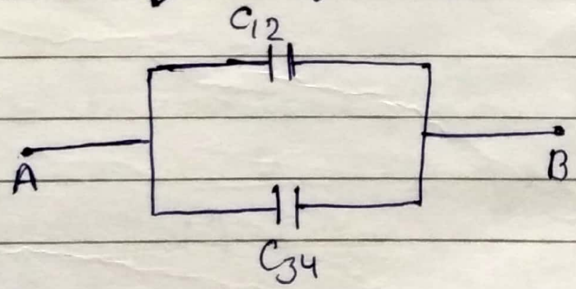


In Series

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$= \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \cancel{\frac{3}{3}} = 1\mu F$$





$$C_{12} = \frac{2 \times 4}{2+4} = \frac{8}{6}$$

$$C_{34} = \frac{3 \times 6}{3+6} = \frac{18}{9}$$

$$C_{eq} = C_{12} + C_{34}$$

$$= \frac{\cancel{8}^{4}}{\cancel{6}_{3}} + \frac{\cancel{18}^{2}}{\cancel{9}}$$

$$= \frac{4}{3} + 2$$

$$= \frac{4+6}{3} = \frac{10}{3}.$$

Imp

Energy Stored in Capacitor :-

$dW = V\,dq$

$W = \int_0^q \frac{q}{C}\,dq = \frac{q^2}{2C}$

$\therefore \boxed{U = \frac{1}{2}\frac{q^2}{C}} \rightarrow ①$

$= \frac{1}{2}\frac{C^2V^2}{C} \quad \therefore q = CV$

$= \boxed{\frac{1}{2}CV^2} \rightarrow ②$

From ①

$U = \frac{1}{2}\frac{q^2}{q/V}$

$\boxed{U = \frac{1}{2}qV} \rightarrow ③$

$dW = V\,dq$

$W = \int_0^q \frac{q}{C} = \frac{q^2}{2C}$

$\boxed{U = \frac{1}{2}\frac{q^2}{C}} \rightarrow ①$

$U = \frac{1}{2}\frac{C^{\cancel{2}}V^2}{\cancel{C}} \Rightarrow U = \frac{1}{2}\frac{CV^2}{1}$

$\boxed{U = \frac{1}{2}CV^2} \rightarrow ②$

From ①

$U = \frac{1}{2}\frac{q^2}{C} \Rightarrow U = \frac{1}{2}\frac{q^{\cancel{2}}}{\cancel{q}/V}$

$\boxed{U = \frac{1}{2}qV} \rightarrow ③$

$u = \frac{U}{\text{V of Cap}} \Rightarrow \frac{\frac{1}{2}CV^2}{A\times d}$

$= \frac{\frac{1}{2}\frac{\epsilon_0 A}{d}\times(Ed)^2}{Ad}$

$u = \boxed{\frac{1}{2}E^2\epsilon_0}$

$\therefore$ Energy stored in per unit volume of the capacities or energy density $u = \frac{U}{\text{V. of cap.}}$

$= \frac{\frac{1}{2}CV^2}{A\times d}$

$= \frac{\frac{1}{2}\times\frac{\epsilon_0 A}{d}\times(Ed)^2}{Ad}$

$\boxed{u = \frac{1}{2}\epsilon_0 E^2}$

Re-distribution of charges :

After the redistribution of charges, suppose the common potential of both sphere become V.

Before redistribution $\rightarrow$ $q_1 = C_1V_1$ & $q_2 = C_2V_2$

$$U_1 = \frac{1}{2}C_1V_1^2 \;\; \& \;\; U_2 = \frac{1}{2}C_2V_2^2$$

$\therefore$ Total energy stored in the system $U = U_1 + U_2 = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 \rightarrow (1)$

Common potential after complete redistribution

$$= \frac{\text{Total Charge}}{\text{Total Capacitance}}$$

$$V = \frac{q_1 + q_2}{C_1 + C_2}$$

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} \rightarrow (2)$$

$\therefore$ ~~Loss of energy~~ $\cancel{= U - U'}$

$$\cancel{= \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 - \left[\frac{C_1V_1 + C_2V_2}{C_1+C_2}\right]}$$

$$\cancel{= \frac{1}{2(C_1+C_2)}\left[(C_1V_1^2 + C_2V_2^2)(C_1V_2)\right] -}$$

$\therefore$ After redistribution of charges, the total energy stored in the system

$$U' = \frac{1}{2}(\text{Eq. Cap.})(\text{common potential})^2$$

$$= \frac{1}{2}(C_1+C_2)\left[\frac{C_1V_1 + C_2V_2}{C_1+C_2}\right]^2$$

$\therefore$ Loss of energy $\Delta U = U - U' = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 - \frac{1}{2}(C_1+C_2)\left[\frac{C_1V_1 + C_2V_2}{C_1+C_2}\right]^2$

$$= \frac{1}{2(C_1+C_2)}\left[(C_1+C_2)(C_1V_1^2 + C_2V_2^2) - (C_1V_1 + C_2V_2)^2\right]$$

