Mathematics - 3

Solu^n of Algebraic and Transcedental Equation

Exact Numbers:- The numbers which have no uncertainity
e.g:- 5, 0, $\frac{5}{2}$, 3.2 etc.

Approximate numbers:- Consider the number $\frac{10}{3} = 3.333----$ which can be approximated as 3.3, 3.33, 3.333---- upto decimal one, two, three places. The no.'s 3.3, 3.33, 3.333 are called approximate numbers to all the given numbers.

Error:- Diff. b/w exact value & approx value is called error

Error = Exact value − Approx. value
= True value − approx. value
= $x - x'$

Absolute Error:- We are always intersted in the magnitude of error & not the sign of error so we take the modulus of error

$$E_a = |x - x'|$$

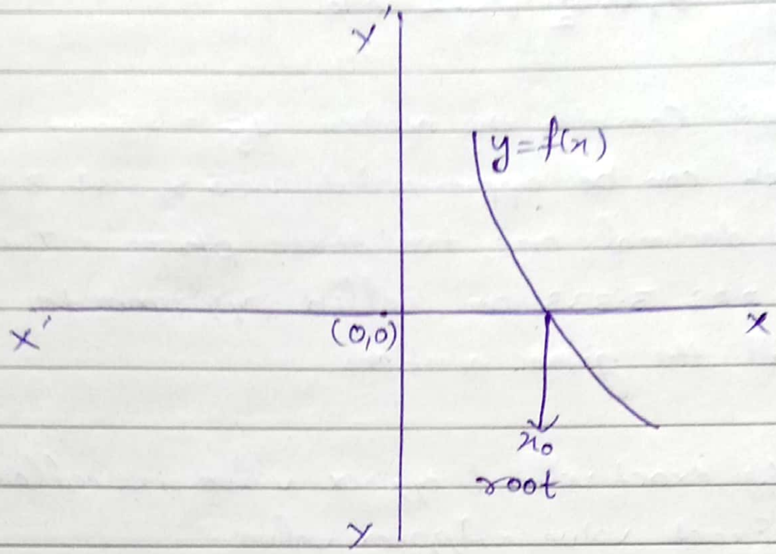
Relative error:- $E_r = \frac{|x - x'|}{x}$

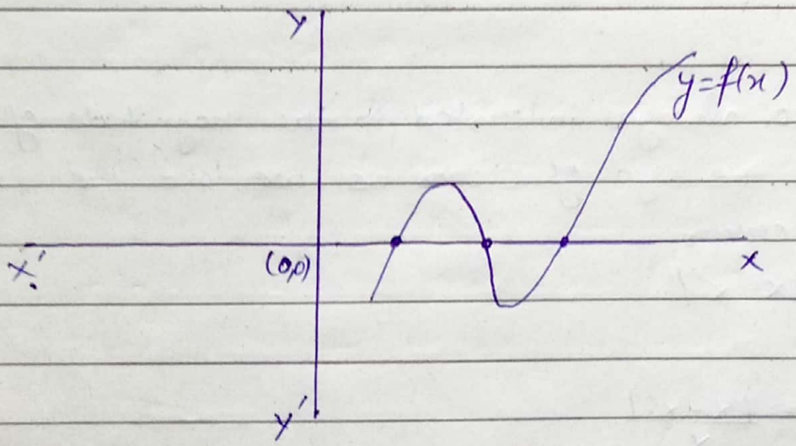
Percentage error:- $E_p = \frac{|x - x'|}{x} \times 100$

Root of an Equation

Algebrically, The real number 'α' is called the real root of the equation $f(x) = 0$
iff $f(\alpha) = 0$

Geometrically :- The point where the curve $y=f(x)$ meets the real axis in rectangular coordinate system is called the real root of the equation $y=f(x)=0$





Algebraic Equation
The eq^n which has only algebraic terms
e.g :- $f(x) = x^3 - 3x + 5$

Transcidental Eq^n :-
The equation which also incorporates some funct^n like trignometric, exponential or logarithm is called ———
e.g :- $f(x) = e^x - \sin x + 2$
$f(x) = \log_e x - \tan x + 2x$

Numerical Methods

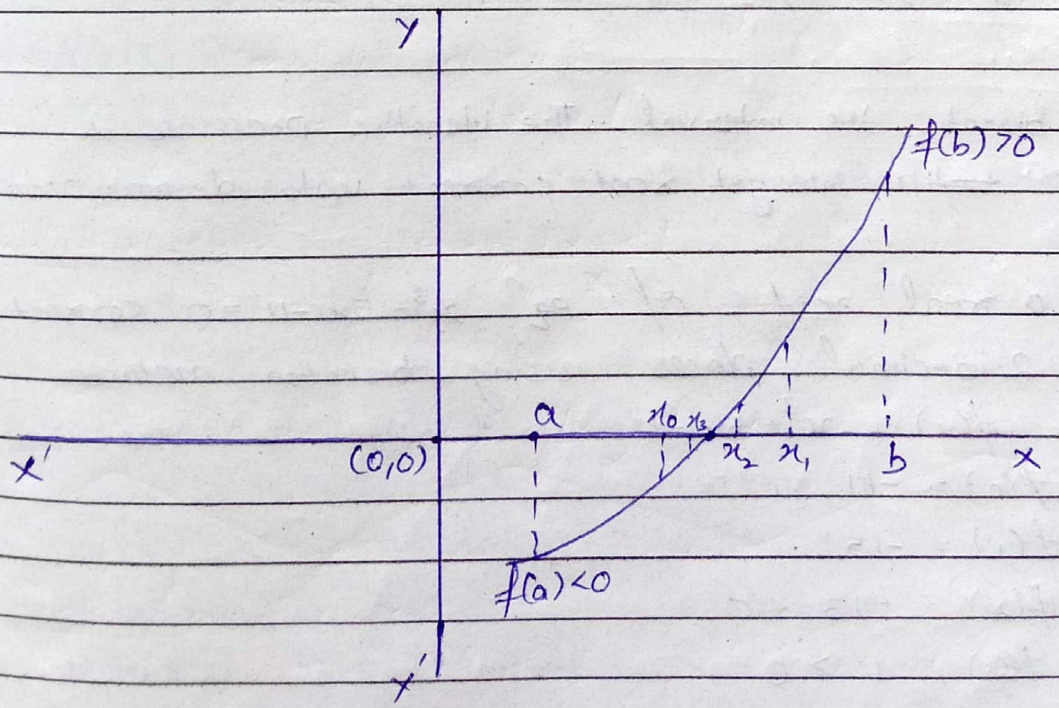
Direct Method

Iterative Method
- Bisection
- Newton's Raphson
- Regula falsi
- Iteration

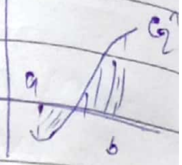
1) Bisection Method to find the root of an equation

Let a funct^n $y=f(x)$ is continuous b/w two points a & b, & $f(a)$, $f(b)$ are of opp. signs i.e $f(a).f(b)<0$ then a real root will lie b/w a & b (Intermediate value theorem)

This will be the point where the curve $y=f(x)$ meets the real axis in rectangular coordinate system.



Consider, $f(x) = 0$ [continuous $f x^n$]
if $f(a) < 0$ & $f(b) > 0$
then root of the eq^n lies b/w a & b [Intermediate Value th.]



1st approx

$$x_1 = \frac{a+b}{2}$$

if $f(x_1) < 0$, then root lies b/w x_1 & b

2nd approximat^n

$$x_2 = \frac{x_1+b}{2}$$

if $f(x_2) > 0$, then root lies b/w x_1 & x_2

3rd approx

$$x_3 = x_1 + x_2$$

& so on — —

We again bisect the interval. The iterative processes is repeated ~~until~~ till we get root correct upto desired accuracy.

