

Books:

- i) Digital Logic Design - moris mano
- ii) Digital Circuit Design - S. Salivahanan.
- iii) Digital Circuit - Anand Kumar.

* Number System:

Number systems are used to represent information or in various formats. in general following four types of system are used.

- i) Binary
- ii) Octal
- iii) Decimal
- iv) Hexadecimal.

* Binary : It has ten numbers 0 and 1 and its Base or radix is 2.

* Octal : It has eight numbers 0 to 7 and its Base is 8.

* Decimal : It has ten numbers 0 to 9 and its Base or Radix is 10.

* Hexadecimal : It has 16 numbers 0 to 9 and six alphabets A to F where

$$A = 10$$

$$C = 12$$

$$E = 14$$

$$B = 11$$

$$D = 13$$

$$F = 15$$

if Base is not there so it is by default in decimal.

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* Dec to Bin :

$$(11)_{10} \Rightarrow (?)_2$$

2	11	1	↑
2	5	1	
2	2	0	
	1		

$$\Rightarrow (1011)_2$$

$$(17)_{10} \Rightarrow (?)_2$$

2	17	1
2	8	0
2	4	0
2	2	0
	1	

$$\Rightarrow (10001)_2$$

$$\Rightarrow \begin{array}{cccccc} 2^4 & 2^3 & 2^2 & 2^1 & 2^0 & \\ 16 & 8 & 4 & 2 & 1 & \end{array}$$

$$11$$

$$11 \Rightarrow$$

$$32 \quad 16 \quad 8 \quad 4 \quad 2 \quad 1$$

$$(1011)_2$$

$$17 \Rightarrow \quad 16 \quad 8 \quad 4 \quad 2 \quad 1$$

$$(10001)_2 \text{ Ans}$$

* Binary $\rightarrow$ Decimal :

(32) 16 (8) 4 (2) 1
1 0 1 1 0 1

write code

= 45 Ans.

Q. $(2 \overset{2}{3} \overset{2}{4})_5$

$(2 \ 1 \ 2)_5$

$(4 \ 7 \ 4)_5$

$(2 \ 0 \ 0 \ 0)_5$

10

$\frac{5}{5}$

$\frac{5}{5}$

$\frac{5}{5}$

0

2 times carry

Q. Write Binary equivalent of the Number from 0 to 15

$A^{\frac{16}{2}}$ $B^{\frac{8}{2}}$ $C^{\frac{4}{2}}$ $D^{\frac{2}{2}}$

0 = 0

0 0 0 0

1 = 1

1 0 0 0

2 = 10

2 0 0 1

3 = 11

3 0 0 1

4 = 1000

4 0 1 0 0 0

5 = 101

5 0 1 0 1 0

6 = 110

6 0 1 1 0 0

7 = 111

7 0 1 1 1 0

8 = 1000

8 1 0 0 0 0

9 = 1001

9 1 0 0 1 0

10 = 1010

10 1 0 1 0 0

11 = 1011

11 1 0 1 1 0

12 = 1100

12 1 1 0 0 0

13 = 1101

13 1 1 0 1 0

14 = 1110

14 1 1 1 0 0

15 = 1111

15 1 1 1 1 0

Negative Dec. to Bin:

$$-8 \quad 32 \quad 16 \quad 8 \quad 4 \quad 2 \quad 1$$

$$1 \quad 0 \quad 0 \quad 0$$

$$(1000)$$

Then 1st complement

$$= 0111$$

Number System Convergence:

(1) Convergence from any Base to Decimal:

$$(i) \quad (ABC)_x = (?)_{10}$$

$$Ax^2 + Bx^1 + Cx^0$$

$$(ii) \quad (ABC.DE)_x = (?)_{10}$$

$$Ax^2 + Bx^1 + Cx^0 + Dx^{-1} + Ex^{-2}$$

Que: Convert the following Number to Decimal:

$$(i) \quad (124.31)_5$$

$$= 1 \times 5^2 + 2 \times 5^1 + 4 \times 5^0 + 3 \times 5^{-1} + 1 \times 5^{-2}$$

$$= (39.64)_{10}$$

$$\begin{aligned}
 & \text{(ii) } (A.3B.12)_{16} \\
 & = 10 \cdot 16^2 + 3 \cdot 16^1 + 11 \cdot 16^0 + 1 \cdot 16^{-1} + 2 \cdot 16^{-2} \\
 & = 2619.0703125_{10} \text{ Ans}
 \end{aligned}$$

(2) Conversion from Decimal to Any Base:

$$(ABC)_{10} = (?)_x$$

x	ABC	R_1	↑
x	Q_1	R_2	
x	Q_2	R_3	
x	Q_3	Q_3	
	0		

$= (Q_3 R_3 R_2 R_1)_x$

$$(ABC.DE)_{10} = (?)_x$$

x	ABC	↑

write it

	$.DE \times x$
	0.80×4
	3.20
	$.20 \times 4$
	0.80
0	0.80×4
	3.20
3	0.20×4
1	1.00

Q. Convert the following numbers.

i) $(246.91)_{10} = (?)_2 = (?)_4 = (?)_8$

ii) $(469)_{10} = (?)_6 = (?)_{16}$

Solⁿ: i) $(246.90)_{10} = (?)_2$

2	246	0	↑
2	123	1	
2	61	1	
2	30	0	
2	15	1	
2	7	1	
2	3	1	
2	1	1	

$$0.90 \times 2 = 1.80$$

$$1 \quad 0.80 \times 2 = 1.60$$

$$1 \quad 0.60 \times 2 = 1.20$$

$$1 \quad 0.20 \times 2 = 0.40$$

$$0$$

$$(11110110.1110)_2$$

$$(246.90)_{10} = (?)_4$$

4	246	2
4	61	1
4	15	3
4	3	3
4	0	0

* Conversion from Binary to any Base that can be represented as 2^n

$$(01011010)_2 = (?)_4 \quad 4 = 2^2 \quad \text{group/pair}$$

$$(11 \cdot 2 \cdot 2)_4$$

$$(110101101110)_2 = (?)_8 \quad 8 = 2^3 \quad 3 \text{ pair}$$

$$(6556)_8$$

$$(011011010110)_2 = (?)_{16} \quad 16 = 2^4 \quad 4 \text{ group}$$

$$(6D56)_{16}$$

* Conversion from Octal to Hexadecimal and Vice-versa
Both related Binary:
Octal $\rightarrow$ Bin $\rightarrow$ Hexadecimal

Q. Conversion the following as indicated:

i) $(345.24)_8 = (?)_{16}$

ii) $(AC9.2B)_{16} = (?)_8$

(1) $(345.24)_8 = (011100101.010100)_2$

$$(011100101.01010000)_2 = (?)_{16}$$

$$= (0E9.90)_{16}$$

$$(2) \quad (AC9 \cdot 2B)_{16} = (?)_8$$

$$(101011001001,000110110)_2$$

Imp:

* Conversion from any Random Base to any another Random Base:

Imp:

Apply to All

$$\text{Rand. B} \Rightarrow \text{Dec} \Rightarrow \text{Rand. Base}$$

Que: Convert the following numbers as indicated.

$$(i) \quad (423 \cdot 13)_5 \rightarrow (?)_6 \rightarrow (?)_7$$

$$(4 \cdot 5^2 + 2 \cdot 5^1 + 3 \cdot 5^0 + 1 \cdot 5^{-1} + 3 \cdot 5^{-2})_{10}$$

$$(4 \times 25 + 10 + 3 + \frac{1}{5} + \frac{3}{25})_{10}$$

$$(100 + 13 + \frac{5+3}{25})_{10}$$

$$(113 + \frac{8}{25})_{10} = (113 + 0.32)_{10}$$

$$= (113.32)_{10}$$

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$$(113.32)_{10} \rightarrow (?)_6$$

6	113	5
6	13	0
3	3	0
	1	

$$0.32 \times 6$$

$$1.92$$

$$0.92 \times 6 = 1.52$$

$$0.52 \times 6 = 3.12$$

$$0.12 \times 6 = 0.72$$

$$0.72$$

1
1
3
0

$$(1005.1130)_6 \text{ Ans}$$

(5 marks)

* Complement method of Subtraction :

In Subtraction of Numbers following two complement methods are used :

- 1) $(r-1)$'s Comp. method
- 2) r 's Comp. method.

Where r denotes the Base or radix.

1) $(r-1)$'s Comp. method :

Let a number B is to be subtracted from another no. A then following steps are used -

Step-1) Take $(r-1)$'s Comp. of the number to be subtracted (i.e. B).

Step-2) add $(r-1)$'s Comp. of number B to A

Step-3) if a carry is generated and around carry
 → if $A > B$ then a carry will be generated at and end around carry.

if $A < B$ then no carry will be generated.

take $(r-1)$'s Comp. of the result and the

Put the negative sign to obtain the final result

2) r 's Comp. method :

Let a no. B is to be subtracted from another no. A then following steps are used

Step-1) Take r 's Comp. of the no. to be subtracted (i.e. B)

Step-2) Add r 's Comp. of number B to A

Step-3) if $A > B$ then a carry will be generated and discard carry

if $A < B$ then no carry will be generated.

Note: The number of digits in the two no. to be subtracted must always be equal.

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take r 's comp. of the result and put the negative sign to obtain the final result.

Comp.

Q Find the respective complement of the following numbers.

(i) $(10101)_2$ $(r-1)$'s r 's
 01010 01011

(ii) $(431)_5$ $(r-1)$'s r 's
 (013) 014

(iii) $(745.23)_8$ $(r-1)$'s r 's
 032.54 032.55

(iv) $(971)_{10}$ $(r-1)$'s r 's
 028 029

(v) $(ABCD5)_{16}$ $(r-1)$'s r 's
 54310 54311

Que. 1 Subtract the following numbers by using the respective complement methods.

(i) $(1101)_2 - (101)_2$

(ii) $(101)_2 - (1101)_2$

(iii) $(926)_{10} - (476)_{10}$

(iv) $(476)_{10} - (926)_{10}$

(v) $(431)_5 - (341)_5$

$$(i) \quad (1101)_2 - (0101)_2$$

using 1's Comp. method

$$1's \text{ comp of } (0101)_2 = (1010)_2$$

adding 1's comp of $(0101)_2 + (1101)_2$

$$\begin{array}{r} 1101 \\ + 1010 \\ \hline \end{array}$$

$$\begin{array}{r} 1101 \\ + 1010 \\ \hline 10111 \end{array}$$

$\rightarrow +1$ end around carry

$$(1000)_2$$

$$\Rightarrow (1101)_2 - (0101)_2 = (1000)_2 \text{ Ans}$$

$$(ii) \quad (0101)_2 - (1101)_2$$

using 1's Comp. method

$$1's \text{ comp of } (1101)_2 = (0010)_2$$

adding 1's comp of $(1101)_2$ to $(0101)_2$

$$\begin{array}{r} 0101 \\ + 0010 \\ \hline 0111 \end{array}$$

$$\text{again find } 1's \text{ comp of } (0111)_2 = 1000$$

and put negative sign.

$$-(1000)_2$$

$$(iii) \quad (926)_{10} - (476)_{10}$$

using 9's complement method.

$$9's \text{ comp of } (476)_{10} = (523)_{10}$$

add 9's comp of $(476)_{10}$ to $(926)_{10}$

$$\begin{array}{r} 926 \\ + 523 \\ \hline 1449 \\ \xrightarrow{+1} \text{end around carry} \\ \hline (450)_{10} \end{array}$$

$$\Rightarrow (926)_{10} - (476)_{10} = (450)_{10}$$

$$(iv) \quad (476)_{10} - (926)_{10}$$

using 9's comp. method

$$9's \text{ Comp. of } (926)_{10} = (073)_{10}$$

add 9's comp. of $(926)_{10}$ to $(476)_{10}$

$$\begin{array}{r} 476 \\ + 073 \\ \hline (549)_{10} \end{array}$$

then again 1's complement and negative sign.

$$-(450)_{10} \text{ Ans}$$

- ① why 9's comp. is first
- ② carry generate add or discard.

