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unit - 1

* Exact d.E:

A differential eq^n of the form

$$M(x,y)\,dx + N(x,y)\,dy = 0$$

is said to be exact if it satisfy the following condition.

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

then we can solve it by the formula

$$\underset{\text{keeping } y \text{ const.}}{\int M\,dx} + \int (\text{term without } x \text{ from } N)\,dy = C.$$

Que: Solve the D.E

$$x\,dx + y\,dy = a^2\frac{(x\,dy - y\,dx)}{x^2+y^2}$$

$$\left(x + \frac{a^2y}{x^2+y^2}\right)dx + \left(y - \frac{a^2x}{x^2+y^2}\right)dy = 0$$

$$M = x + \frac{a^2y}{x^2+y^2}, \quad N = y - \frac{a^2x}{x^2+y^2}$$

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$$\frac{\partial M}{\partial y} = a^2\left(\frac{(x^2+y^2).1 - y \times 2y}{(x^2+y^2)^2}\right)$$

$$= a^2\frac{(x^2-y^2)}{(x^2+y^2)^2}$$

$$\frac{\partial N}{\partial x} = -a^2\left(\frac{(x^2+y^2) - x.2x}{(x^2+y^2)^2}\right)$$

$$= -a^2\left(\frac{y^2-x^2}{(x^2+y^2)^2}\right)$$

$$= a^2\frac{(x^2-y^2)}{(x^2+y^2)^2}$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \quad (\text{exact})$$

then soln

$$\int \left(x + \frac{a^2 y}{x^2+y^2}\right)dx + \int y\,dy = c$$

$$\frac{x^2}{2} + a^2 y \times \frac{1}{y} \tan^{-1}\frac{x}{y} + \frac{y^2}{2} = c$$

$$\frac{x^2}{2} + a^2\tan^{-1}\frac{x}{y} + \frac{y^2}{2} = c$$

$$\left(\frac{x^2+y^2}{2}\right) + a^2\tan^{-1}\frac{x}{y} = c$$

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Que Find the value of α for which the D.E

$$(xy^2 + \alpha x^2 y)\,dx + (x+y)x^2\,dy = 0$$

is Exact. Solve the eqn for this value of α.

$$M = xy^2 + \alpha x^2 y \qquad N = x^3 + x^2 y$$

$$\frac{\partial M}{\partial y} = 2xy + \alpha x^2 \qquad \frac{\partial N}{\partial x} = 3x^2 + 2xy$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$2xy + \alpha x^2 = 3x^2 + 2xy$$

$$\alpha = \frac{3x^2}{x^2} = 3 \text{ Ans}$$

$$(xy^2 + 3x^2 y)\,dx + (x^3 + y x^2)\,dy = 0$$

Soln of the d.E.

$$\frac{y^2 x^2}{2} + 3y\frac{x^3}{3} + 0 = c$$

$$\frac{x^2 y^2}{2} + y x^3 = c$$

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Equations Reducible to Exact form

diff. eq^n which is not an exact di.E but may become exact by multiplying by a suitable function known as intragating factor (I.F).

Following rules are used to finding out the integrating factor.

Rule:1

if $Mdx + Ndy = 0$ is a homogenious D.E

then

$$I.F = \frac{1}{Mx + Ny}, \quad Mx + Ny \neq 0$$

Rule:2 if $Mdx + Ndy = 0$ is of the form

$$f_1(xy) \cdot y dx + f_2(xy) \cdot x dy = 0$$

then

$$I.F = \frac{1}{Mx - Ny}, \quad Mx - Ny \neq 0$$

Rule:3 In $Mdx + Ndy = 0$ if

$$\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = f(x)$$

then $I.F = e^{\int f(x) dx}$.

Rule-4 In $Mdx + Ndy = 0$ if

$$\frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) = f(y)$$

then $I.F = e^{\int f(y)\,dy}$.

Rule-5 let $x^h \cdot y^k$ be the I.F of the D.E. and then multiply by m and n and equal $\frac{\partial m}{\partial y} = \frac{\partial N}{\partial x}$ and find h and k.