$$= \frac{1}{2(C_1+C_2)}\left[C_1^2+V_1^2+C_1C_2V_2^2+C_1C_2V_1^2+C_2^2V_2^2-C_1^2V_1^2\right.$$
$$\left.-C_1^2V_1^2-C_2V_2^2-2C_1C_2V_1V_2\right]$$

$$= \frac{C_1C_2}{2(C_1+C_2)}\left[V_2^2+V_1^2-2V_1V_2\right]$$

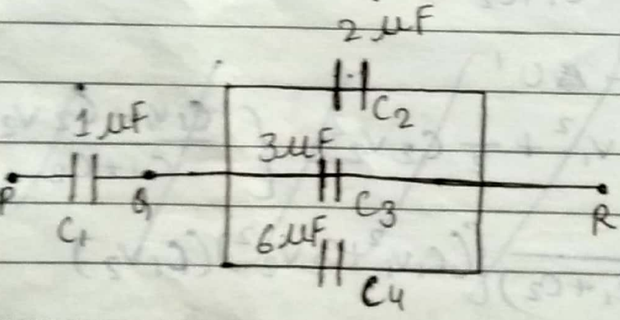
$$\Delta U = \frac{C_1C_2}{2(C_1+C_2)}(V_1-V_2)^2 > 0$$

$\therefore\ U - U' > 0$

$\therefore\ U > U'$

$\therefore$ Loss of energy takes place during the redistribution of charges.

Q. In the fig. energy stored in C_4 is 27J. Calculate the total energy stored in the system.



$$U_4 = \frac{1}{2}C_4V^2 = 27J$$

$$\frac{1}{2}\times 6\times 10^{-6}\times V^2 = 27$$

Energy stored in C_2,

$$U_2 = \frac{1}{2}\times 2\times 10^{-6}\times 9\times 10^{6} = 9J$$

Energy stored in C_3,

$$U_3 = \frac{1}{2}\times 3\times 10^{-6}\times 9\times 10^{6} = 13.5\,J,$$

$\therefore$

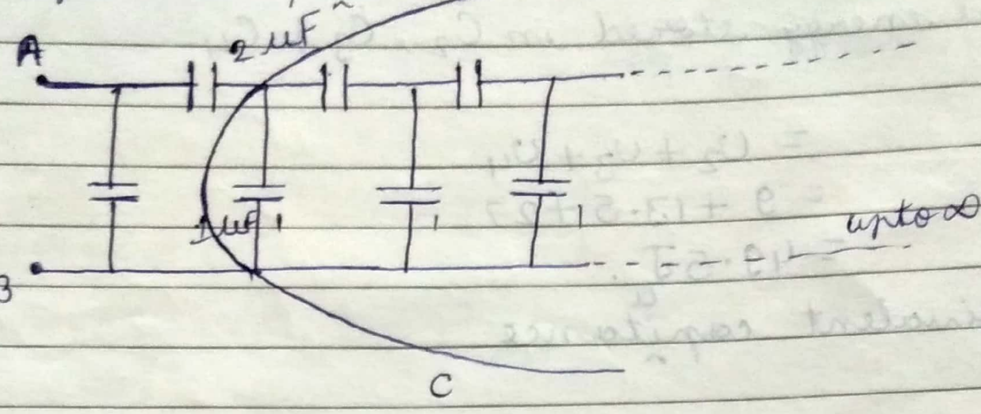
Total energy stored in C_2, C_3 & C_4

$$= U_2 + U_3 + U_4$$
$$= 9 + 13.5 + 27$$
$$= 49.5\,J.$$

Equivalent capitance

A 800 pF capacitor is charged by a 100 V battery. After some time the battery is disconnected. The capaci