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(v) $(431)_5 - (341)_5$

using 4's comp. method

4's comp. of $(341)_5$

using 8's comp. method:

(i) $(1101)_2 - (0101)_2$

using 2's comp. method

2's comp. of $(0101)_2$ is $(1011)_2$

add 2's comp. in $(1101)_2$

$$\begin{array}{r} 1101 \\ + 1011 \\ \hline 10000 \end{array} = (1000)_2$$

~~1000~~ 1000

(ii) $(0101)_2 - (1101)_2$

using 2's comp. method

2's comp. of $(1101)_2$ is $(0011)_2$

add 2's comp. in $(0101)_2$

$$\begin{array}{r} 0100 \\ + 0011 \\ \hline (0111)_2 \end{array}$$

take 2's comp. is $(1001)_2$

final result is $-(1001)_2$

- (iii) $(926)_{10} - (476)_{10}$
 using 10's comp. method
 10's comp. of $(476)_{10} = (524)_{10}$
 add 10's comp. in $(926)_{10}$

$$\begin{array}{r} 926 \\ + 524 \\ \hline 1450 \end{array}$$

discarded carry $\rightarrow$ $(450)_{10}$ Ans

- (iv) $(476)_{10} - (926)_{10}$
 using 10's comp. meth.
 10's comp of $(926)_{10} = (074)_{10}$
 add 10's comp. in $(476)_{10}$

$$\begin{array}{r} 476 \\ + 074 \\ \hline 550 \end{array}$$

Again 10's comp of $(550)_{10} = (450)_{10}$

result = $(476)_{10} - (450)_{10}$

- (v) $(431)_5 - (341)_5$
 using 5's comp. method
 5's comp of $(341)_5 = (104)_5$
 add 10's comp of $(341)_5$ in $(431)_5$

$$\begin{array}{r} 431 \\ + 104 \\ \hline 1040 \end{array}$$

Discard carry $\rightarrow$ $(040)_5$ Ans

Q Subtract the following no.s by using 1's and 2's comp

$$(29)_{10} - (18)_{10}$$

$$(18)_{10} - (29)_{10}$$

Solⁿ: $(29)_{10} - (18)_{10}$

using 2's comp

$$(29)_{10} = (11101)_2$$

32 16 8 4 2 1

$$(18)_{10} = (10010)_2$$

$$(11101)_2 - (10010)_2$$

using 2's comp.

$$2's \text{ comp. of } (10010)_2 = (01101)_2$$

add 2's comp in $(11101)_2$

$$\begin{array}{r} 11101 \\ + 01101 \\ \hline (101110)_2 \\ \xrightarrow{+1} \\ \hline (10000)_2 \end{array}$$

$$(10000)_2 \text{ Ans}$$

$$\begin{array}{r} 11101 \\ + 01101 \\ \hline (101010)_2 \\ \xrightarrow{+1} \\ \hline (01011)_2 \end{array}$$

$$\Rightarrow (01011)_2$$

$$(11101)_2 - (10010)_2 = (01011)_2$$

$$(29)_{10} - (18)_{10} = (11)_{10}$$

$$(18)_{10} - (29)_{10} =$$

using 1's Comp.

$$(11101)_2 - (10010)_2$$

1's comp. of $(10010)_2$ is $(\cancel{0}1101)_2$

add 1's comp in $(11101)_2$

$$\begin{array}{r} 11101 \\ + 01101 \\ \hline 01010 \end{array}$$

→ +1

$$\underline{(01011)_2}$$

$$(10010)_2 - (11101)_2$$

1's comp of $(11101)_2$ is $(00010)_2$

add 1's comp in $(10010)_2$

$$\begin{array}{r} 10010 \\ + 00010 \\ \hline 10100 \end{array}$$

Since no carry generated therefore 1's comp of the result

$$(10100)_2 \text{ 1's is } (01011)_2$$

$$= -(01011)_2$$

$$(10010)_2 - (11101)_2 = -(01011)_2$$

$$(18)_{10} - (29)_{10} = -(11)_{10}$$

7/8/19

* Boolean Algebra :

Boolean Algebra is used to represent the relationship b/w inputs and Output in Algebraic form.

* Truth table :

It represents the tabular representation of the relationship b/w inputs and Outputs. It has two columns. One for input another for outputs. A truth table can be of any of the following types

- i) 2 - variable
- ii) 3 - variable
- iii) 4 - variable
- iv) 5 - variable

2 variable = 4 comb.

3 variable = 8 comb.

2 variable			3 variable			
I/P			O/P			
A	B		A	B	C	O/P
0	0	0	0	0	0	0
0	0	1	0	0	1	1
0	1	0	0	1	0	0
0	1	1	0	1	1	1
1	0	0	1	0	0	0
1	0	1	1	0	1	1
1	1	0	1	1	0	0
1	1	1	1	1	1	1

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4 variable $\Rightarrow$ 16 combination

	I/P				O/P
	A	B	C	D	
0	0	0	0	0	
1	0	0	0	1	
2	0	0	1	0	
3	0	0	1	1	
4	0	1	0	0	
5	0	1	0	1	
6	0	1	1	0	
7	0	1	1	1	
8	1	0	0	0	
9	1	0	0	1	
10	1	0	1	0	
11	1	0	1	1	
12	1	1	0	0	
13	1	1	0	1	
14	1	1	1	0	
15	1	1	1	1	

Imp.

Boolean Expression :

In order to write Boolean Expression from a truth table a variable is represented either as $\bar{A}$ or as A .

A normal form $\bar{A}$ complemented form

The boolean expressions are of following two types.

- i) Sum of products (SOP) form
- ii) Products of sum (POS) form.

* SOP form :

This form is obtained by adding the Product terms. The terms of SOP form are called as "MINTERMS".

Example of SOP form are

$$Y = \bar{A}B + A\bar{B} + A\bar{B}\bar{C}$$

$$Y = \bar{A}\bar{B} + A + \bar{B}C$$

The value of Output at minterms is always a logic '1'

* POS form :

The Product of Sum form is obtained by multiplying the Sum terms. The terms of a POS form are called as "MAXTERMS".

Examples of POS form are -

$$Y = (A + \bar{B})(A + B)$$

$$Y = (\bar{A} + B)(\bar{A} + \bar{C})$$

The value of output at maxterms is always a logic '0'.

* Expression from truth table :

$$\text{SOP} \Rightarrow A = 1$$

$$\bar{A} = 0$$

$$\text{POS} \Rightarrow A = 0$$

$$\bar{A} = 1$$

	A	B	Y
0	0	0	0
1	0	1	1
2	1	0	0
3	1	1	1

SOP = max term

POS = min term

$$\text{SOP} \Rightarrow Y = \bar{A}B + AB$$

$$\text{POS} \Rightarrow Y = (A+B) \cdot (\bar{A}+B)$$

Ques: Write Boolean Expression in SOP and POS for the following truth table.

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	1
6	1	1	0	0
7	1	1	1	1

$$\text{SOP} \Rightarrow \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}C + ABC$$

$$\text{POS} = (A+B+C)(A+\bar{B}+C)(\bar{A}+B+C)(\bar{A}+\bar{B}+C)$$

Σ	A	B	C	D	Y
0	0	0	0	0	0
1	0	0	0	1	1
2	0	0	1	0	0
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	1
6	0	1	1	0	0
7	0	1	1	1	1
8	1	0	0	0	0
9	1	0	0	1	1
10	1	0	1	0	0
11	1	0	1	1	1
12	1	1	0	0	0
13	1	1	0	1	1
14	1	1	1	0	0
15	1	1	1	1	1

$$\text{SOP} = \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}CD + \bar{A}B\bar{C}D + \bar{A}BCD + \\ A\bar{B}\bar{C}D + A\bar{B}CD + ABC\bar{D} + ABCD$$

$$\text{POS} = (A+B+C+D)(A+B+\bar{C}+D)(A+\bar{B}+C+D) \\ (A+\bar{B}+\bar{C}+D)(\bar{A}+B+C+D)(\bar{A}+B+\bar{C}+D) \\ (\bar{A}+\bar{B}+C+D)(\bar{A}+\bar{B}+\bar{C}+D)$$

* Truth table from Boolean Express:

$$Y = AB + \bar{A}B$$

min term = 1

	A	B	Y
0	0	0	0
1	0	1	1
2	1	0	0
3	1	1	1

$$Y = A + \bar{A}B$$

Canonical

max term = 0

	A	B	Y
0	0	0	0
1	0	1	1
2	1	0	0
3	1	1	1

* Canonical form:

It is a type of Boolean form in which each term has each possible variables.

Sol

$$Y = AB + \bar{A}C + ABC$$

$$Y = AB(C + \bar{C}) + \bar{A}C(B + \bar{B}) + ABC$$

$$= ABC + AB\bar{C} + \bar{A}BC + \bar{A}\bar{B}C + ABC$$

$$\begin{matrix} 111 & 110 & 011 & 001 & 110 \end{matrix}$$

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	1
7	1	1	1	1

POS:

$$\begin{aligned}
 Y &= A(A+B) \\
 &= (A+B\bar{B})(A+\bar{B}) \\
 &= (A+B)(A+\bar{B})(A+\bar{B}) \\
 &= (A+B)(A+\bar{B})
 \end{aligned}$$

	A	B	Y
0	0	0	0
1	0	1	0
2	1	0	1
3	1	1	1

Ques Obtain the truth table for the following boolean Express.