Q.) Find a real root of eq^n $x^3 - 5x - 11 = 0$ correct upto 2 decimal places using bisection method

Sol^n let $f(x) = x^3 - 5x - 11$

Now $f(0) = -11$
$f(1) = -15$
$f(2) = -13 < 0$
$f(3) = 1 > 0$

So a real root lies b/w 2 & 3
Table of calculation is as follows:

a $f(a)<0$	b $f(b)>0$	$x_0 = \frac{a+b}{2}$	$f(x_0) = x_0^3 - 5x_0 - 11$
2	3	2.5	-7.875
2.5	3	2.75	-3.953
2.75	3	2.875	-1.611
2.875	3	2.9375	-0.340
2.9375	3	2.96875	0.3213
2.9375	2.96875	2.953125	-0.12
2.953125	2.96875	2.9609375	0.1525
2.953125	2.9609375	2.957031	0.0712
2.953125	2.957031	2.955078	0.02978
2.953125	2.955078	2.9541015	0.00909
2.953125	2.9541015	2.95361325	-0.0012

So the root correct upto two decimal places 2.95

Another method

$f(2.9) = -1.16$

a $f(a)<0$	b $f(b)>0$	$x_0 = \frac{a+b}{2}$	$f(x_0) = x_0^3 - 5x_0 - 11$
2.9	3	2.95	-0.0776
2.95	3	2.975	0.4556
2.95	2.975	2.9625	0.18
2.95	2.9625	2.95625	0.054
2.95	2.95625	2.953125	-0.012
2.953125	2.95625	2.9546875	0.021
2.953124	2.9546875	2.95390625	0.0049

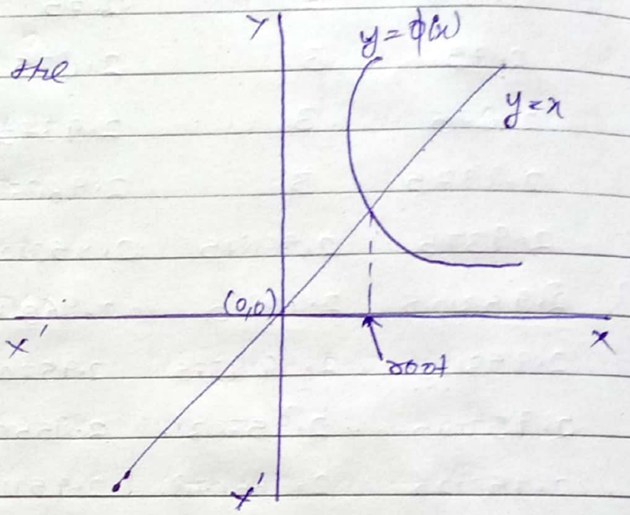
Iteration Method

Let a funct^n $y = f(x)$ is continuous b/w two points 'a' & 'b' and $f(a)$, $f(b)$ are of opp. signs i.e. $f(a) \cdot f(b) < 0$, then atleas one real root of eq^n $f(x) = 0$ lies b/w 'a' & 'b'

In this method, we first write the given eq^n $f(x) = 0$ in the form

$x = \phi(x)$ —— (1)

then the abscissa of the pt. of intersection of line $y = x$ & curve $y = \phi(x)$ gives the desired root.



Let initial approx root is x_0 then first approx to the root is $x_1 = \phi(x_0)$

Second approx root is $x_2 = \phi(x_1)$

Proceeding in this way, we get

$\boxed{x_{n+1} = \phi(x_n)}$ —— (2)

This process is repeated till x_{n+1} & x_n are to be the same upto desired accuracy.

Note 1 → 1) The seq. of approximation $x_0, x_1, x_2, \ldots x_n$ will converge to the desired root $\alpha \in (a,b)$ provided $\boxed{|\phi'(x)| < 1}$ $\forall$ $x \in (a,b)$.

2) By using Iteration method, we can find the root of an equation given in terms of an infinite series.

Q.1 Find a real root of eq^n $x^3-5x-11=0$ —— (1)
Correct upto 2 decimal places using iteration method

Solu^n :- Let $f(x) = x^3-5x-11$

$f(0) = -11$

$f(1) = -15$

$f(2) = -13 < 0$

$f(3) = 1 > 0$

so a real root of eq^n lies b/w 2 & 3

Now we will write the given eq^n (1) in the form $x=\phi(x)$

$$x^3 = 5x+11$$

$$x = (5x+11)^{1/3} = \phi(x)$$

$$|\phi'(x)| = \left|\frac{5}{3(5x+11)^{2/3}}\right|$$

$$|\phi'(2)| = 0.21$$

$$|\phi'(3)| = \frac{5}{3(26)^{2/3}}$$

$$= 0.187$$

Thus

$|\phi(x)| < 1 \ \forall \ x \in (2,3)$ (Convergence Condition of iteration method)

Now the form $x=\phi(x)$ as (2) is correct

Let $x_{n+1} = \phi(x_n)$

$= (5x_n+11)^{1/3}$

let $x_0 = 3$

$x_1 = (5x_0+11)^{1/3} = 2.9624$

$x_2 = (5x_1+11)^{1/3} = 2.9553$

$x_3 = (5x_2+11)^{1/3} = 2.9539$

$$x_4 = (5x_3+11)^{1/3} = 2.9537$$
$$x_5 = (5x_4+11)^{1/3} = 2.9538$$

let $x_0 = 2$ (krna kisi ek value se hi hai)

$$x_1 = (5x_0+11)^{1/3} = 2.7589$$
$$x_2 = (5x_1+11)^{1/3} = 2.9159$$
$$x_3 = (5x_2+11)^{1/3} = 2.9464$$
$$x_4 = (5x_3+11)^{1/3} = 2.9522$$
$$x_5 = (5x_4+11)^{1/3} = 2.9533$$
$$x_6 = (5x_5+11)^{1/3} = 2.95361$$

Q-) Find a real root of eq^n $\cos x - 3x + 1 = 0$ —①
Correct upto 4 decimal places using Iteration method

Solu^n :- let $f(x) = \cos x - 3x + 1 = 0$

$$f(0) = 2 > 0$$
$$f(1) = -1.45 < 0$$

So a real root lies b/w 0 & 1

The given eq^n ① in the form $x = \phi(x)$ can be written as:

$$x = \frac{1}{3}(1+\cos x) = \phi(x) \text{ 'say'}$$

$$|\phi'(x)| = \left|\frac{-\sin x}{3}\right| = \left|\frac{\sin x}{3}\right| < 1 \ \forall \ x \in (0,1)$$

(sin x is a bounded funct^n, it's modulus value lie b/w 0 & 1)

let $x_0 = 1$

$$x_{n+1} = \frac{1}{3}(1+\cos x_n)$$

$$x_1 = \frac{1}{3}(1+\cos x_0)$$
$$= 0.513434$$

$x_2 = \frac{1}{3}(1+\cos x_1) = 0.623687$

$x_3 = \frac{1}{3}(1+\cos x_2) = 0.603910$

$x_4 = \frac{1}{3}(1+\cos x_3) = 0.607707$

$x_5 = \frac{1}{3}(1+\cos x_4) = 0.606986$

$x_6 = \frac{1}{3}(1+\cos x_5) = 0.6071236$

$x_7 = 0.6070974$
$x_8 = 0.607102$
$x_9 = 0.607101$

So root is 0.6071

$f(x) = \cos x - 3x + 1$
$f(0.6071) = 0.0000058$

Q) Find a real root of the eq^n $3x^3+10x^2+10x+7=0$ by bisection method.