Que:- Solve the D.E

$$(y - xy^2)\,dx - (x + x^2y)\,dy = 0$$

$$(1 - xy)\,y\,dx - (1 - xy)\,x\,dy = 0$$

$$m = y - xy^2, \quad N = -x - x^2y.$$

$$\frac{\partial m}{\partial y} = 1 - 2xy \qquad \frac{\partial N}{\partial x} = -1 - 2xy$$

$$\frac{\partial m}{\partial y} \neq \frac{\partial N}{\partial x} \quad \text{(Not exact)}$$

Rule-2 $(1 - xy)\,y\,dx - (1 - xy)\,x\,dy = 0$

$$I.F = \frac{1}{xy - x^2y^2 + (xy + x^2y^2)} = \frac{1}{2x^2y^2}$$

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$$I.F = -\frac{1}{2x^2y^2}$$

$$I.F = \frac{1}{Mx - Ny} = \frac{1}{xy - x^2y^2 + xy + x^2y^2}$$

$$I.F = \frac{1}{2xy}$$

multiplying by $\frac{1}{2xy}$ in eq^n ①

$$\frac{1}{2}\left(\frac{1}{x} - y\right)dx - \frac{1}{2}\left(\frac{1}{y} + x\right)dy = 0$$

$$\left(\frac{1}{x} - y\right)dx - \left(\frac{1}{y} + x\right)dy = 0$$

$$\int\left(\frac{1}{x} - y\right)dx + \int\left(-\frac{1}{y}\right)dy = c$$

$$\log x - yx + -\log y = c$$

$$\log\frac{x}{y} - yx = \log c$$

$$\log\frac{x}{cy} = xy$$

$$\frac{x}{cy} = e^{xy}$$

$$x = cye^{xy}$$ Ans

Q. $(x\sec^2 y - x^2\cos y)dy = (\tan y - 3x^4)dx$

$(x\sec^2 y$

$(x\sec^2 y - x^2\cos y)dy - (\tan y - 3x^4)dx = 0$

$M = -\tan y + 3x^4 \qquad N = x\sec^2 y - x^2\cos y$

$\frac{\partial M}{\partial y} = -\sec^2 y \quad , \quad \frac{\partial N}{\partial x} =$

$\frac{\partial N}{\partial x} = \sec^2 y - 2x\cos y$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = -\sec^2 y - \sec^2 y + 2x\cos y$$
$$= -2\sec^2 y + 2x\cos y.$$

$$\Rightarrow \frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{-2\sec^2 y + 2x\cos y}{x(\sec^2 y - x\cos y)}$$
$$= \frac{-2}{x} = f(x)$$

$$I.F = e^{\int -\frac{2}{x}dx} = e^{-2\log x} = x^{-2} = \frac{1}{x^2}$$

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multiply x^{-2} in eq ①

$$\left(\frac{x\sec^2 y - x^2\cos y}{x^2}\right)dy - \left(\frac{\tan y - 3x^4}{x^2}\right)dx = 0$$

$$\left(\frac{\sec^2 y}{x} - \cos y\right)dy - \left(\frac{\tan y}{x^2} - 3x^2\right)dx = 0$$

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Q. Solve the D.E.

$$y(1+xy)dx + x(1+xy+x^2y^2)dy = 0 \quad -(i)$$

$M = y(1+xy)$ $\quad N = x(1+xy+x^2y^2)$

$M = y + xy^2$ $\quad N = x + x^2y + x^3y^2$

$\frac{\partial m}{\partial y} = 1 + 2xy$ $\quad \frac{\partial N}{\partial x} = 1 + 2xy + 3x^2y^2$

$$\frac{\partial m}{\partial y} \neq \frac{\partial N}{\partial x} \text{ (Not exact).}$$

$$I.F. = \frac{1}{Mx - Ny}$$

$$= \frac{1}{\cancel{xy} + \cancel{x^2y^2} - \cancel{xy} - \cancel{x^2y^2} - x^3y^3}$$

$$I.F = -\frac{1}{x^3y^3}$$

multiply I.F in eq^n (1)

$$\frac{y(1+xy)dx}{x^3y^3} + \frac{x(1+xy+x^2y^2)dy}{x^3y^3} = 0$$

$$(x^{-3}y^{-2} + x^{2}y^{-1})dx + (x^{-2}y^{-3} + x^{-1}y^{-2} + y^{-1})dy = 0$$

Rule 2, 3

not algebric fu → Solve eq^n ③

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Solution

$$\int (x^{-3}y^{-2} + x^{-2}y^{-1})dx + \int \left(\frac{1}{y}\right) dy = 0$$

$$-y^{-2}\frac{x^{-2}}{2} - y^{-1}x^{-1} + \log y = c$$

$$\frac{1}{2}\frac{1}{x^2y^2} + \frac{1}{xy} - \log y = c$$

Ans

Que: Solve the d.E

$$(3y - 2xy^3)dx + (4x - 3x^2y^2)dy = 0$$

$$\frac{\partial m}{\partial y} = 3 - 6xy^2 \qquad \frac{\partial N}{\partial x} = 4 - 6xy^2$$

$$\frac{\partial m}{\partial y} \neq \frac{\partial N}{\partial x}$$

$$\left(\frac{\partial m}{\partial y} - \frac{\partial N}{\partial x}\right) = -1$$

Rule 3 and 4 is not applicable.