(i) $Y = AB + AC + C$ (SOP)

(ii) $Y = (A+B)(A+\bar{C})(B)$

$$(i) Y = \bar{A}B + A\bar{C} + C$$

$$Y = \bar{A}B(C + \bar{C}) + A\bar{C}(B + \bar{B}) + C(A + \bar{A})(B + \bar{B})$$

$$= \bar{A}BC + \bar{A}B\bar{C} + A\bar{B}C + A\bar{B}\bar{C} +$$

$$+ ABC + A\bar{B}C + A\bar{B}C + A\bar{B}\bar{C}$$

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	1
3	0	1	1	1
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	1

$$(ii) Y = (\bar{A} + B)(A + \bar{C})(B)$$

$$= (\bar{A} + B + (C + \bar{C}))(A + \bar{C} + (B + \bar{B}))(B + (A + \bar{A}) + (C + \bar{C}))$$

$$= (\bar{A} + B + C)(\bar{A} + B + \bar{C})(A + B + \bar{C})(A + \bar{B} + \bar{C})$$

$$(\bar{A} + B + C)(\bar{A} + B + \bar{C})(A + B + \bar{C})(A + \bar{B} + \bar{C})$$

	A	B	C	Y
0	0	0	0	0
1	0	0	1	0
2	0	1	0	1
3	0	1	1	0
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	1

A

Q.1 Find min terms for the following Boolean Expression (30P)

$$Y = \bar{A}B + A\bar{C} + B\bar{C}\bar{D}$$

$$Y = \bar{A}B + A\bar{C} + B\bar{C}\bar{D}$$

$$= \bar{A}B(C + \bar{C})(D + \bar{D}) + A\bar{C}(B + \bar{B})(D + \bar{D}) + B\bar{C}\bar{D}(A + \bar{A})$$

$$= \bar{A}B(C + \bar{C})D + \bar{A}B(C + \bar{C})\bar{D} + A\bar{C}(B + \bar{B})D + A\bar{C}(B + \bar{B})\bar{D}$$

$$+ B\bar{C}\bar{D}A + B\bar{C}\bar{D}\bar{A}$$

$$= \bar{A}B\bar{C}D + \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}\bar{D} + A\bar{B}\bar{C}\bar{D} + A\bar{B}\bar{C}\bar{D}$$

$$\begin{array}{cccccc} 0111 & 0101 & 0110 & 0100 & 1101 & 1001 \\ 7 & 5 & 6 & 4 & 13 & 9 \end{array}$$

$$+ A\bar{B}\bar{C}\bar{D} + A\bar{B}\bar{C}\bar{D} + B\bar{C}\bar{D}A + B\bar{C}\bar{D}\bar{A}$$

$$\begin{array}{cccc} 1100 & 1000 & 1101 & 1100 \\ 12 & 8 & 13 & 12 \end{array}$$

$$7, 6, 5, 4, 13, 2, 9, 8, 14$$

$$Y = \sum m(4, 5, 6, 7, 8, 9, 12, 13, 14)$$

Q.2 Obtain max terms for the following Expression

$$Y = \bar{A}B + A\bar{C} + B\bar{C}\bar{D}$$

$$Y = \prod M(0, 2, 3, 10, 11, 15)$$

$$y = \bar{y}$$

Ques

Same

Dated: _____

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Obtain the complemented / inverse / reverse form of the following expression

SOP $\Rightarrow$ POS

$$Y = (A\bar{B} + \bar{A}C)$$

Convert SOP $\rightarrow$ POS
POS $\rightarrow$ SOP

$$\overline{A+B} \Rightarrow \text{for open bar}$$

$$+ \rightarrow \cdot \quad A \rightarrow \bar{A}$$

$$\cdot \rightarrow + \quad \bar{A} \rightarrow A$$

$$Y = \overline{(A\bar{B} + \bar{A}C)}$$

$$= \overline{(A\bar{B}) \cdot (\bar{A}C)}$$

$$= (\bar{A}+B) \cdot (A+\bar{C})$$

POS $\Rightarrow$ SOP

$$Y = (\bar{A}+B) \cdot (A+\bar{C}) \cdot (\bar{A}+\bar{B})$$

$$= \overline{(\bar{A}+B)} + \overline{(A+\bar{C})} + \overline{(\bar{A}+\bar{B})}$$

$$= (A \cdot \bar{B}) + (\bar{A} \cdot C) + (A \cdot B)$$

* Logic gates : or design
Logic gates are used to implement a digital circuits.

Logic gates are classified into three categories
1) Basic Gates
2) Universal Gates
3) Special purpose Gates.

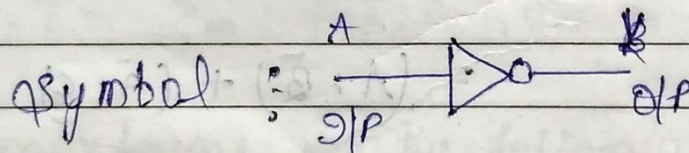
1) Basic Gates :

These are further classified into three categories.

- i) NOT
- ii) AND
- iii) OR

i) NOT Gate :

It is a single input, single output gate used to complement the input applied to it.



truth table :

i/p	o/p
A	Y
0	1
1	0

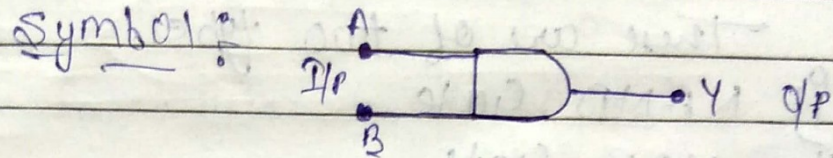
Boolean Expression :

$$Y = \bar{A}$$

(ii) AND Gate : (multiply) output

It is a multiple input single output logic Gate, used to multiply the input applied to it.

Symbol :

Truth table

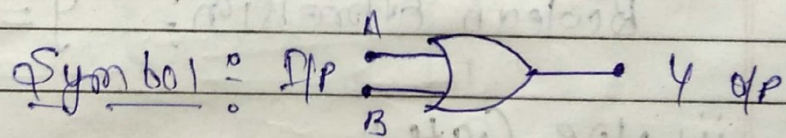
	i/p		o/p	
	A	B	Y	
0	0	0	0	
1	0	1	0	
2	1	0	0	
3	1	1	1	

(iii) Boolean Expression :

$$Y = A \cdot B$$

(iii) OR Gate :

It is a multiple input single output logic Gate, used to add the inputs applied to it.

Truth table

	i/p		o/p	
	A	B	Y	
0	0	0	0	
1	0	1	1	
2	1	0	1	
3	1	1	1	

Boolean Expression : $Y = A + B$

1
0
1
1
Generate
digit.

② Universal Gates:

These gates are used to realise ~~the~~ any Boolean function and i.e. why name of universal.

These are of two types.

- i) NAND Gate
- ii) NOR Gate

i) NAND Gate:

It is a multiple i/p single o/p logic gate. used to perform the complement of AND Gate.

Symbol:  4 o/p

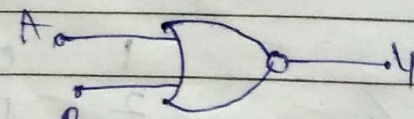
Truth table

	i/p		o/p
	A	B	Y
0	0	0	1
1	0	1	1
2	1	0	1
3	1	1	0

Boolean Expression: $Y = \overline{AB}$

ii) NOR Gate:

It is a multiple i/p single o/p logic gate used to perform the complement of OR Gate.

Symbol:  4 o/p

Truth table:

	A	B	Y
0	0	0	1
1	0	1	0
2	1	0	0
3	1	1	0

Boolean Expression: $Y = (A+B)$ ② Special purpose Gates:

These are of two types:

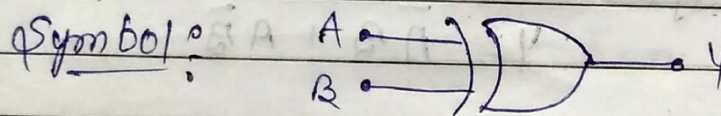
i) XOR Gate

ii) XNOR Gate

i) XOR Gate: $A \oplus B$

It is an two I/P single O/P logic Gate.

Symbol:



Truth table:

	A	B	Y
0	0	0	0
1	0	1	1
2	1	0	1
3	1	1	0

If both I/P is same output = 0

Boolean Expression: $Y = \bar{A}B + A\bar{B}$

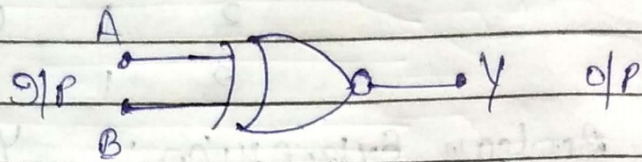
$$= (A \oplus B)$$

XOR Gate.

(iii) XNOR Gate :

It is an two ~~or~~ single op^o Gate.

Symbol :



Truth table :

	A	B	Y
0	0	0	1
1	0	1	0
2	1	0	0
3	1	1	1

same = 1

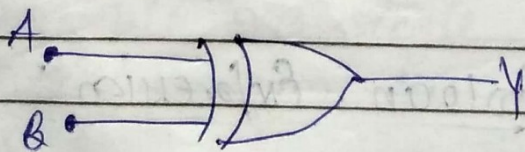
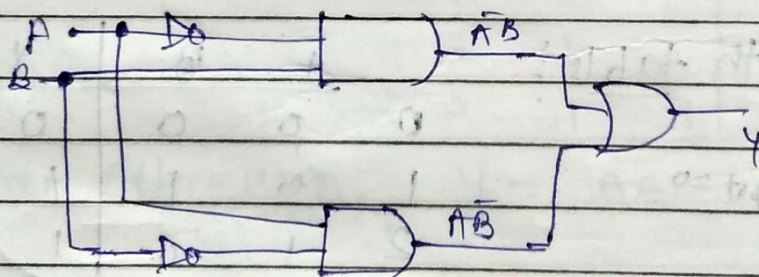
Boolean

Expression $Y = \bar{A} \bar{B} + AB = (A \odot B)$

Q.

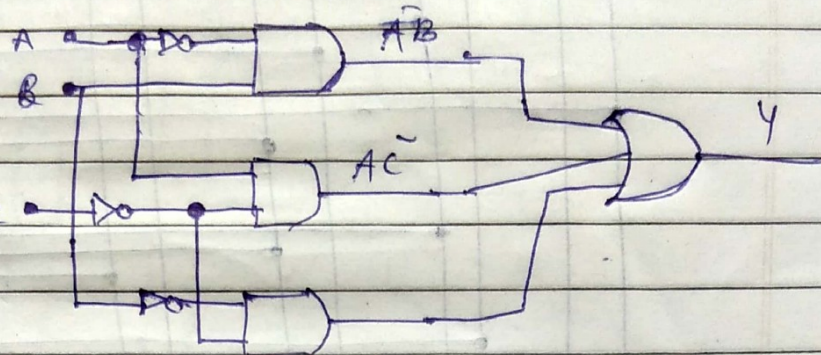
Implement the following function by using Logic gates

$$Y = \bar{A} B + A \bar{B}$$



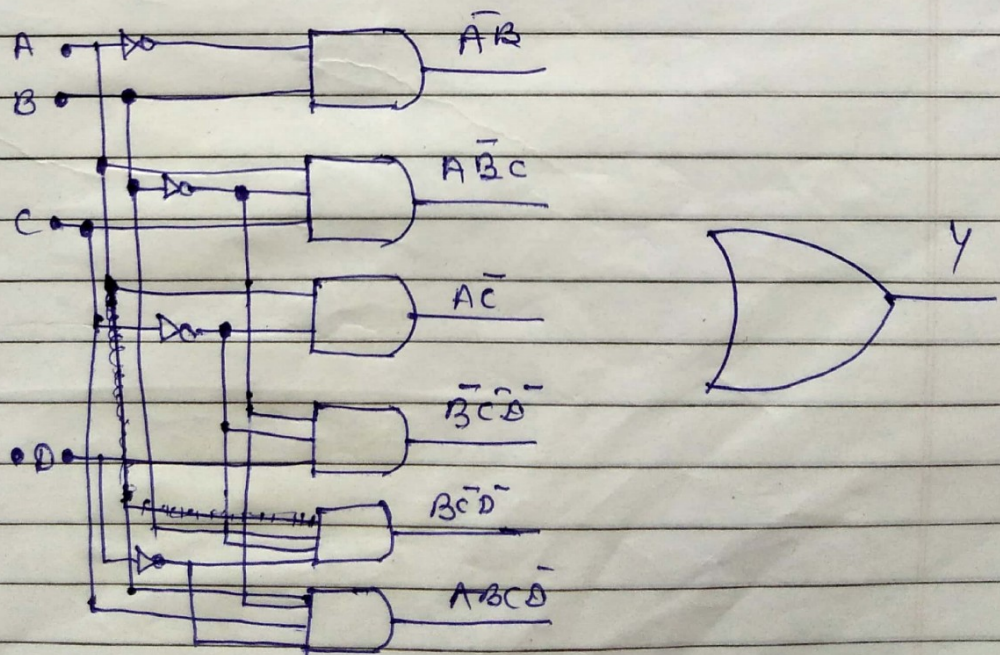
Q. Implement the following function by using logic gates