Sol^n let $f(x) = 3x^3+10x^2+10x+7$
$f(0) = 7$
$f(-1) = 4$
$f(-2) = 3$
$f(-3) = -14$
so one root of the eq^n lies b/w -2 and -3
$a=-2,\ b=-3$

No. of approx.	a	b	$x_n = \frac{a+b}{2}$	$f(x_n) = 3x_n^3 + 10x_n^2 + 10x_n + 7 = 0$
1	-2	-3	-2.5	$f(x_1) = -ve$
2	-2	-2.5	-2.25	$f(x_2) = +ve$ (0.95)
3	-2.25	-2.5	-2.375	$f(x_3) = -ve$ (0.53)
4	~~-2.375~~ -2.25	-2.375	~~-2.4375~~ -2.3125	$f(x_4) = +ve$ (0.25)
5	-2.3125	-2.375	-2.34375	$f(x_5) = -ve$ (0.12)

Q-2 Find a real root of the eq^n $x \log_{10} x = 1.2$

Q-1 Find a +ve real root of $x - \cos x = 0$ by bisection method, correct upto 4 decimal places b/w 0 & 1.

Iteration Method

Q-1 $x = e^{-x}$

$\phi x = e^{-x}$

$\phi'(x) = -e^{-x}$

$|\phi'(x)| = e^{-x}$

$f(x) = x - e^{-x} = 0$

$f(0) = -1$

$f(1) = 1 - \frac{1}{e} = +ve$

Hence one root lies b/w 0 & 1

$x_{n+1} = \phi(x_n)$

$x_0 = \frac{0+1}{2} = 0.5$

$x_1 = \phi(x_0)$

$= e^{-x_0}$

$= e^{-0.5}$

$= 0.6061$

$x_2 = \phi(x_1) = e^{-x_1}$

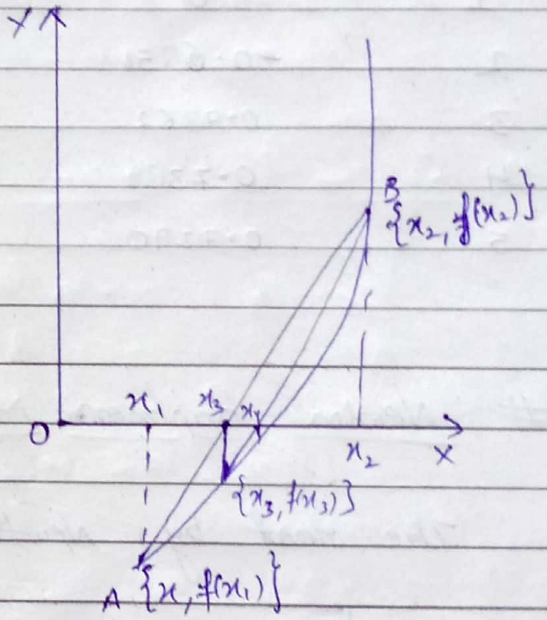
$x_2 = e^{-.6061} = 0.54547$

$x_3 = e^{-0.54547}$

$x_3 = 0.57956$

Regular - falsi Method

Consider an equation $f(x) = 0$ —(i)



Eq^n of the cordAis of the curve $f(x)$ is $y - f(x_1) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}(x - x_1)$

Cord 1st passes through the x-axis sat the pt. $(x_3, 0)$ lies eq^n (i)

$$0 - f(x_1) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}(x_3 - x_1)$$

$$-\frac{(x_2 - x_1)}{f(x_2) - f(x_1)} f(x_1) = x_3 - x_1$$

$$x_3 = x_1 - \frac{(x_2 - x_1)}{f(x_2) - f(x_1)} f(x_1)$$

$$x_3 = \frac{x_1 f(x_2) - \cancel{x_1 f(x_1)} - x_2 f(x_1) + \cancel{x_1 f(x_1)}}{f(x_2) - f(x_1)}$$

$$\boxed{x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}}$$

$$x_{n+1} = \frac{x_{n-1} f(x_n) - x_n f(x_{n-1})}{f(x_n) - f(x_{n-1})}$$ (General formula)

0.0000

Q.) To obtain a real root of the eqⁿ $x - \cos x = 0$ by reguler falsi method upto four decimal places

Solⁿ

$f(x) = x - \cos x = 0$

$f(0) = -1$

$f(1) = 0.45969$

Root lie b/w 0 & 1

$x_0 = 0, \quad x_1 = 1$

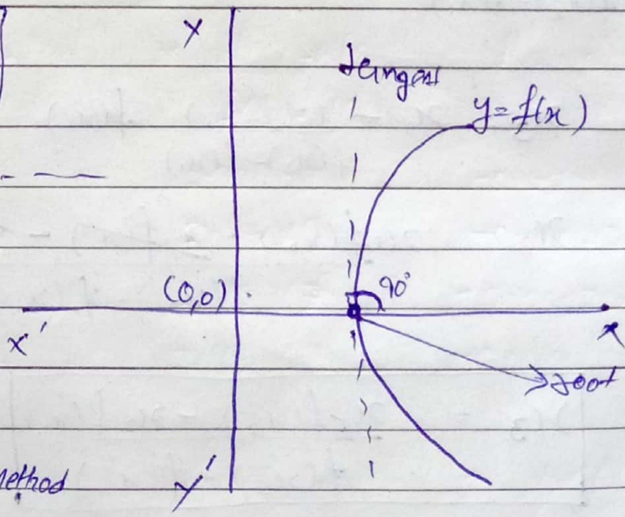
No. of approximation	x_0	x_1	$f(x_0)$	$f(x_1)$	x_n	$f(x_{n+1})$
1	0	1	-1	0.4596	$x_2 = 0.6851$	-0.0892
2	0.6851	1	-0.0892	0.4596	$x_3 = 0.7362$	-0.0048
3	0.7362	1	-0.0048	0.4596	$x_4 = 0.7389$	-0.0003
4	0.7389	1	-0.0003	0.4596	$x_5 = 0.7390$	-0.0001
5	0.7390	1	-0.0001	0.4596	$x_6 = 0.7390$	-0.0001

Newton Raphson Method

The root by Newton-Raphson Method is given by

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

where $n = 0, 1, 2, 3, \ldots$



Here $f'(x_n)$ is the tangent to the curve at the point x_n

Now in Newton Raphson Method the tangent to the curve while crossing the real axis must be approximately vertical i.e. $(f'(x_n) \to \infty)$

Convergence condition of Newton-Raphson Method

The sequence of approximat^n $x_0, x_1, x_2 - - - x_n$ will converge to the desired root $\alpha \in (a,b)$ provided

$$\boxed{|f(x) f''(x)| < |f'(x)|^2}$$

Q.1 Find a real root of eq^n $x^3 - 5x - 11 = 0$ —① Correct upto 4 decimal places using newton raphson method.

solu^n:- let $f(x) = x^3 - 5x - 11$

$f(0) = -11$

$f(1) = -15$

$f(2) = -13 < 0$

$f(3) = 1 > 0$

so real root of the eq^n lies b/w 2 & 3

$f'(x) = 3x^2 - 5$

Now Newton raphson formula is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{(x_n^3 - 5x_n - 11)}{3x_n^2 - 5}$$

$$x_{n+1} = \frac{3x_n^3 - \cancel{5x_n} - x_n^3 + \cancel{5x_n} + 11}{3x_n^2 - 5}$$

$$x_{n+1} = \frac{2x_n^3 + 11}{3x_n^2 - 5} \quad \text{—②}$$

let $x_0 = 3$

put $n = 0$ in ②

$$x_1 = \frac{2x_0^3 + 11}{3x_0^2 - 5} = 2.954545$$

$$x_2 = \frac{2x_1^3 + 11}{3x_1^2 - 5} = 2.953672$$

$$x_3 = \frac{2x_2^3 + 11}{3x_1^2 - 5} = 2.953671$$

$\therefore$ Root correct upto 4 decimal places is 2.9537

Note:- Newton raphson method has a convergence of order 2 or quadratic convergence.