Rule 5

let $x^h y^k$ be the integrated factor of eq^n ①

then multiplying

$x^h y^k$ in eq^n ①

Rule-5

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$$(3yx^hy^k - 2xy^3x^hy^k)dx + (4xx^hy^k - 3x^2y^2x^hy^k)dy = 0$$

$$M = 3y^{1+k}x^h - 2x^{1+h}y^{3+k}$$

$$N = 4x^{1+h}y^k - 3x^{2+h}y^{2+k}$$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

Since eq^n is exact

$$\frac{\partial M}{\partial y} = 3x^h(1+k)y^k - 2x^{1+h}(3+k)y^{k+2}$$

$$\frac{\partial N}{\partial x} = 4y^k(1+h)x^h - 3y^{2+k}(2+h)x^{h+1}$$

$$\cancel{3x^h(1+k)y^k - 2x^{1+h}(3+k)y^{k+2} = 4y^k(1+h)x^h - 3y^{2+k}(2+h)x^{1+h}}$$

Comparision 1st term

$$3x^hy^k(1+k) = 4y^kx^h(1+h)$$

$$3(1+k) = 4(1+h) \Rightarrow 3+k = 4+h$$

$$\boxed{k-h=1}$$

$$2x^{1+h}(3+k)y^{k+2} = 3(2+h)y^{2+k}x^{h+1}$$

$$2(3+k) = 3(2+h)$$

$$6+2k = 6+6h$$

$4k$

P and Q are fn of y and cons

* Linear form:

$$\frac{dy}{dx} + Py = Q \quad \Big| \quad \frac{dx}{dy} + Px = Q$$

P and Q are the function of x and constant

$$\frac{dy}{dx} + Py = Qy^n$$

Bernoulli's eqn

or $$\frac{dx}{dy} + Px = Qx^n.$$

Que: $\tan y \frac{dy}{dx} + \tan x = \cos y \cos^2 x.$

$$\frac{\tan y}{\cos y}\frac{dy}{dx} + \frac{\tan x}{\cos y} = \cos^2 x.$$

$\sec y = v$

$$\sec y \tan y \frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dv}{dx} + \tan x . v = \cos^2 x$$

Linear d.E of 1st order in v

$$I.F = e^{\int P.dx} = e^{\int \tan x \, dx}$$

$$= e^{\log \sec x} = \sec x$$

$V \times I.F = \int Q \cdot I.F\, dx + C$

Sol

~~$V \cdot \sec x = \int (\tan x \sec x) dx + C$~~

$V \sec x =$

$V \times \sec x = \int \cos^2 x \cdot \sec x\, dx + C$

$\sec y \sec x = \sin x + C$

Q $3\frac{dy}{dx} + \frac{3y}{x} = 2x^4 y^4$

$3\frac{dy}{dx} + \left(\frac{3}{x}\right) y = 2x^4 y^4$

divide both side by y^4.

$\frac{3}{y^4}\frac{dy}{dx} + \left(\frac{3}{x}\right)\frac{1}{y^3} = 2x^4$

$\frac{1}{y^3} = v$

$\frac{-3}{dy^4}\frac{dy}{dx} = \frac{dv}{dx}$

$-\frac{dv}{dx} + \left(\frac{3}{x}\right) v = 2x^4$

$\frac{dv}{dx} - \left(\frac{3}{x}\right) v = -2x^4$

$I.f = e^{\int \frac{-3}{x} dx} = e^{(-3\log x)} = x^{-3}$

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$$I.f = x^{-3}$$

$$V.I.f = \int (Q.If).dx + c$$

$$\frac{V}{x^3} = \int \left(-2x^4 . \frac{1}{x^3}\right) dx + c$$

$$\frac{V}{x^3} = -\frac{\cancel{2}x^2}{\cancel{2}} + c$$

$$\frac{V}{x^3} = -x^2 + c$$

$$\frac{1}{y^3 x^3} = -x^2 + c \quad \text{Ans}$$

$$\frac{1}{y^3 x^3} + x^2 = c$$

Bernoulli Form:

$$\frac{dy}{dx} + Py = Qy^n$$

$$\frac{1}{y^n}\frac{dy}{dx} + \frac{P}{y^{n-1}} = Q$$

let $\frac{1}{y^{n-1}} = v$

$$y^{1-n} = v$$

$$(1-n)y^{-n}\frac{dy}{dx} = \frac{dv}{dx}$$

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$$\frac{1}{y^n}\frac{dy}{dx} = \frac{1}{(1-n)}\frac{dv}{dx}$$

$$\frac{1}{(1-n)}\frac{dv}{dx} + P.V = Q$$

$$\frac{dv}{dx} + (1-n)PV = (1-n)Q$$

$$\frac{dv}{dx} + P_1 V = Q_1 \quad \text{(Linear D.E)}$$
(Linear in v).

Q. $$y\log y\frac{dx}{dy} + x - \log x = 0$$

$$\frac{dx}{dy} + \frac{x}{y\log y} - \frac{\log x}{y\log y} = 0$$

$$\frac{dx}{dy} + \left(\frac{1}{y\log y}\right)x = \frac{\log x}{y\log y}$$

$$\frac{1}{\log x}\frac{dx}{dy} + \frac{1}{(y\log y)\log x} \times x = \frac{1}{y\log y}$$

let $\dfrac{x}{\log x} = v$

$$\frac{\log x\frac{dx}{dy} - 1.\frac{dx}{dy}}{(\log x)^2} = \frac{dv}{dy}$$

$$\frac{(\log x - 1)\frac{dx}{dy}}{(\log x)^2} = \frac{dv}{dy}$$

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Q. $y \log y \frac{dx}{dy} + x - \log y = 0$

$$\frac{dx}{dy} + \frac{x}{y \log y} = \frac{\log y}{y \log y}$$

$$\frac{dx}{dy} + \frac{x}{(y \log y)} = \frac{1}{y}$$

$$P = \frac{1}{y \log y}, \quad Q = \frac{1}{y}$$

$$IF = e^{\int P\,dx} =$$

$$= e^{\int \frac{1}{y \log y} dy} = e^{\log \log y} = \log y$$

$\log y = t$

$\frac{1}{y}\frac{dy}{dt}$

$$I.F = \log y$$

$$x \cdot \log y = \int \frac{1}{y} \times (\log y)\, dy + c$$

$$x \log y = \frac{1}{2}(\log y)^2 + c$$

Ans

not 1st degree ⇒ non-linear d.E.

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Q. $x^2 y - x^3 \frac{dy}{dx} = y^4 \cos x$

$$x^3 \frac{dy}{dx} - x^2 y = -y^4 \cos x$$

$$\frac{dy}{dx} - \frac{1}{x} y = -y^4 \frac{\cos x}{x^3}$$

$$\cancel{\frac{1}{\cos x}\frac{dy}{dx} + \left(\frac{-1}{x\cos x}\right) dy = -y^4}$$

$$\frac{1}{y^4}\frac{dy}{dx} - \left(\frac{1}{x}\right)\frac{1}{y^3} = -\frac{\cos x}{x^3}$$

$$\frac{1}{y^4}\frac{dy}{dx} + \left(\frac{-1}{x}\right)\frac{1}{y^3} = -\frac{\cos x}{x^3}$$

let $\frac{1}{y^3} = v$

$$-\frac{3}{y^4}\frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{1}{y^4}\frac{dy}{dx} = -\frac{1}{3}\frac{dv}{dx}$$

$$-\frac{1}{3}\frac{dv}{dx} + \left(\frac{-1}{x}\right) v = -\frac{\cos x}{x^3}$$

$$\cancel{I.F = e^{\int \frac{1}{3x}} = e^{\int \log x^4} = x^4 = \frac{1}{x}}$$

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$$I.F = \frac{1}{x}$$

$$v \cdot \frac{1}{x} = \int \left(-\frac{\cos x}{x^3} \cdot \frac{1}{x}\right) dx + v.$$

$$\frac{1}{y^3 x} = \cancel{}$$

$$\frac{dv}{dx} + \left(\frac{3}{x}\right)v = \frac{3\cos x}{x^3}$$

$$I.F = e^{\int \frac{3}{x} dx}$$

$$IF = x^3$$

$$v \cdot x^3 = \int \left(\frac{3\cos x}{\cancel{x^3}} \times \cancel{x^3}\right) dx + C$$

$$\frac{1}{y^3} \cdot x^3 = 3\int \cos x \, dx + C$$

$$\frac{1}{y^3} x^3 = 3\sin x + C$$

Q.1 $\left(y + \frac{1}{3}y^3 + \frac{1}{2}x^2\right) dx + \frac{1}{4}(1+y^2)x\,dy = 0$