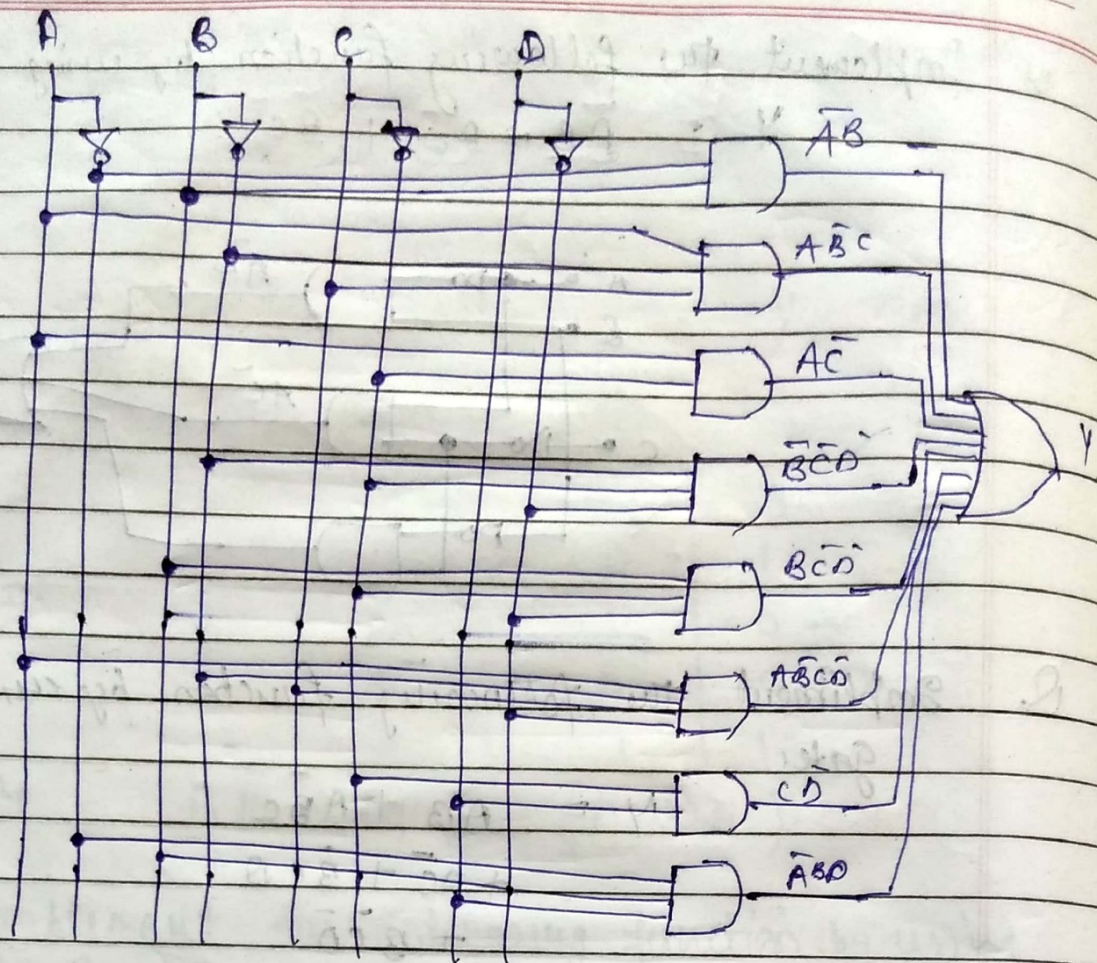
$$Y = \bar{A}B + A\bar{C} + \bar{B}C$$



Q. Implement the following function by using logic gates

$$Y = \bar{A}B + A\bar{B}C + A\bar{C} + \bar{B}\bar{C}\bar{D} + B\bar{C}\bar{D} + A\bar{B}C\bar{D} + \bar{C}\bar{D} + \bar{A}B\bar{D}$$



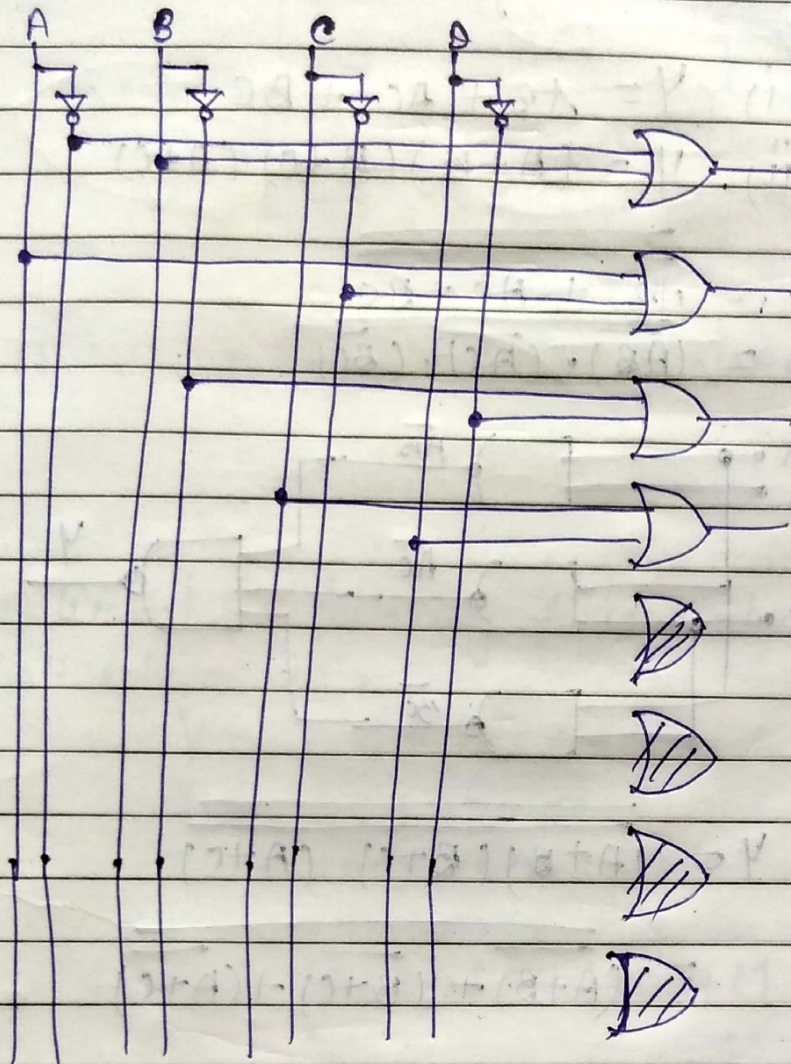


Q.

Dated : _____

Page : _____

Q. $Y = (\bar{A} + B)(A + \bar{C})(\bar{B} + D)(C + D)$



$(\bar{A} + B)(A + \bar{C}) = (\bar{A} \cdot A) + (\bar{A} \cdot \bar{C}) + (B \cdot A) + (B \cdot \bar{C})$

$(\bar{A} \cdot A) + (\bar{A} \cdot \bar{C}) + (B \cdot A) + (B \cdot \bar{C})$

Q. Implement the following function by using NAND Gate only.

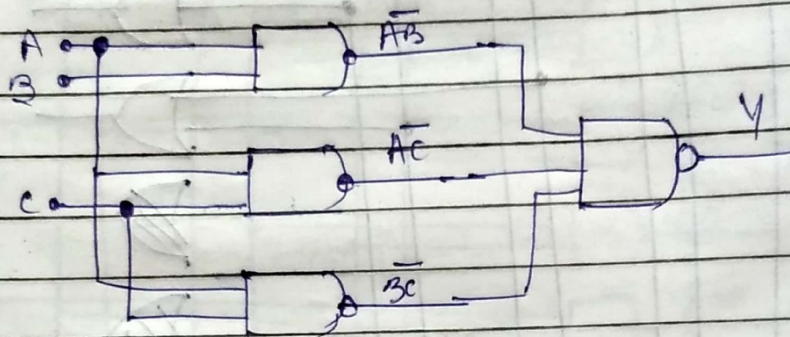
$$(i) Y = AB + AC + BC$$

$$(ii) Y = (A+B)(B+C)(A+C)$$

use
double
Complement

$$(i) Y = \overline{\overline{AB + AC + BC}}$$

$$Y = (\overline{AB}) \cdot (\overline{AC}) \cdot (\overline{BC})$$



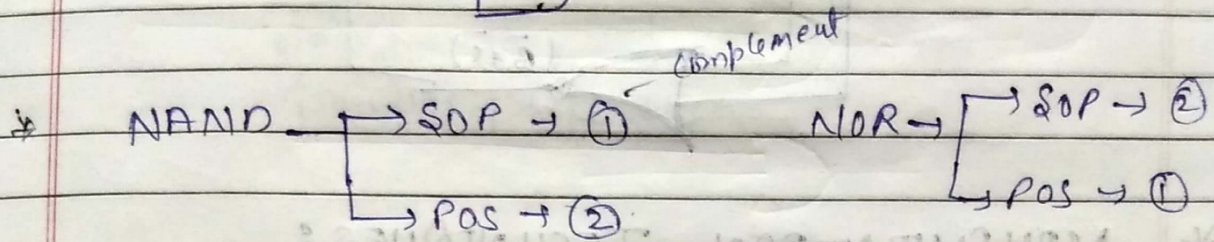
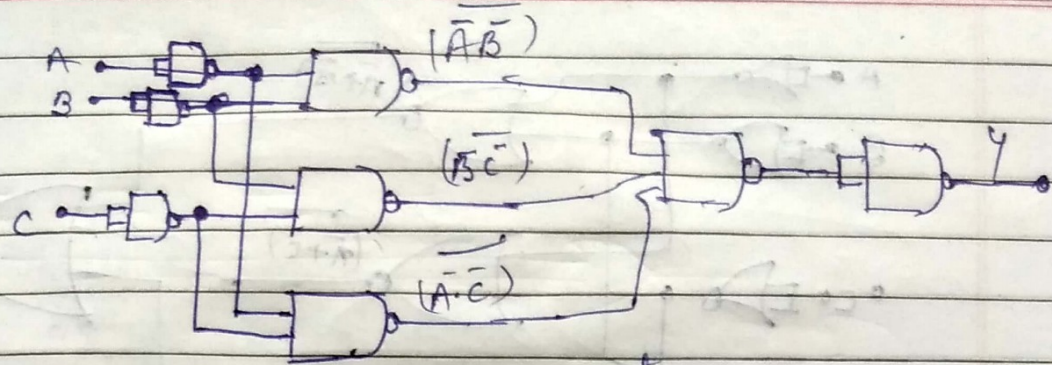
$$(ii) Y = \overline{\overline{(A+B)(B+C)(A+C)}}$$

$$= \overline{(\overline{A+B}) + (\overline{B+C}) + (\overline{A+C})}$$



$$= (\overline{A} \cdot \overline{B}) + (\overline{B} \cdot \overline{C}) + (\overline{A} \cdot \overline{C})$$