$f(2.9537) = 0.00005$

Regular falsi method

formula $x = \frac{a f(b) - b f(a)}{f(b) - f(a)}$

OR

$$x_{n+1} = \frac{x_{n-1} f(x_n) - x_n f(x_{n-1})}{f(x_n) - f(x_{n-1})}$$

Q1

a	$f(a)<0$	b	$f(b)>0$	$x = \frac{af(b) - bf(a)}{f(b) - f(a)}$	$f(x_0) = x_0^3 - 5x_0 - 11$
2	-13	3	1	2.92857	-0.5258
2.92857	-0.52587	3	1	2.95319	-0.01023
2.95319	-0.01023	3	1	2.95366	-0.00025
2.95366	-0.00025	3	1	2.95367	-0.00004

Note:- The convergence of regular falsi method is 1.618

Interpolation

Suppose we are given the following values of $y=f(x)$ for a set of values of x

$$y=f(x)$$

$$\begin{array}{lccccc} x : & x_0 & x_1 & x_2 & ---- & x_n \\ y=f(x) : & y_0 & y_1 & y_2 & ---- & y_n \end{array}$$

Thus the process of finding the value of y corresponding to any value of $x=x_i$ b/w x_0 and x_n is called interpolation while the process of computing the value of the funct^n ~~bet~~ $y f(x)$ outside the given range is called the extrapolation

Finite Differences

The calculus of finite differences deals with the changes that take place in the value of the funct^n due to finite changes in the independent variable

Suppose we are given a set of values (x_i, y_i); $i=0,1,2---n$ for any function $y=f(x)$. A value of independent variable x is called argument and the corresponding value of dependent variable y is called entry

Suppose that the function $y=f(x)$ is tabulated for the equally spaced values $x=x_0, x_0+h, x_0+2h, x_0+3h, ---- x_0+(n-1)h$ giving $y_0, y_1, y_2 ---- y_{n-1}$ To determine the values of $f(x)$ or $f'(x)$ for some intermediate values of x. Following 3 types of differences are useful:

① Forward differences
② Backward ,,
③ Central ,,
④ Divided ,,

① Forward Differences:- Let $y_0, y_1, ----- y_n$ are n+1 entries corresponding to $x_0, x_1, x_2 ---- x_n$ then the differences $y_1 - y_0$, $y_2 - y_1$, $y_3 - y_2$, ----- $y_n - y_{n-1}$

These differences ~~is~~ are said to be first forward differences and it is denoted as $\Delta y_0, \Delta y_1, \Delta y_2, \Delta y_{n-1}$

The operator Δ (del) → first forward diff. operator

$$\Delta y_0 = y_1 - y_0$$

$$\Delta y_{n-1} = y_n - y_{n-1}$$

2nd ~~follower~~ forward diff. are.

$$\Delta^2 y_0 = \Delta(\Delta y_0)$$
$$= \Delta(y_1 - y_0)$$
$$= \Delta y_1 - \Delta y_0$$
$$= y_2 - y_1 - (y_1 - y_0)$$
$$\Delta^2 y_0 = y_2 - 2y_1 + y_0$$

Similarly

$$\Delta^2 y_1 = y_4 - 2y_3 + y_2$$
$$\vdots$$
$$\Delta^2 y_{n-2} = y_n - 2y_{n-1} + y_{n-2}$$

3rd forward differences are.

$$\Delta^3 y_0 = \Delta(\Delta^2 y_0)$$
$$= \Delta(y_2 - 2y_1 + y_0)$$
$$= \Delta y_2 - 2\Delta y_1 + \Delta y_0$$
$$= y_3 - y_2 - 2(y_2 - y_1) + (y_1 - y_0)$$
$$\Delta^3 y_0 = y_3 - 3y_2 + 3y_1 - y_0$$

$$\Delta^3 y_1 = y_4 - 3y_3 + 3y_2 - y_1$$
$$\vdots$$
$$\Delta^3 y_{n-3} = y_n - 3y_{n-1} + 3y_{n-2} - y_{n-3}$$

Forward Difference Table

x	y	Δy_0	$\Delta^2 y_0$	$\Delta^3 y$
x_0	$\boxed{y_0}$	$y_1 - y_0 = \boxed{\Delta y_0}$	$\Delta y_1 - \Delta y_0 = \boxed{\Delta^2 y_0}$	$\Delta^2 y_1 - \Delta^2 y_0 = \boxed{\Delta^3 y_0}$
x_1	y_1	$y_2 - y_1 = \Delta y_1$	$\Delta y_2 - \Delta y_1 = \Delta^2 y_1$	
x_2	y_2	$y_3 - y_2 = \Delta y_2$		
x_3	y_3			

Q-) Construct the F.D.T given that

x : 5 10 15 20 25 30

y : 9962 9848 9659 9397 9063 8660

and point out the values of 2nd forward diff of y_{10} $(\Delta^2 y_{10})$ or $\Delta^4 y_5$

Soln

x	y	Δy_0	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	Δ^5
5	9962	-114	-75	2	$\boxed{-1}$	3
10	9848	-189	$\boxed{-73}$	1	2	
15	9659	-262	-72	3		
20	9397	-334	-69			
25	9063	-403				
30	8660					

$\Delta^2 y_{10} = -73$

$\Delta^4 y_5 = -1$

Backward Differences

let $y_0, y_1, y_2 \ldots y_n$ are $(n+1)$ entries corresponding to the equidistant arguments $x_0, x_1, x_2 \ldots x_n$ then the differences $y_1 - y_0, y_2 - y_1, y_3 - y_2 \ldots y_n - y_{n-1}$ are denoted as $\nabla y_1, \nabla y_2, \nabla y_3 \ldots \nabla y_n$. These differences are said to be the first backward differences and operator ∇ is said to be first backward operator.

2nd backward diff are.

$$\nabla^2 y_2 = \nabla(\nabla y_2)$$
$$= \nabla(y_2 - y_1)$$

$$= \nabla y_2 - \nabla y_1$$
$$= y_2 - y_1 - (y_1 - y_0)$$
$$\nabla^2 y_2 = y_2 - 2y_1 + y_0$$
$$\nabla^2 y_3 = y_3 - 2y_2 + y_1$$

$$\nabla^2 y_n = y_n - 2y_{n-1} + y_{n-2}$$

Backward Diff. Table

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$
x_0	y_0	$y_1 - y_0 = \nabla y_1$	$\nabla y_2 - \nabla y_1 = \nabla^2 y_2$	$\nabla^2 y_3 - \nabla^2 y_2 = \boxed{\nabla^3 y_3}$
x_1	y_1	$y_2 - y_1 = \nabla y_2$	$\nabla y_3 - \nabla y_2 = \boxed{\nabla^2 y_3}$	
x_2	y_2	$y_3 - y_2 = \boxed{\nabla y_3}$		
x_3	$\boxed{y_3}$			

Central Differences

The differences $y_1 - y_0$, $y_2 - y_1$, $y_3 - y_2$ — $y_n - y_{n-1}$ are said to central differences & denoted as $\delta y_{1/2}$, $\delta y_{3/2}$, $\delta_{5/2}$ — $\delta y_{(n-\frac{1}{2})}$ (These are called first central differences)
δ is first central difference operator.