Q. Find equation capaciatiance between A & B.



$$\frac{2C}{2+C} + 1 = C$$

$$\frac{2C+C+2}{2+C} = C$$

$$3C+2 = C(C+2)$$

$$3C+2 = C^2+2C$$

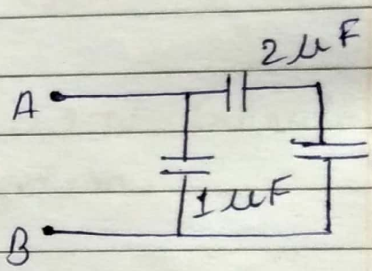
$$C^2 = C-2=0.$$

$$C = \frac{1 \pm \sqrt{1-4(1)(-2)}}{2(1)}$$

$$= \frac{1 \pm \sqrt{1+8}}{2} = \frac{1 \pm 3}{2}$$

$$= \frac{1+3}{2} \text{ or } \frac{1-3}{2} \; \alpha$$

$$= 2\,\mu F$$

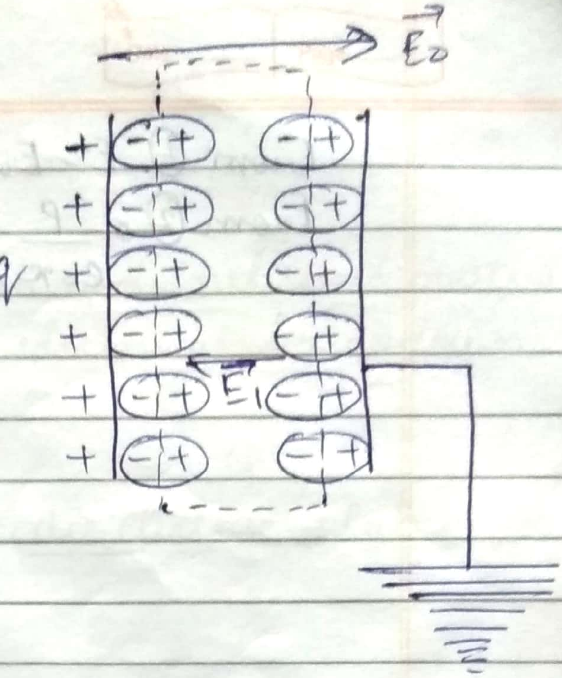


Affect of di-electric on the capacitance of a capacitor:-

Reduced electric field due to die-electric polarization.

Nett electric field $= \vec{E} = \vec{E_0} - \vec{E_1}$

$$K = \frac{\vec{E_0}}{\vec{E} - \vec{E_1}}$$



$\downarrow E = \downarrow V d$ → Const

$\uparrow C = \frac{q}{V\downarrow}$ → Same.

$C_m = K C_V$

$C_V = \frac{\epsilon_0 A}{d}$

$= \frac{\epsilon_0 \, K \cdot A}{d}$

$\boxed{C_M = C_V K}$

* Di-electric polarization :-

To ~~prove~~ show: $K = 1 + \chi$

Polarization density (P) :-

The induced dipole moment developed per unit volume of a di-electric when placed in an external electric field is called as polarization density (P).

Mathematically,

$$P = \frac{q_i \cancel{d}}{A \cancel{d}}$$

$$= \frac{q_i}{A}$$

$$= \sigma_i$$

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Electric Susceptibility (χ)

If E is not large then experimentally it has been found that

$$P \propto E$$

$$P = \epsilon_0 \chi E \rightarrow (2)$$

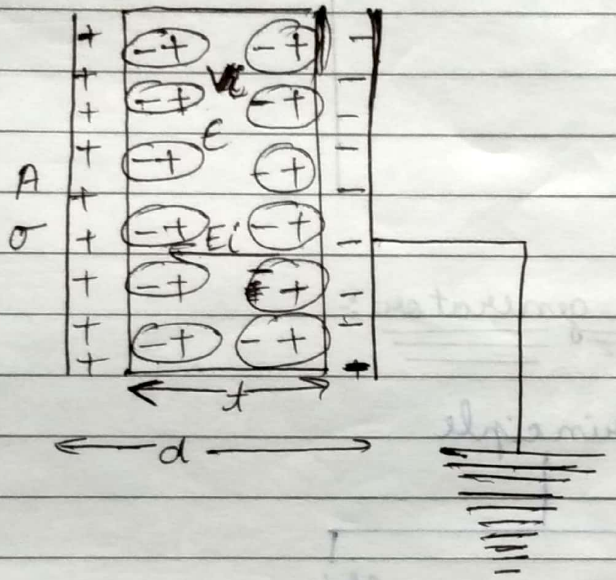
From ① $E = E_0 - E_1$

From ② : $\frac{P}{\epsilon_0 \chi} = E_0 - E_i$

* Di-electric strength:

Max. Value of electric field in which a di-electric material can may withstand without electrical breakdown.
SI unit : Newton per Coulomb.

* Capacitance of a capacitor with di-electric slab :



Net potential b/w the plates $V = V_0 + V_i = E_0(d-t) + E_i t$

$$= \frac{\sigma}{\epsilon_0}(d-t) + \frac{\sigma}{\epsilon} t$$

$$= \frac{q/A}{\epsilon_0}(d-t) + \frac{q/A}{\epsilon_0 K} t$$

$$V = \frac{q}{A\epsilon_0}\left[d - t + \frac{t}{K}\right]$$

$$\frac{q}{V} = \frac{\epsilon_0 A}{d - t + \frac{t}{k}}$$

$$\therefore \boxed{C = \frac{\epsilon_0 A}{d - t + t/k}}$$

Special Case: $\frac{\epsilon_0 A}{d-t}$
For a metallic slab $K = \infty$ $\therefore C = \frac{\epsilon_0 A}{d-t}$

** Effect of di-electric on C, q, U, E, V :-

After Disconnecting a battery		Without disconnecting a battery
C :-	$\uparrow$	
q :-	$=$	
E :-	$\downarrow$	
V :-	$\downarrow$	
U :-	$\downarrow$	

Van De Graaf generator :-

Principle
- Corone discharge
- Skin effect