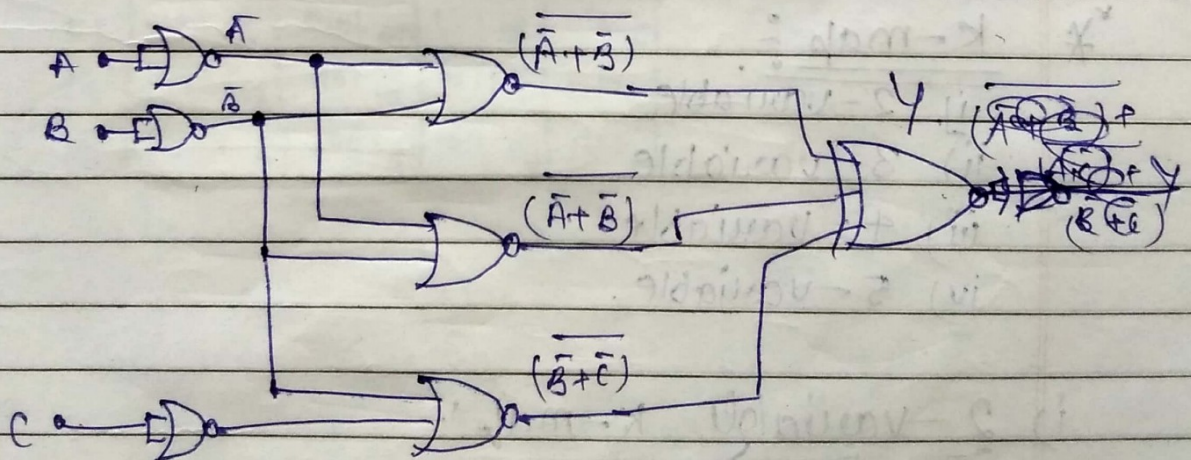
$$= (\overline{A \cdot B}) \cdot (\overline{B \cdot C}) + (\overline{A \cdot C})$$

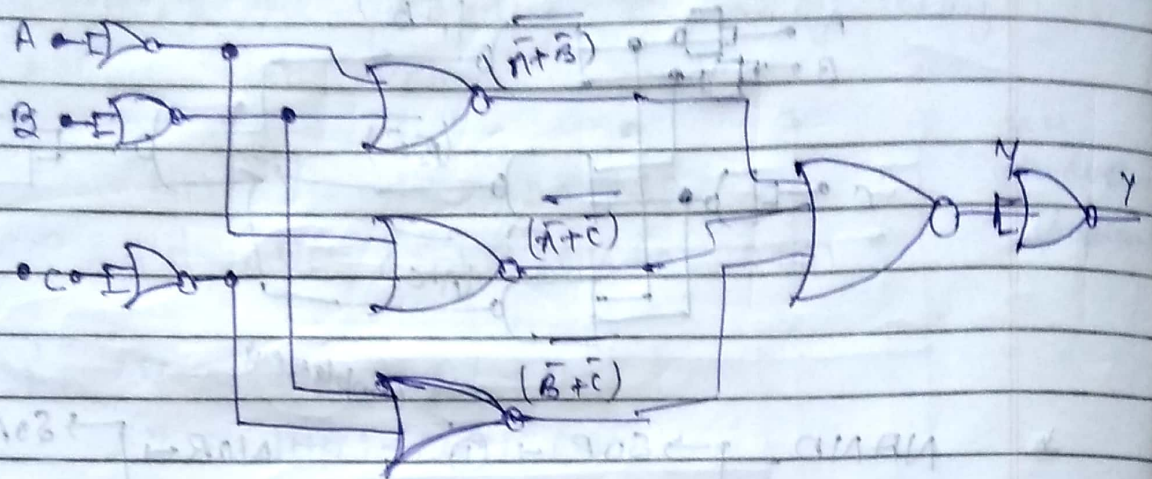


Q. Implement the following function by using NOR gate only.

(i) $Y = AB + AC + BC$ = add.

$$\begin{aligned}
 Y &= \overline{\overline{AB + AC + BC}} \\
 &= (\overline{AB}) \cdot (\overline{AC}) \cdot (\overline{BC}) \\
 &= (\overline{A+B}) \cdot (\overline{A+C}) \cdot (\overline{B+C}) \\
 &= \overline{(\overline{A+B}) + (\overline{A+C}) + (\overline{B+C})}
 \end{aligned}$$





* MINIMIZATION TECHNIQUES :-

minimization techniques are used to minimize a boolean expression so as to reduce the no. of logic gates required to design it. following three methods are used to minimize a boolean expression

- 1) Boolean Algebra
- 2) Karnaugh Map (K-map)
- 3) Quine - McClusky (Q-map) method
(Tabular method)

* K-map :-

- i) 2-variable
- ii) 3-variable
- iii) 4-variable
- iv) 5-variable

i) 2-variable K-map :-

	0	1
0	00 0	01 1
1	10 2	11 3

(18) 3-variable k-map:

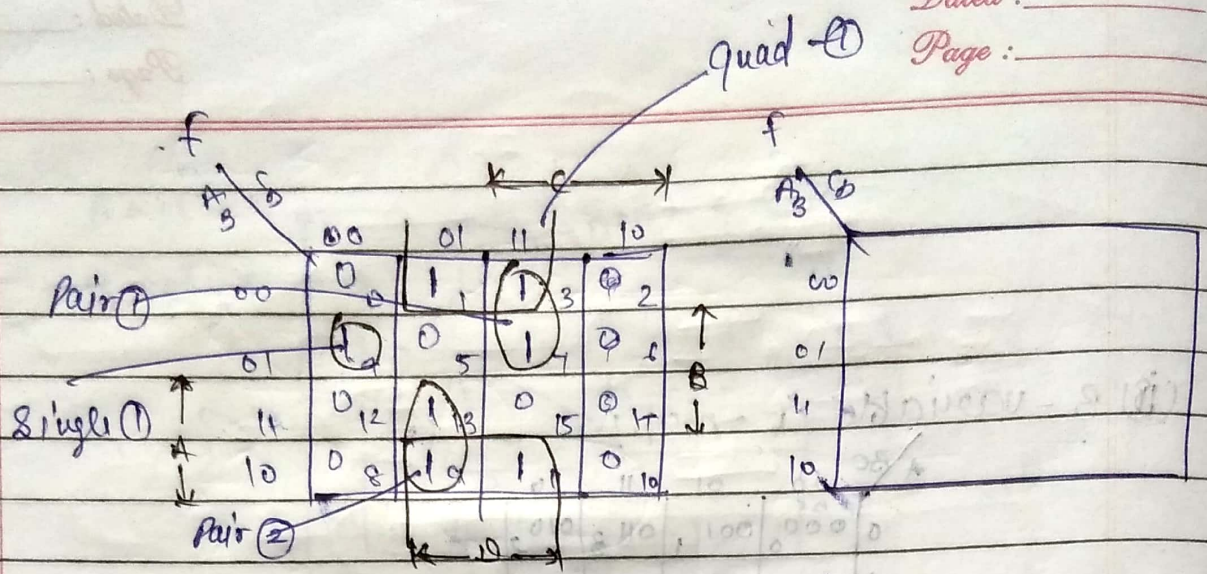
A \ BC	00	01	11	10
0	000 ₀	001 ₁	011 ₃	010 ₂
1	100 ₄	101 ₅	111 ₇	110 ₆

(19) 4-variable k-map:

A \ BC	00	01	11	10
00	0000 ₀	0001 ₁	0011 ₃	0010 ₂
01	0100 ₄	0101 ₅	0111 ₇	0110 ₆
11	1100 ₁₂	1101 ₁₃	1111 ₁₅	1110 ₁₄
10	1000 ₈	1001 ₉	1011 ₁₁	1010 ₁₀