2nd central differences are:-

$$\delta^2 y_1 = \delta y_{3/2} - \delta y_{1/2}$$

$$\delta^2 y_2 = \delta y_{5/2} - \delta y_{3/2}$$

Central diff. Table

x	y	δy	$\delta^2 y$	$\delta^3 y$
x_0	y_0	$y_1 - y_0 = \delta y_{1/2}$	$\delta y_{3/2} - \delta y_{1/2} = \delta^2 y_1$	$\delta^2 y_2 - \delta^2 y_1 = \delta^3 y_{3/2}$
x_1	y_1	$y_2 - y_1 = \delta y_{3/2}$	$\delta y_{5/2} - \delta y_{3/2} = \delta^2 y_2$	
x_2	y_2	$y_3 - y_2$ $\delta y_{5/2}$		
x_3	y_3			

Other Diff. Operator

① shift operator

It is denoted by E

$E f(x) = f(x+h)$

$E^2 f(x) = f(x+2h)$

$E^3 f(x) = f(x+3h)$

$E^n f(x) = f(x+nh)$

or $E y_0 = y_1$

$E^2 y_1 = y_3$

Inverse shift Operator

It is denoted by E^{-1}

$E^{-1} f(x) = f(x-h)$

$E^{-2} f(x) = f(x-2h)$

or

$E^{-1} y_1 = y_0$

$E^{-2} y_2 = y_0$

Average Operator

It is denoted by μ & defined as

$\mu f(x) = \frac{1}{2}\left[f\left(x+\frac{h}{2}\right) + f\left(x-\frac{h}{2}\right)\right]$

Note: Among all operators $E, \Delta, \nabla, \delta, \mu$, shift operator E is the fundamental operator of calculus, they $\Delta, \nabla, \delta, \mu$ can be written as in the forms of E

Relation b/w operators

(i) $\Delta = E - 1$

We have

$\Delta y_0 = y_1 - y_0$

$\Delta y_0 = E y_0 - y_0$

$\Delta y_0 = (E-1) y_0$

$\boxed{\Delta = E - 1}$ or $\boxed{E = 1 + \Delta}$

$\Delta^5 y_2 = \Delta^3 (\Delta^2 y_2)$

$= \Delta^3 (y_4 - 2y_3 + y_2)$

$\Delta^5 y_2 = (E-1)^5 y_2$

$= ({}^5C_0 E^5 (-1)^0 + {}^5C_1 E^4 (-1)^1 + {}^5C_2 E^3 (-1)^2 + {}^5C_3 E^2 (-1)^3 + {}^5C_4 E^1 (-1)^4 + {}^5C_5 E^0 (-1)^5) y_2$

$= (E^5 - 5E^4 + 10E^3 - 10E^2 + 5E - 1) y_2$

$= y_7 - 5y_6 + 10y_5 - 10y_4 + 5y_3 - y_2$

(ii) $\nabla = 1 - E^{-1}$

We have

$\nabla y_1 = y_1 - y_0$

$\nabla y_1 = y_1 - E^{-1} y_1$

$\nabla y_1 = (1 - E^{-1}) y_1$

$\boxed{\nabla = 1 - E^{-1}}$

or

$\boxed{E^{-1} = 1 - \nabla}$

$\frac{1}{E} = 1 - \nabla$

$\boxed{E = (1 - \nabla)^{-1}}$

(iii) $\delta = E^{1/2} - E^{-1/2}$

We have

$$\delta y_{\frac{1}{2}} = y_1 - y_0$$

$$= E^{1/2} y_{\frac{1}{2}} - E^{-\frac{1}{2}} y_{\frac{1}{2}}$$

$$\delta \cancel{y_{1/2}} = (E^{1/2} - E^{-1/2}) \cancel{y_{\frac{1}{2}}}$$

$$\delta = E^{1/2} - E^{-1/2}$$

(iv) $\mu = \frac{1}{2}\left[E^{1/2} + E^{-1/2}\right]$

(v) $E = e^{hD}$

where $D = \frac{d}{dx}$

We have

$$Ef(x) = f(x+h)$$

$$= f(x) + \frac{h f'(x)}{1!} + \frac{h^2}{2!} f''(x) + \frac{h^3}{3!} f'''(x).$$

(Expansion by taylor series)

$$= f(x) + h\,Df(x) + \frac{h^2}{2!} D^2 f(x) + \cdots$$

$$Ef(x) = f(x) + \frac{(hD)f(x)}{1!} + \frac{(hD)^2}{2!} f(x) + \cdots$$

$$E = 1 + \frac{hD}{1!} + \frac{(hD)^2}{2!} + \frac{(hD)^3}{3!} + \cdots$$

$$\boxed{E = e^{hD}}$$

$$1 + \Delta = e^{hD}$$

$$\boxed{\Delta = e^{hD} - 1}$$

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Differences of a Polynomial

n^{th} differences of a polynomial of n^{th} degree are constant and all higher order differences are zero when the value of the independent variable are at equal interval

$$y = 6x^3 + 6x^2 + 5x + 3$$
$$\Delta y = \Delta(6x^3 + 6x^2 + 5x + 3)$$
$$\Delta y = hD(6x^3 + 6x^2 + 5x + 3)$$
$$\Delta y = h(18x^2 + 12x + 5)$$
$$\Delta(\Delta y) = h^2(36x + 12)$$
$$\Delta^3 = h^3 \times 36$$

$$\left\{ \begin{array}{c} \Delta f(x) = hD(f(x)) \\ \Delta = hD \end{array} \right\}$$

Q-) Evaluate (i) $\Delta \tan^{-1} x$ (ii) $\Delta^2 \cos 2x$ where h is the interval of differences.

Sol (i) $\Delta f(x) = f(x+h) - f(x)$

$$= \tan^{-1}(x+h) - \tan^{-1} x$$
$$= \tan^{-1} \frac{(x+h) - x}{1 + (x+h)x}$$
$$= \tan^{-1} \frac{h}{1 + x(x+h)}$$
$$= \tan^{-1} \frac{h}{1 + x^2 + xh}$$

(ii) $\Delta^2 \cos 2x = \Delta(\Delta \cos 2x)$

$= \Delta\{\cos 2(x+h) - \cos 2x\}$

$= \Delta\left\{2\sin\left(\frac{2x+2h+2x}{2}\right)\sin\left(\frac{2x-2x-2h}{2}\right)\right\}$

$= \Delta\left\{2\sin(2x+h)\sin(-h)\right\}$

$= -2\sin h \, \Delta \sin(2x+h)$

$= -2\sin h \left\{\sin\{2(x+h)+h\} - \sin(2x+h)\right\}$

Q1 Construct a backward diff. Table for $y = \log x$, given that

x: 10 20 30 40 50

y: 1 1.3010 1.4771 1.6021 1.6990

and find the value of $\nabla^3 \log_{40}$ & $\nabla^4 \log 50$

Sol:

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
10	1	.3010	−0.1249		
20	1.3010	.1761	−0.0511	.0738	−.0508
30	1.4771	.1250	−0.281	0.0230	
40	1.6021	.0969			
50	1.6990				

$\nabla^3 \log_{40} = 0.078$ $\qquad$ $\nabla^4 \log_{50} = -0.0508$

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
10	1	.3010	−0.1249	$y - 1.5283$	$6.3576 - 4y$
20	1.3010	.1761	$y - 1.6532$	$4.8293 - 3y$	
30	1.4771	$y - 1.4771$	$3.1761 - 2y$		
40	y	$1.6990 - y$			
50	1.6990				

Since In this problem we are given 4 entries so we obtain a cubic polynomial (3 degree polynomial) & put y term $= 0$

2nd method to find missing term

$\Delta^4 y_0 = 0$

$(E-1)^4 y_0 = 0$ $\quad [\Delta = E-1]$

$({}^4c_0 E^4 - {}^4c_1 E^3 + {}^4c_2 E^2 - {}^4c_3 E + {}^4c_4) y_0 = 0$

$E^4 y_0 - 4E^3 y_0 + 6E^2 y_0 - 4E y_0 + y_0 = 0$

$y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 = 0$

With usual notations, prove that

Q-1 $\quad \Delta^n\left(\frac{1}{x}\right) = (-1)^n \frac{n!\, h^n}{x(x+h)(x+2h)\cdots(x+nh)}$

Sol^n $\quad f(x) = \frac{1}{x}$

$f(x+h) = \frac{1}{x+h}$

$\Delta f(x) = f(x+h) - f(x)$

$= \frac{1}{x+h} - \frac{1}{x}$

$\Delta\left(\frac{1}{x}\right) = \frac{-h}{x(x+h)}$

$\Delta^2\left(\frac{1}{x}\right) = \Delta\left(\frac{-h}{x(x+h)}\right)$

$= -h\Delta\left(\frac{1}{x(x+h)}\right)$

$= (-\cancel{h})\Delta\left\{\frac{1}{x} - \frac{1}{x+h}\right\} \times \frac{1}{\cancel{h}}$

$= -1\,\Delta\left\{\Delta\left(\frac{1}{x}\right) - \Delta\left(\frac{1}{x+h}\right)\right\}$

$$= -1\left[\left(\frac{1}{x+h}-\frac{1}{x}\right)-\left(\frac{1}{x+2h}-\frac{1}{x+h}\right)\right]$$

$$= -1\left[\frac{-h}{x(x+h)}-\frac{-h}{(x+h)(x+2h)}\right]$$

$$\Delta^2\left(\frac{1}{x}\right) = (-1)(-h)\left[\frac{1}{x(x+h)}-\frac{1}{(x+h)(x+2h)}\right]$$

$$\Delta^2\left(\frac{1}{x}\right) = (-1)^2 h\left[\frac{x+2h-x}{x(x+h)(x+2h)}\right]$$

$$\Delta^2\left(\frac{1}{x}\right) = (-1)^2 h\left[\frac{2h}{x(x+h)(x+2h)}\right]$$

$$\Delta^2\left(\frac{1}{x}\right) = \frac{(-1)^2\, 2!\, h^2}{x(x+h)(x+2h)}$$

Proceed upto n, we get

$$\Delta^n\left(\frac{1}{x}\right) = \frac{(-1)^n\, n!\, h^n}{x(x+h)(x+2h)\cdots(x+nh)}$$

Newton's forward interpolation formula for equal interval

let $y_0, y_1, y_2 \cdots y_n$ are $(n+1)$ entries corresponding to $x_0, x_1, x_2 \cdots x_n$ equal / equidistant argument

$$y_p = y_0 + P\Delta y_0 + \frac{P(P-1)}{2!}\Delta^2 y_0 + \frac{P(P-1)(P-2)}{3!}\Delta^3 y_0 + \cdots \infty$$

where $p = \frac{x-x_0}{h} \Rightarrow x = x_0 + ph$

Newton's Backward interpolation formula for equal interval

$$y_p = y_n + P\nabla y_n + \frac{P(P+1)}{2!}\nabla^2 y_n + \frac{P(P+1)(P+2)}{3!}\nabla^3 y_n + \cdots \infty$$

where $p = \frac{x-x_n}{h}$

Q) From the following table, find the value of $e^{0.24}$

x:	0.1	0.2	0.3	0.4	0.5
e^x:	1.10517	1.22140	1.34986	1.49182	1.69872

Soln forward diff. table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0.1	1.10517	0.11623	0.01223	0.00127	0.00017
0.2	1.22140	0.12846	0.01350	0.00144	
0.3	1.34986	0.14196	0.01494		
0.4	1.49182	0.15690			
0.5	1.69872				

Here $h = 0.1$ &

$$p = \frac{x - x_0}{h}$$

$$p = \frac{x - .1}{.1}$$

Given $x = 0.24$

$$p = \frac{.24 - .1}{.1} = \frac{.14}{.1}$$

$$\boxed{p = 1.4}$$

$$y_p = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$

$$y_{(1.4)} = 1.10517 + 1.4 \times 0.11623 + \frac{1.4(1.4-1)}{2!} \times 0.01223 + \frac{1.4(1.4-1)(1.4-2)}{3!} \times 0.00127$$

$$+ \frac{1.4(1.4-1)(1.4-2)(1.4-3)}{4!} \times 0.00017$$

$$y_{(1.4)} = 1.2712490 = e^{0.24}$$

Q. From the table, estimate the no. of students who obtained marks b/w 40 & 45

marks :	30-40	40-50	50-60	60-70	70-80
No. of std. :	31	42	51	35	31

Solⁿ Forward Diff. Table

No. of students marks less than (x)	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
40	31	42	9	-25	37
50	73	51	-16	12	
60	124	35	-4		
70	159	31			
80	190				

$h=10$

$h=10$

$p = \frac{x-x_0}{h}$ $\qquad p = \frac{x-40}{10}$ $\qquad x=45$

$p = \frac{45-40}{10} = \frac{5}{10} = 0.5$

$$y_{(0.5)} = 31 + \frac{0.5(0.5-1)9}{2} + 0.5\times 42 + \frac{0.5(0.5-1)(0.5-2)(-25)}{6}$$

$$+ \frac{0.5(0.5-1)(0.5-2)(0.5-3)}{24}\times 37$$

$y_{(0.5)} = 31 + 21 - 0.125 - 1.5625 - 1.4453125$

$y_{(0.5)} = 47.867 = 48$

No. of students who got the marks b/w 40 & 45 = 48-31
= 17

Q) Population of town was as given. Estimate the populatⁿ for the year 1925.

year(x) :	1891	1901	1911	1921	1931
population(y) : (thousand)	46	66	81	93	101

Solⁿ Backward Diff Table

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1891	46				
1901	66	20			
1911	81	15	-5	2	$\boxed{-3}$
1921	93	12	-3	$\boxed{-1}$	
1931	$\boxed{101}$	$\boxed{8}$	$\boxed{-4}$		

$h = 10$

$$p = \frac{x - x_n}{h} = \frac{1925 - 1931}{10} = \frac{-6}{10} = -0.6$$

$$y_p = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \frac{p(p+1)(p+2)(p+3)}{4!}\nabla^4 y_n$$

$$y_p = 101 + 8(-0.6) - \frac{0.6(1-0.6)}{2}(-4) - \frac{0.6(1-0.6)(2-0.6)}{6}(-1)$$

$$-\frac{0.6(1-0.6)(2-0.6)(3-0.6)}{24}(-3)$$

$$y_p = 101 - 4.8 + 0.48 + 0.056 + 0.1008$$

$$y_p = 96.8368$$

Imp.

Q1 Find the cubic polynomial which takes the following values

x :	0	1	2	3
$f(x)$:	1	2	1	10

Sol^n

x	$f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$
0	1	1	-2	12
1	2	-1	10	
2	1	9		
3	10			

$p = \frac{x-0}{1}$ $\quad p = x$

$$y(x) = 1 + x + \frac{x(x-1)(-2)}{2} + \frac{x(x-1)(x-2)(12)}{6}$$

$$y(x) = 1 + x - (x^2 - x) + (x^2 - x)(2x - 4)$$

$$y(x) = 1 + x - x^2 + x + 2x^3 - 4x^2 - 2x^2 + 4x$$

Divided difference for unequal intervals

let $y_0, y_1, y_2 \ldots y_n$ are $(n+1)$ entries corresponding to unequal arguments $x_0, x_1, x_2 \ldots x_n$, then first divided difference for the arguments x_0, x_1 is given by,

$$\boxed{\frac{\Delta y_0}{x_1} = \frac{y_1 - y_0}{x_1 - x_0} = [x_0, x_1]}$$

Similarly divided diff. for the arguments x_1, x_2

$$\boxed{\frac{\Delta y_1}{x_2} = \frac{y_2 - y_1}{x_2 - x_1} = [x_1, x_2]}$$

Similarly for arguments x_2, x_3

$$\boxed{\frac{\Delta y_2}{x_3} = \frac{y_3 - y_2}{x_3 - x_2} = [x_2, x_3]}$$

second divided difference for x_0, x_1, x_2

$$\underset{x_1, x_2}{\Delta^2} y_0 = [x_0, x_1, x_2]$$

$$= \frac{[x_1, x_2] - [x_0, x_1]}{x_2 - x_0}$$

$$\boxed{\underset{x_1, x_2}{\Delta^2} y_0 = \frac{\underset{x_2}{\Delta} y_1 - \underset{x_1}{\Delta} y_0}{x_2 - x_0}}$$