8 + 2 = 10

0	00
0	01
0	11
0	10
1	00
1	01
1	11
1	10



$$f = \sum m(1, 3, 4, 7, 9, 11, 13)$$

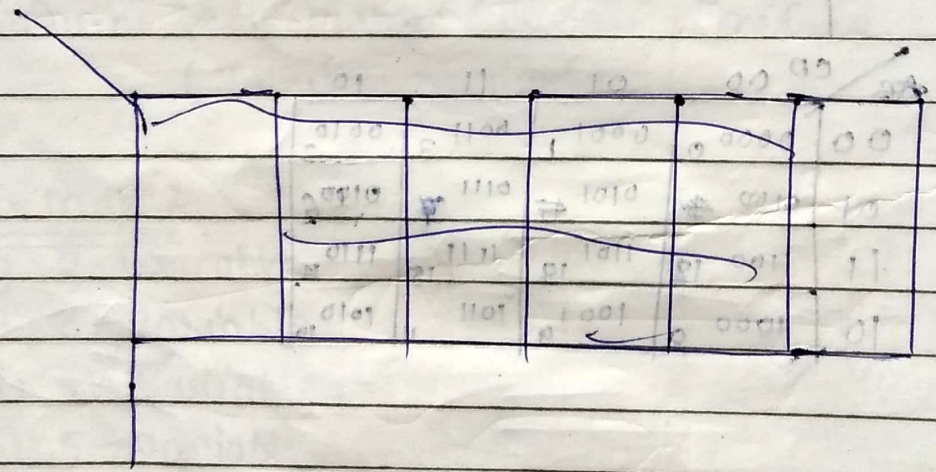
Quad 1 : $\bar{B}D$

Pair 1 : $\bar{A}CD$

Pair 2 : $A\bar{C}D$

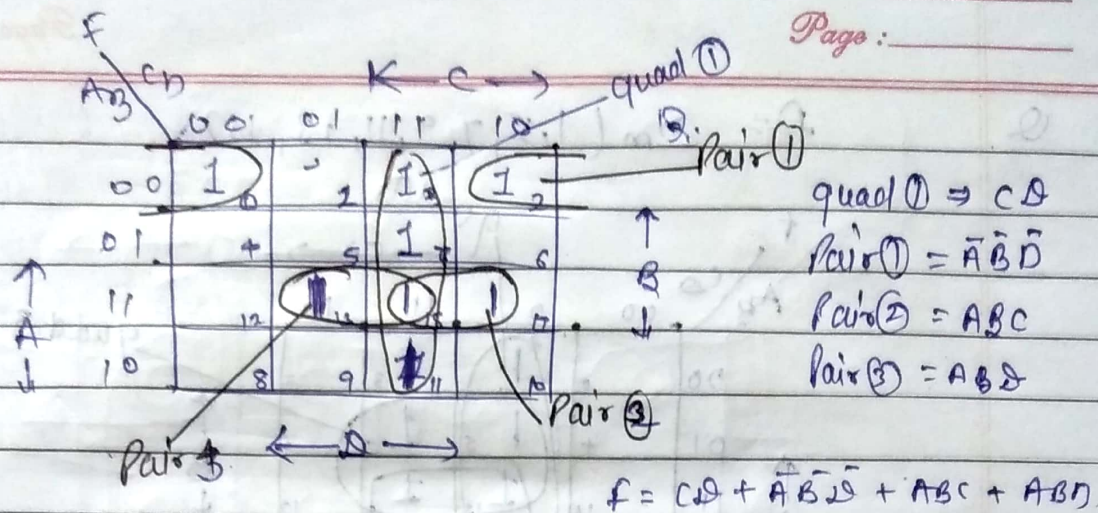
Single 1 : $\bar{A}B\bar{C}\bar{D}$

$$f = \bar{B}D + \bar{A}CD + A\bar{C}D + \bar{A}B\bar{C}\bar{D}$$

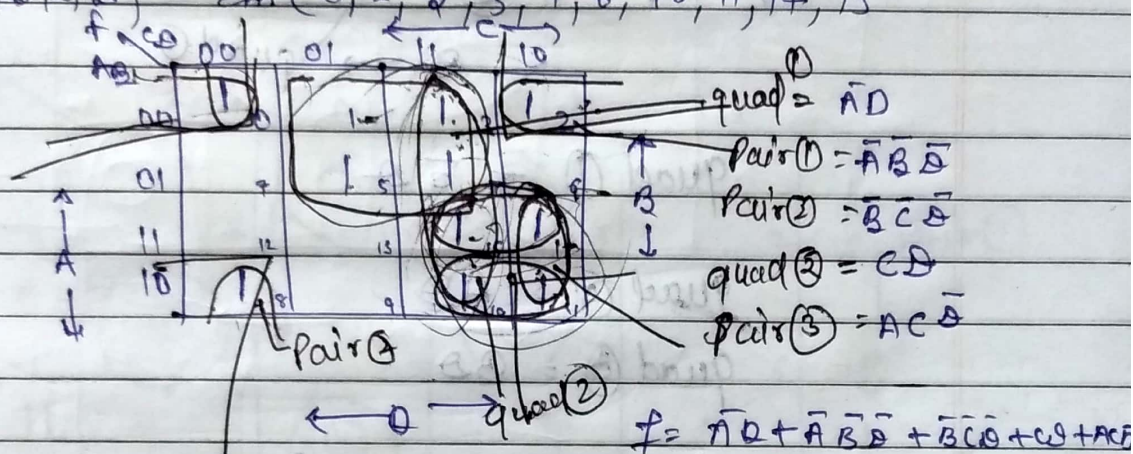


Q.1 $f(A, B, C, D) = \sum m(0, 2, 3, 7, 11, 13, 14, 15)$ Dated: _____

Page: _____



Q.2 $f(A, B, C, D) = \sum m(0, 2, 3, 5, 7, 8, 10, 11, 14, 15)$

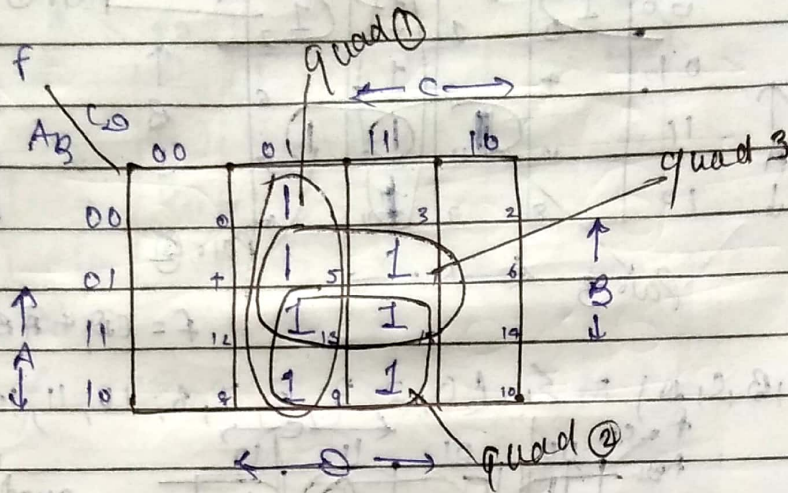


00	01	11	10
00	01	11	10
01	01	11	10
11	01	11	10

00	01	11	10
00	01	11	10
01	01	11	10
11	01	11	10

Q

$$f = \sum m(1, 5, 7, 9, 11, 13, 15)$$



$$\text{quad 1} = \bar{c}d$$

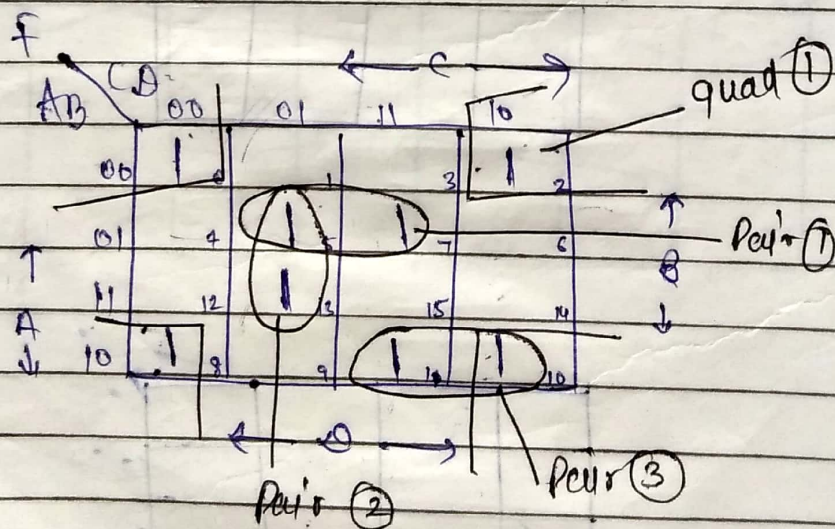
$$\text{quad 2} = Ad$$

$$\text{quad 3} = Bd$$

$$f = \bar{c}d + Ad + Bd$$

Q.