Similarly $x_1, x_2 ; x_3$

$$\underset{x_2, x_3}{\Delta^2} y_1 = [x_1, x_2, x_3]$$

$$= \frac{[x_2, x_3] - [x_1, x_2]}{x_3 - x_1}$$

$$\boxed{\underset{x_2 x_3}{\Delta^2} y_1 = \frac{\underset{x_3}{\Delta} y_2 - \underset{x_2}{\Delta} y_1}{x_3 - x_1}}$$

Third divided difference for x_0, x_1, x_2, x_3

$$\underset{x_1, x_2, x_3}{\Delta^3} y_0 = \frac{[x_1, x_2, x_3] - [x_0, x_1, x_2]}{x_3 - x_0} = \frac{\underset{x_2, x_3}{\Delta^2} y_1 - \underset{x_1, x_2}{\Delta^2} y_0}{x_3 - x_0}$$

Divided diff table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
x_0	y_0				
		$\frac{y_1 - y_0}{x_1 - x_0} = \underset{x_1}{\Delta} y_0$			
			$\frac{\Delta y_1 - \Delta y_0}{x_2 - x_0} = \Delta^2 y_0$		
x_1	y_1			$\frac{\Delta^2 y_1 - \Delta^2 y_0}{x_3 - x_0} = \Delta^3 y_0$	
		$\frac{y_2 - y_1}{x_2 - x_1} = \Delta y_1$			
			$\frac{\Delta y_2 - \Delta y_1}{x_3 - x_1} = \Delta^2 y_1$		$\frac{\Delta^3 y_1 - \Delta^3 y_0}{x_4 - x_0} = \Delta^4 y_0$
x_2	y_2				
		$\frac{y_3 - y_2}{x_3 - x_2} = \Delta y_2$		$\frac{\Delta^2 y_2 - \Delta^2 y_1}{x_4 - x_1} = \Delta^3 y_1$	
			$\frac{\Delta y_3 - \Delta y_2}{x_4 - x_2} = \Delta^2 y_2$		
x_3	y_3				
		$\frac{y_4 - y_3}{x_4 - x_3} = \Delta y_3$			
x_4	y_4				

Property of the Divided Difference

(1) Divided difference is symmetric
we have, $[x_0, x_1] = \frac{y_1 - y_0}{x_1 - x_0} = \frac{y_0 - y_1}{x_0 - x_1} = [x_1, x_0]$

(2) n^{th} divided difference of a n degree polynomial is constant.

Newton's divided difference interpolation formula

let $y_0, y_1, y_2 --- y_n$ are $(n+1)$ entries corresponding to unequal arguments $x_0, x_1, x_2 --- x_n$, then,

$$y(x) = y_0 + (x - x_0)\Delta y_0 + (x-x_0)(x-x_1)\Delta^2 y_0 + (x-x_0)(x-x_1)(x-x_2)\Delta^3 y_0 + --- n.$$

Lagrange's Interpolation formula for unequal intervals

$$y(x) = \frac{(x-x_1)(x-x_2)---(x-x_n)}{(x_0-x_1)(x_0-x_2)---(x_0-x_n)} y_0 + \frac{(x-x_0)(x-x_2)----(x-x_n)}{(x_1-x_0)(x_1-x_2)----(x_1-x_n)} y_1 + \frac{(x-x_0)(x-x_1)----(x-x_n)}{(x_2-x_0)(x_2-x_1)----(x_2-x_n)} y_2 + ----\infty$$

Inverse Lagrange's Interpolation formula

It is used if we obtein value of x corresponding to any value of y, then

$$x(y) = \frac{(y-y_1)(y-y_2)----(y-y_n)}{(y_0-y_1)(y_0-y_2)---(y_0-y_n)} x_0 + \frac{(y-y_0)(y-y_2)----(y-y_n)}{(y_1-y_0)(y_1-y_2)---(y_1-y_n)} x_1 + ----$$

Q-1) Evaluate $\Delta^2\left(\frac{5x+12}{x^2+5x+6}\right)$, the interval differencing being unity

Sol^n we have

$$\frac{5x+12}{(x+2)(x+3)} = \frac{A}{x+2} + \frac{B}{x+3}$$

$$\frac{5x+12}{\cancel{(x+2)(x+3)}} = \frac{A(x+3)+B(x+2)}{\cancel{(x+2)(x+3)}}$$

$$5x+12 = A(x+3)+B(x+2) \quad \text{--- (1)}$$

$$\cancel{5x+12 = Ax+3A}$$

put $x+2=0$ in (i)

$$-10+12 = A(-2+3)$$

$$\boxed{A=2}$$

put $x+3=0$ in eq (1)

$$-15+12 = B(-1)$$

$$\boxed{B=3}$$

$$\frac{5x+12}{(x+2)(x+3)} = \frac{2}{x+2} + \frac{3}{x+3}$$

$$\Delta\left(\frac{5x+12}{x^2+5x+6}\right) = \Delta\left(\frac{2}{x+2}\right) + \Delta\left(\frac{3}{x+3}\right)$$

$$= 2\left\{\Delta\left(\frac{1}{x+2}\right)\right\} + 3\left\{\Delta\left(\frac{1}{x+3}\right)\right\}$$

we have $\Delta f(x) = f(x+h) - f(x)$

here $h=1$ (unity)

$$\Delta f(x) = f(x+1) - f(x)$$

$$= 2\left\{\left(\frac{1}{x+3} - \frac{1}{x+2}\right)\right\} + 3\left(\frac{1}{x+4} - \frac{1}{x+3}\right)$$

$$= 2\left(\cancel{\frac{x+2-x-3}{(x+3)(x+2)}}\right) + 3\left(\cancel{\frac{\quad}{(x+4)}}\right)$$

$$\Delta\left(\frac{5x+12}{x^2+5x+6}\right) = \frac{3}{x+4} - \frac{1}{x+3} - \frac{2}{x+2}$$

$$\Delta^2\left(\frac{5x+12}{x^2+5x+6}\right) = 3\Delta\left(\frac{1}{x+4}\right) - \Delta\left(\frac{1}{x+3}\right) - 2\Delta\left(\frac{1}{x+2}\right)$$

Formula

$$\Rightarrow \Delta\left(f(x).g(x)\right) = f(x+h)\,\Delta g(x) + g(x).\Delta f(x)$$

$$\Rightarrow \Delta\left(\frac{f(x)}{g(x)}\right) = \frac{g(x).\Delta f(x) - f(x)\,\Delta g(x)}{g(x).g(x+h)}$$

Q-1 (i) $\Delta\left(e^{ax}\log bx\right)$ (ii) $\Delta\left(\frac{2^x}{(x+1)!}\right)$; $h=1$

Soln (i) $\Delta\left(\underset{I}{e^{ax}}\ \underset{II}{\log bx}\right) = e^{a(x+h)} . \Delta \log bx + \log bx\ \Delta e^{ax}$

$$= e^{a(x+h)}\{\log b(x+h) - \log bx\} + \log bx\{e^{a(x+h)} - e^{ax}\}$$

$$= e^{a(x+h)}\log b(x+h) - e^{ax}\log bx$$

$$= e^{ax}\{e^{ah}\log b(x+h) - \log bx\}$$

(ii) $$\Delta\left(\frac{2^x}{(x+1)!}\right) = \frac{(x+1)!\,\Delta 2^x - 2^x\Delta(x+1)!}{(x+1)!\,(x+2)!}$$

$$= \frac{(x+1)!\,(2^{x+1} - 2^x) - 2^x\left((x+2)! - (x+1)!\right)}{(x+1)!\,(x+2)!}$$

$$= \frac{(x+1)!\,2^{x+1} - \cancel{(x+1)!\,2^x} - 2^x(x+2)! + \cancel{2^x(x+1)!}}{(x+1)!\,(x+2)!}$$

$$= \frac{(x+1)!\,2^{x+1} - 2^x(x+2)!}{(x+1)!\,(x+2)!}$$

$$= \frac{\cancel{(x+1)!}\left(2^{x+1} - 2^x(x+2)\right)}{\cancel{(x+1)!}\,(x+2)!}$$

$$= \frac{2^{x+1} - 2^x(x+2)}{(x+2)!}$$

$$= \frac{2^x(\cancel{2} - x - \cancel{2})}{(x+2)!}$$

$$= \frac{-x2^x}{(x+2)!}$$

Q1 Evaluate n^{th} diff. of (i) $\Delta^n \sin(ax+b)$ (ii) $\Delta^n \cos(ax+b)$

Sol^n

$$\Delta^n \cos(ax+b) = \Delta^{n-1} \Delta \cos(ax+b)$$
$$= \Delta^{n-1}\left\{\cos\{a(x+h)+b\} - \cos(ax+b)\right\}$$
$$= \Delta^{n-1}\left\{2\sin\left(\frac{ax+ah+b+ax+b}{2}\right).\sin\left(\frac{ax+b-ax-ah-b}{2}\right)\right\}$$
$$= \Delta^{n-1}\left\{-2\sin\left(\frac{2ax+2b+ah}{2}\right)\sin\frac{ah}{2}\right\}$$
$$= \Delta^{n-1}(-1)\left\{2\sin\left(ax+b+\frac{ah}{2}\right)\sin\left(\frac{ah}{2}\right)\right\}$$
$$= -2\sin\left(\frac{ah}{2}\right)\Delta^{n-2}.\Delta\sin\left(ax+b+\frac{ah}{2}\right)$$
$$\cancel{= (-1)2\sin\frac{ah}{2}\Delta^{n-2}\left\{\sin\left(a(x+h)+b+\frac{ah}{2}\right)-\sin\left(ax+b+\frac{ah}{2}\right)\right\}}$$
$$= (-1)2\sin\left(\frac{ah}{2}\right)\Delta^{n-2}\Delta\cos\left\{(ax+b)+\left(\frac{ah-\pi}{2}\right)\right\}$$

Q-2 Find the 3rd divided diff. with arguments 2, 4, 9, 10 of the funct^n $f(x) = x^3 - 2x$.

Sol^n

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
2	4	$\frac{56-4}{4-2} = 26$		
4	56	$\frac{711-56}{9-4} = 131$	$\frac{131-26}{9-2} = 15$	$\frac{23-15}{10-2} = 1$
9	711	269	$\frac{269-131}{10-4} = 23$	
10	980			

Q) Prove that 3^{rd} divided diff. of

(i) $\underset{bcd}{\Delta^3}\left(\frac{1}{a}\right) = -\frac{1}{abcd}$

(ii) Show that the n^{th} ~~derivati~~ divided differences $[x_0, x_1, x_2 --- x_n]$ for $U_x = \frac{1}{x}$ is $\frac{(-1)^n}{x_0, x_1, x_2 --- x_n}$

Soln

x	$y=\frac{1}{x}$	Δy	$\Delta^2 y$	$\Delta^3 y$
a	$\frac{1}{a}$			
		$\frac{\frac{1}{b}-\frac{1}{a}}{b-a} = \frac{-1}{ab}$		
b	$\frac{1}{b}$		$\frac{\frac{1}{ab}-\frac{1}{bc}}{c-a} = \frac{1}{abc}$	
		$\frac{\frac{1}{c}-\frac{1}{b}}{c-b} = \frac{-1}{bc}$		$\frac{\frac{1}{bcd}-\frac{1}{abc}}{d-a} = \frac{-1}{abcd}$
			$\frac{\frac{1}{bc}-\frac{1}{cd}}{d-b} = \frac{1}{bcd}$	
c	$\frac{1}{c}$	$\frac{\frac{1}{d}-\frac{1}{c}}{d-c} = \frac{-1}{cd}$		
d	$\frac{1}{d}$			

Q) Using Newton's divided diff. formula find a polynomial funcn satisfying the following data

x:	-4	-1	0	2	5
$f(x)$:	1245	33	5	9	1335

Hence find $f(1)$.

Soln

x	$f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
-4	1245				
		-404			
-1	33		94		
		-28		-14	
0	5		10		3
		2		13	
2	9		88		
		442			
5	1335				

Newton's divided diff. interpolation formula

$$y(x) = y_0 + (x-x_0)\Delta y_0 + (x-x_0)(x-x_1)\Delta^2 y_0 + (x-x_0)(x-x_1)(x-x_2)\Delta^3 y_0$$
$$+ (x-x_0)(x-x_1)(x-x_2)(x-x_3)\Delta^4 y_0$$

$$f(x) = 1245 + (x+4)(-404) + (x+4)(x+1)94 + (x+4)(x+1)(-14x)$$
$$+(x+4)(x+1)(x)(x-2)3$$

$$f(x) = 1245 - 404x - 1616 + 94(x^2+5x+4) - 14x(x^2+5x+4)$$
$$+ 3(x^2-2x)(x^2+5x+4)$$

$$f(x) = 1245 - 404x - 1616 + 94x^2 + 470x + 376 - 14x^3 - 70x^2 - 56x$$
$$+ 3(x^4+5x^3+4x^2-2x^3-10x^2-8x)$$

$$f(x) = 3x^4 - 5x^3 + 6x^2 - 14x + 5$$

Q1 Given the following table, find f(x) as a polynomial in powers of $(x-5)$

x:	0	2	3	4	7	9
$f(x)$:	4	26	58	112	466	922

Sol

x	$f(x)$	$\Delta f(x_0)$	$\Delta^2 f(x_0)$	$\Delta^3 f(x_0)$	$\Delta^4 f(x_0)$	$\Delta^5 f(x_0)$
0	4					
		11				
2	26		7			
		32		1		
3	58		11		0	
		54		1		0
4	112		16		0	
		118		1		
7	466		22			
		228				
9	922					

Newton's divided diff. interpolation formula

$$y(x) = y_0 + (x-x_0)\Delta y_0 + (x-x_0)(x-x_1)\Delta^2 y_0 + (x-x_0)(x-x_1)(x-x_2)\Delta^3 y_0$$

$$y(x) = 4 + 11x + 7x(x-2) + x(x-2)(x-3)$$
$$= 4 + 11x + 7x^2 - 14x + x(x^2 - 3x - 2x + 6)$$
$$= 4 + 7x^2 - 3x + x^3 - 5x^2 + 6x$$
$$y(x) = x^3 + 2x^2 + 3x + 4$$

To express it in power of $(x-5)$ we use synthetic division method

$$y(x) = x^3 + 2x^2 + 3x + 4$$

5	1	2	3	4
		+5	+35	+190
5	1	7	38	D=194
		+5	+60	
5	1	12	C=98	
		+5		
	1	B=17		
	A=1			

$$f(x) = (x-5)^3 + 17(x-5)^2 + 98(x-5) + 194$$

Q1 Compute the value of $f(x)$ for $x = 2.5$ from the following table using lagrange's interpolation formula.

x:	1	2	3	4
$f(x)$:	1	8	27	64

Sol^n^ $$f(x) = \frac{(x-2)(x-3)(x-4)}{(1-2)(1-3)(1-4)}1 + \frac{(x-1)(x-3)(x-4)}{(2-1)(2-3)(2-4)}8 + \frac{(x-1)(x-2)(x-4)}{(3-1)(3-2)(3-4)}27$$

$$+ \frac{(x-1)(x-2)(x-3)}{(4-1)(4-2)(4-3)}64$$

~~$f(x) = (2.5-2)(2.5-3)(2.5-4)$~~

put $x = 2.5$

$$f(x) = \frac{0.5(-0.5)(-1.5)}{(-1)(-2)(-3)} + \frac{(1.5)(-0.5)(-1.5)}{(-1)(-2)} \times 8 + \frac{(1.5)(0.5)(-1.5)}{2\times(-1)} \times 27$$

$$+ \frac{(1.5)(0.5)(-0.5)}{3\times 2} \times 64$$

$f(x) = -0.0625 + 4.5 + 15.1875 - 4$

$f(x) = 15.625$