$$f = \sum m(0, 2, 5, 7, 8, 10, 11, 13)$$



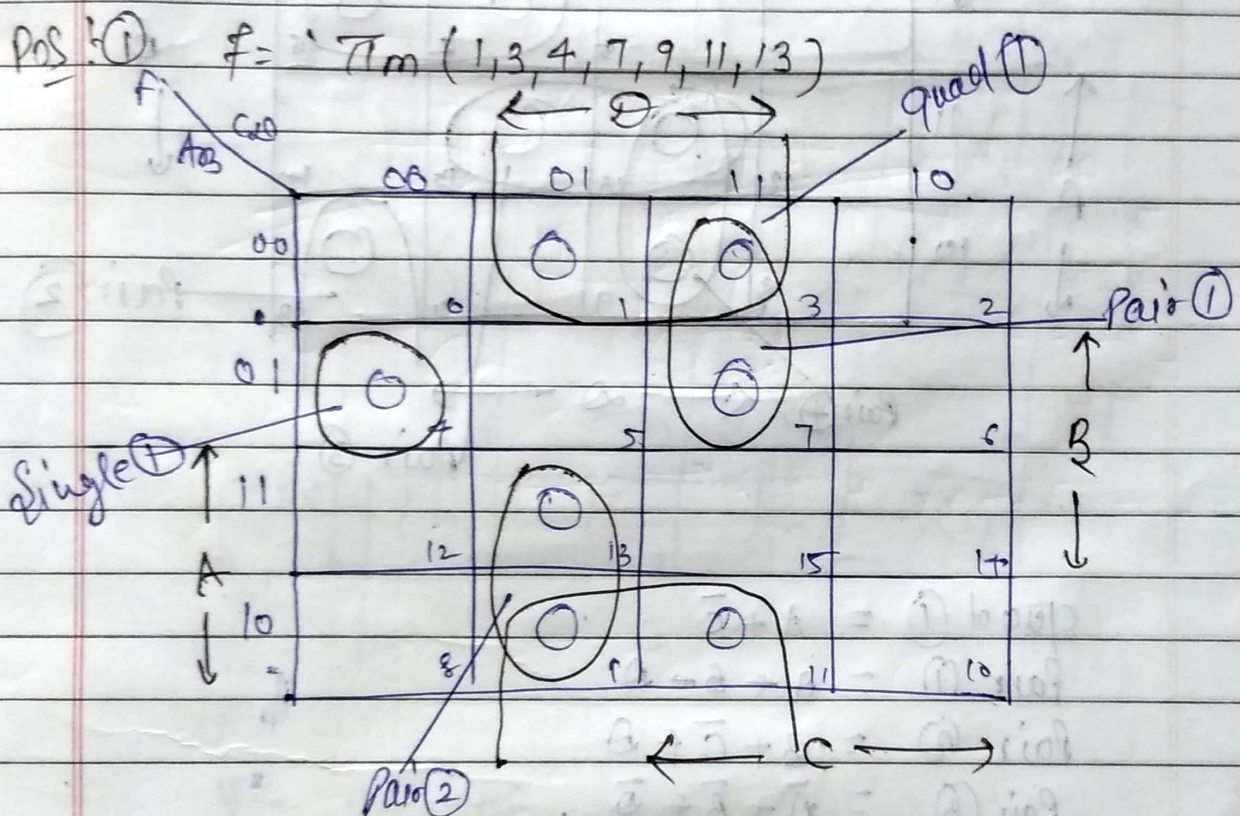
$$\text{quad ①} = \bar{B}\bar{D}$$

$$\text{pair ①} = \bar{A}B\bar{D}$$

$$\text{pair ②} = B\bar{C}\bar{D}$$

$$\text{pair ③} = A\bar{B}C$$

$$F = \bar{B}\bar{D} + \bar{A}B\bar{D} + B\bar{C}\bar{D} + A\bar{B}C$$



$$\text{quad ①} = \bar{A} + \bar{B} + \bar{D}$$

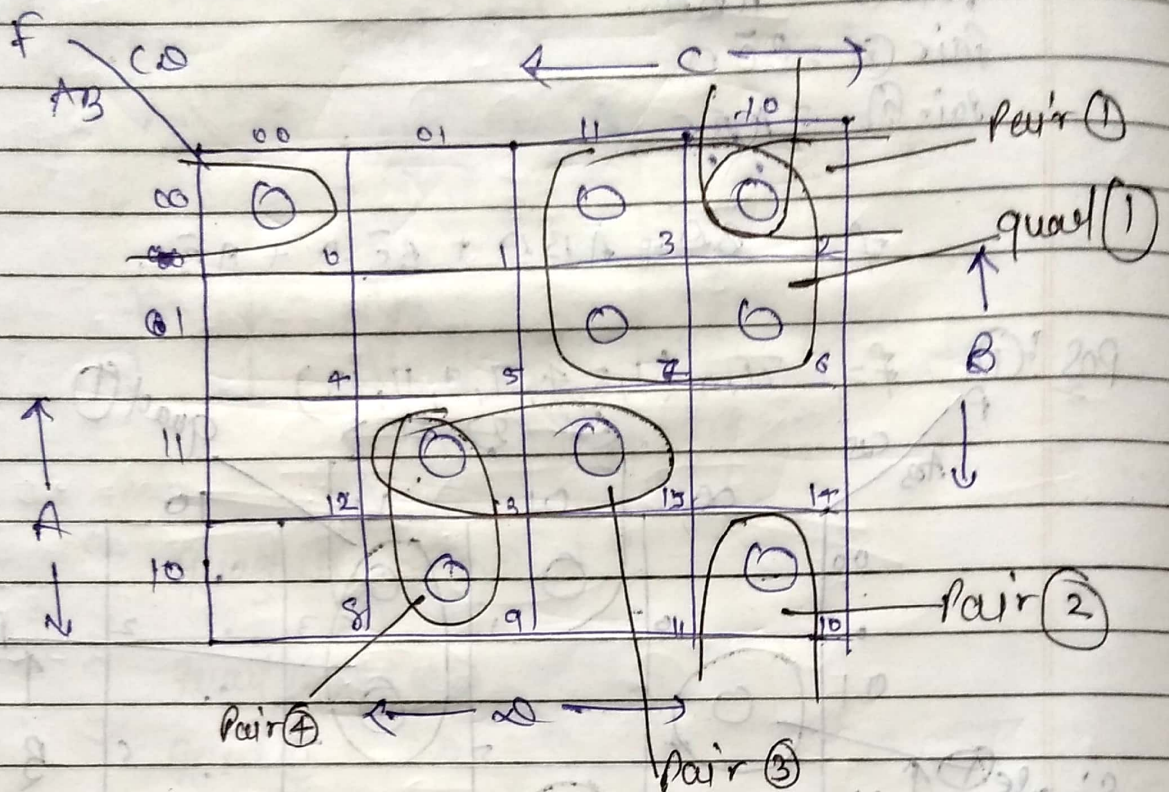
$$\text{pair ①} = A + \bar{C} + \bar{D}$$

$$\text{pair ②} = \bar{A} + C + \bar{D}$$

$$\text{Single ①} = A + \bar{B} + C + \bar{D}$$

$$F = (\bar{B} + \bar{D})(A + \bar{C} + \bar{D})(\bar{A} + C + \bar{D})(A + \bar{B} + C + \bar{D})$$

Q.2 $f = \Pi m(0, 2, 3, 6, 7, 9, 10, 13, 15)$



$$\text{quad (1)} = A + \bar{C}$$

$$\text{Pair (1)} = A + B + D$$

$$\text{Pair (2)} = B + \bar{C} + D$$

$$\text{Pair (3)} = \bar{A} + \bar{B} + \bar{D}$$

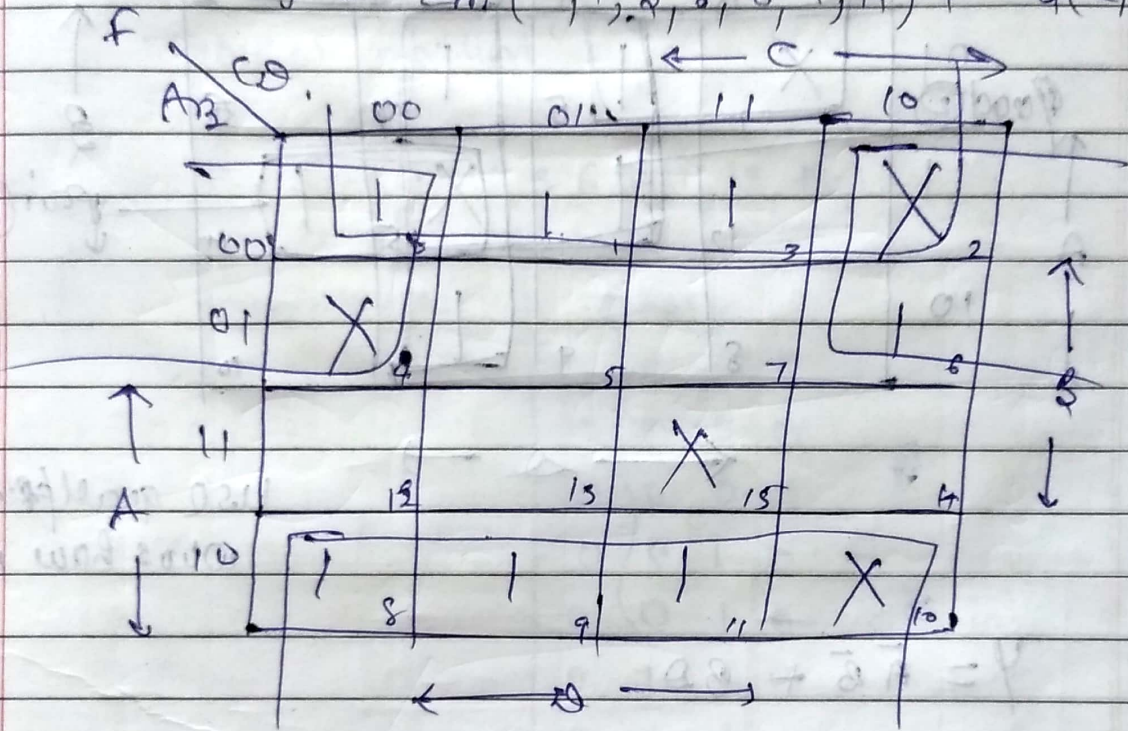
$$\text{Pair (4)} = \bar{A} + C + \bar{D}$$

$$f = (A + \bar{C})(A + B + D)(B + \bar{C} + D)(\bar{A} + \bar{B} + \bar{D})(\bar{A} + C + \bar{D})$$

* Don't Cares

Ques Simplify the following Boolean Expression by using K map

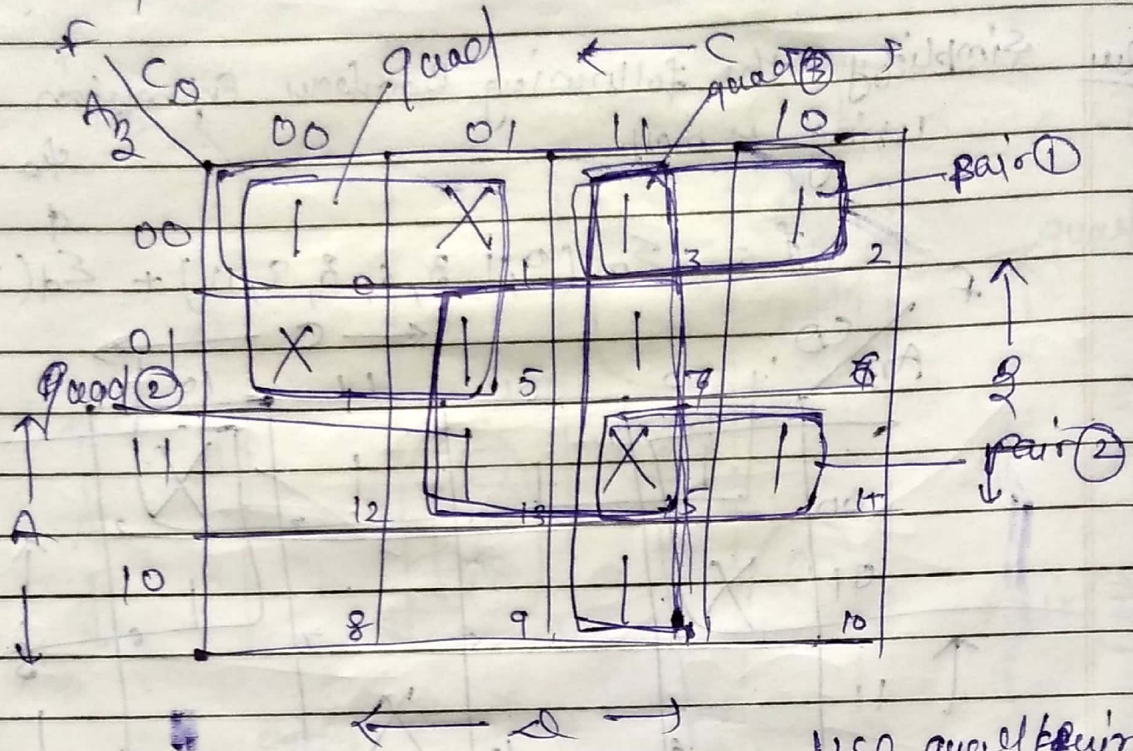
$$f = \sum m(0, 1, 3, 6, 8, 9, 11) + \sum d(2, 4, 10, 15)$$



$$Y = \bar{B} + \bar{A}\bar{D}$$

Q.2

$$f = \sum m(0, 2, 3, 5, 7, 11, 13, 14) + \sum d(1, 4, 15)$$



Use quad pair who have max 1

$$Y = \bar{A}\bar{B} + BD$$

Dated : _____

Page : _____

min terms	Binary
	A B C D
0	0 0 0 0
1	0 0 0 1
2	0 0 1 0
3	0 0 1 1
6	0 1 1 0
7	0 1 1 1
11	1 0 1 1
13	1 1 0 1
15	1 1 1 1

min terms	→
implicants	→
↓	
	1 0 0 0
	0 0 1 0
	1 0 1 0
	0 1 1 0
	1 1 1 0

Dated : _____

Page : _____

Group	Integers	Binary	First Reduction	Second Reduction
0	0 ✓	0 0 0 0	(0,1) 0000 -	0 0 - -
1	1 ✓	0 0 0 1	(0,2) 0000 - 0	0 - 1 -
2	2 ✓	0 0 1 0	(1,3) 0000 - 1	
	3 ✓	0 0 1 1	(2,3) 0001 -	
	6 ✓	0 1 1 0	(2,6) 0000 - 10	
3	7 ✓	0 1 1 1	(3,7) 0000 - 11	
	11 ✓	1 0 1 1	(3,11) 0000 - 0 1 1	
	13 ✓	1 1 0 1	(6,7) 0000 - 11	
	14 ✓	1 1 1 0	(6,14) 0000 - 110	

Q: $f = \sum m(0, 1, 2, 5, 7, 9, 11, 12)$

min terms	Binary
	A B C D
0	0 0 0 0
1	0 0 0 1
2	0 0 1 0
5	0 1 0 1
7	0 1 1 1
9	1 0 0 1
11	1 0 1 1
12	1 1 0 0

Group	Min terms	Binary	First Reduction	Second Reduction
0	0 ✓	0 0 0 0	(0,1) 0 0 0 +	
1	1 ✓	0 0 0 1	(0,2) 0 0 - 0	
	2 ✓	0 0 1 0	(1,5) 0 - 0 1	
2	5 ✓	0 1 0 1	(1,9) - 0 0 1	
	9 ✓	1 0 0 1	(5,7) 0 1 - 1	
	12	1 1 0 0	(9,11) 1 0 - 1	
3	7 ✓	0 1 1 1		
	11 ✓	1 0 1 1		

Dated: _____

Page: _____

Minterms →	0	1	2	5	7	9	11	12
Implicants ↓								
(0,1)	X	X						
(0,2) *	X		(X)					
(1,5)		X		X				
(1,9) *		X				X		
(5,7) *				X	(X)			
(9,11) *						X	(X)	
12 *								(X)

$$Y = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + A\bar{B}\bar{C} + \bar{B}\bar{C}D + A\bar{B}D$$

*

Que: 11 $f = \sum m(0, 1, 3, 7, 8, 9, 11, 15)$

minterms	Binary
	A B C D
0	0 0 0 0
1	0 0 0 1
3	0 0 1 1
7	0 1 1 1
8	1 0 0 0
9	1 0 0 1
11	1 0 1 1
15	1 1 1 1

Group	minterms	Binary	First Red.	Sec Reduction	Third Red
0	0 ✓	0 0 0 0	(0,1) ✓ 000-	(0,1,8,9) -00-	
1	1 ✓	0 0 0 1	(0,8) ✓ -000	(1,3,9,11) -0-1	
	8 ✓	1 0 0 0	(1,3) ✓ 00-1	(3,11,7,15) --11	
2	3 ✓	0 0 1 1	(1,9) ✓ -001		
	9 ✓	1 0 0 1	(8,9) ✓ 100-		
3	7 ✓	0 1 1 1	(2,7) 0-11		
	11 ✓	1 0 1 1	(3,11) ✓ -011		
4	15 ✓	1 1 1 1	(9,11) ✓ 10-1		
			(7,15) ✓ -111		
			(11,15) ✓ 1-11		

Third Red.

Dated: _____

Page: _____

miniterms $\rightarrow$	0	1	3	7	8	9	11	15
Implicants $\downarrow$								
(0, 1, 8, 9) *	(X)	X			(X)	X		
(1, 3, 9, 11)		X	X			X	X	
(3, 11, 7, 15) *			X	X			X	X
(3, 7)			X	X			X	X
(11, 15)								

$$Y = \bar{B}\bar{C} + CD$$

② $f = \sum m(0, 5, 8, 9, 10, 11, 14, 15)$

miniterms	Binary	
	A B C D	
0	0 0 0 0	(0, 0)
5	0 1 0 1	(0, 5)
8	1 0 0 0	(0, 8)
9	1 0 0 1	(0, 9)
10	1 0 1 0	(0, 10)
11	1 0 1 1	(0, 11)
14	1 1 1 0	(0, 14)
15	1 1 1 1	(0, 15)

Group	minterms	Binary	First Red.	Sec. Red
0	0 ✓	0 0 0 0	(0,8) - 0 0 0 -	(0,8 , 4,12) 1
1	8 ✓	1 0 0 0	(8,9) 1 0 0 -	
2	5 ✓	0 1 0 1	(9,11) 1 0 - 1	
	9 ✓	1 0 0 1	(11,15) 1 - 1 1	
3	10	0 0 0 1 0	(14,15) 1 1 1 -	
	11 ✓	1 0 1 1		
	14 ✓	1 1 1 0		
+	15 ✓	1 1 1 1		

minterms →	0	5	8	9	10	11	14	15
Implicants ↓								
(0,8) *	(X)		X					
(8,9)			X	X				
(9,11) *				X		X		
(11,15)						X		X
(14,15) *							(X)	X
(5) *		(X)						

$$Y = \bar{B}\bar{C}\bar{D} + A\bar{B}D + ABC + \bar{A}B\bar{C}D$$

* * Error Detection & Correction Codes:

When digital data is transmitted from a source to destination then due to faulty gates or noise in the transmission medium, the data may get encounters with errors.

When an error occurs a logic "0" may change to logic "1" and logic "1" to logic "0".

In order to detect and correct the error in the transmitted data following error detection and correction codes are used.

(1/5)

1) Parity method.

2) Check-sum method.

(3/5)

3) Hamming Code method

1) Parity method: In Parity method an additional bit known as Parity bit is added with the data to be transmitted. The value of Parity bit is selected in such a way that the total no. of 1 in the data including Parity bit ~~becomes~~ becomes even or odd. depending upon the value of parity bit the Parity method are of following two type:

i) even Parity methods

ii) odd Parity methods

i) even parity method :

In this method the value of parity bit is selected in such a way that the total no. of 1's in the data including parity bit becomes even.

(ii) odd parity method :

In this method the value of parity bit is selected in such a way that the total no. of 1's in the data including parity bit becomes odd.

In both even as well as odd parity methods a parity check circuit is used at the receiver. This circuit checks the received data and if the received no. of 1's do not match the parity method used to transmit the data then a flag is raised which indicates that the received data is corrupted.

The even and odd parity for a 4-bit data can be written as follows -

* Drawback :

data	data	even parity bit	odd parity bit
0 0 0 0	0 0 0 0	0	1
0 0 0 1	0 0 0 1	1	0
0 0 1 0	0 0 1 0	1	0
0 0 1 1	0 0 1 1	0	1
0 1 0 0	0 1 0 0	1	0
0 1 0 1	0 1 0 1	0	1
0 1 1 0	0 1 1 0	0	1
0 1 1 1	0 1 1 1	1	0
1 0 0 0	1 0 0 0	1	0
1 0 0 1	1 0 0 1	0	1
1 0 1 0	1 0 1 0	0	1
1 0 1 1	1 0 1 1	1	0
1 1 0 0	1 1 0 0	0	1
1 1 0 1	1 1 0 1	1	0
1 1 1 0	1 1 1 0	1	0
1 1 1 1	1 1 1 1	0	1

* Hamming Code method

Hamming code method performs both the correction as well as error detection of error in digital data. In this method K Parity bits are added with the data (m bits) to be transmitted. the value of K satisfy the following relation

$$m < 2^K$$

where m represents message bits

K represents Parity bits

message bit < 2

Parity bit

$m < 2^k$

Dated: _____

Page: _____

Smart

26/08/19

Q. Generate a seven bit ~~ham~~ even parity hamming code for the data '1011'.

No. of message bits = 4

" " Parity bits = 3

Total bits = 7

	001	010	011	100	101	110	111
Bit positions:	1	2	3	4	5	6	7
	p_1	p_2	m_1	p_3	m_2	m_3	m_4
message Bits:			1		0	1	1

Parity Bits
Come

2^0

2^1

2^2

2^3

Parity Bits : 0 1 0 0

Hamming Code : 0 1 1 0 0 1 1

First Parity check:

m_1, m_2, m_4
0 1 0 1

Total no. of 1's at bit positions (1, 3, 5, 7)
is even therefore $p_1 = 0$

Second Parity check:

Total no. of 1's at bit positions (2, 3, 6, 7)
is odd therefore $p_2 = 1$

Third Parity check:

Total no. of 1's at bit positions (4, 5, 6, 7)
is even therefore $p_3 = 0$

7 bit even parity hamming code for the
message 1011 = 0110011

Ques Generate a ~~Seven~~ ^{Seven} bit odd parity hamming code for the data 1001.

Solⁿ No of message bits = 4
 " " Parity bits = 3
 Total bits = 7.

	001	010	011	100	101	110	111
Bit Positions :	1	2	3	4	5	6	7
	P_1	P_2	m_1	P_3	m_2	m_3	m_4
message Bits :			1		0	0	1
Parity Bits :	1	1		0			
Hamming code :	1	1	1	0	0	0	1

First Parity check:

Total no. of 1's at bits Position (1, 3, 5, 7)
 is even therefore $P_1 = 1$

Second Parity check:

Total no. of 1's at bits Position (2, 3, 6, 7)
 is even therefore $P_2 = 1$

Third Parity check:

Total no. 1's at bits Position (4, 5, 6, 7)
 is odd therefore $P_3 = 0$

7 Bit odd Parity Hamming code for the message 1001 $\Rightarrow 1110001$.

Ques If the Seven Bit Even Parity Hamming code is

0 1 0 0 0 1 1

detect and correct error if any.

	001	010	011	100	101	110	111
Bit Positions:	1	2	3	4	5	6	7
Received code:	0	1	0	0	0	1	1

First Parity check :-

The total no. of 1's at Bit Positions (1, 3, 5, 7) is odd therefore this group is in error and the error code

$$C_1 = 1$$

Second Parity check:-

The total no. of 1's at Bit positions (2, 3, 6, 7) is odd therefore this group is in error and the error code

$$C_2 = 1$$

Third Parity check:-

The total no. of 1's at Bit positions (4, 5, 6, 7) is even therefore there is no error in this group and the error code

$$C_3 = 0$$

The error code for the Bit position in error is

$$\begin{array}{ccc} C_3 & C_2 & C_1 \\ 0 & 1 & 1 \end{array} = 3 \text{ (Decimal)}$$

→ Error position

Then for Bit position 3 is in error. it is a logic "0" bit should be logic "1"

therefore the correct Hamming code =

$$= 0 \ 1 \ 1 \ 0 \ 0 \ 1 \ 1$$

* Binary Codes

The Binary codes are classified into following categories.

- i) 8-4-2-1
- ii) 5-2-2-1
- iii) Gray code
- iv) BCD code
- v) Excess-3

Q. Convert the following no. to Gray code.

Ex: $(101101011)_2$

XOR

$$\begin{array}{cccccccc} 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ \oplus & & & & & & & & \\ 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{array}$$

↓

$$(10101111011)_2$$

$$\begin{array}{l} \text{diff} = 1 \\ \text{same} = 0 \end{array}$$

$$(101101011)_2$$

$$G = (11011110)_G$$

Q. Convert following Gray codes to Binary

$$\begin{array}{ccc}
 (101101)_G \Rightarrow (8)_2 & & (101100101)_G \Rightarrow (?)_2 \\
 \downarrow & & \downarrow \\
 (110110)_2 & & (110111001)_2
 \end{array}$$

* BCD Codes = (Binary Coded decimal)

It is a special form of Binary code in which each decimal digit is represented by four bit Binary equivalent.

$$0 \rightarrow 0000$$

$$5 \rightarrow 0101$$

$$10 \rightarrow 00010000$$

$$15 \rightarrow 00010101$$

$$16 \quad 8 \quad 4 \quad 2 \quad 1$$

* Excess -3

$$(1011)_2 = (?)_{\text{Ex-3}}$$

$$(25)_{10} = (?)_{\text{Ex-3}} \Rightarrow (11100)_{\text{Ex-3}}$$

$$(1010)_{\text{Ex-3}} = (?)_2 \Rightarrow (0111)_2$$

$$(1011)_2 = (110)_{\text{Ex-3}}$$

SBG STUDY