Q.2 $(x - y^2)\,dx + 2xy\,dy = 0$

Q.3 $(x^3 - 3xy^2)\,dx + (y^3 - 3x^2y)\,dy = 0$, $y(0) = 1$.

Q.4 $y(1+xy)\,dx + x(1+xy+x^2y^2)\,dy = 0$

Q.5 $(2x^2y^4e^y + 2xy^3 + y)\,dx + (x^2y^4e^y - x^2y^2 - 3x)\,dy = 0$

Q.6 $(x^2y - 2xy^2)\,dx - (x^3 - 3x^2y)\,dy = 0$

Q.7 $(y^3 - 2yx^2)\,dx + (2xy^2 - x^3)\,dy = 0$

Q.8 $x(3y\,dx + 2x\,dy)\,dx + 8y^4(y\,dx + 3x\,dy) = 0$

* Differential eq^n of 1st order but not first degree!

The most General form of the differential equation of 1st order but not of the first degree are given

$$P_0 p^n + P_1 p^{n-1} + P_2 p^{n-2} \cdots P_n = 0$$

where $p = \frac{dy}{dx}$ and $P_0, P_1 \cdots P_n$

are the functions of x and y.

Such type of differential equations can be solve by the following methods / rules / cases

1) Solvable for p
2) Solvable for y
3) Solvable for x.

(1) Solvable for p. [Linear form convert in p.]

Consider

$$P_0 p^{n} + P_1 p^{n-1} + \cdots P_n = 0 \quad \text{(i)}$$

Now eq^n (1) can be written as

$$\left(p - f_1(x,y)\right)\left(p - f_2(x,y)\right) \cdots$$
$$\left(p_n - f_n(x,y)\right) = 0 \quad \text{(ii)}$$

Now

$$\left(p - f_1(x,y)\right) = 0$$

$$p = f_1(x,y)$$
$$\frac{dy}{dx} = f_1(x,y)$$

which is linear d.E of 1st order

solution of above eq^n is

$$\phi(x,y,c)$$

similarly $\phi_2(x,y,c)$

$$\phi(x,y,c)$$

Hence the required General solution of eq^n (1) is

$$\phi_1(x,y,c) \cdot \phi_2(x,y,c) \cdot \phi_3(x,y,c) \cdots$$

$$\cdots \phi_n(x,y,c)$$

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Ex: Solve $x^2 = 1 + p^2$

$$p^2 = x^2 - 1$$

$$p = \pm\sqrt{x^2-1}$$

$$\int \frac{dy}{dx} = \int \sqrt{x^2-1}$$

$$(p - \sqrt{x^2-1})(p + (\sqrt{x^2-1})) = 0$$

$$\int \frac{dy}{dx} = \pm\int \sqrt{x^2-1}$$

$$y = \pm\left[\frac{x}{2}\sqrt{x^2-1} - \frac{1}{2}\log x + \sqrt{x^2-1} + c\right]$$

$$y = \mp\left[\frac{x}{2}\sqrt{x^2-1} - \frac{1}{2}\log(x + \sqrt{x^2-1} + c\right] = 0$$

Que $xy(p^2+1) = (x^2+y^2)p$

$$xyp^2 + xy = (x^2+y^2)p$$

$$xyp^2 - (x^2+y^2)p + xy = 0$$

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$$p = \frac{+(x^2+y^2) \pm \sqrt{(x^2+y^2) - 4xy \cdot xy}}{2xy}$$

$$= \frac{+(x^2+y^2) \pm \sqrt{(x^2+y^2)^2 - 4x^2y^2}}{2xy}$$

$$p = \frac{(x^2+y^2) \pm \sqrt{(x^2-y^2)^2}}{2xy}$$

$$p = \frac{x^2+y^2 \pm (x^2-y^2)}{2xy}$$

$$p = \frac{2x^2}{2xy} = \frac{x}{y} \Rightarrow \frac{dy}{dx} = \frac{x}{y}$$

$$\int y\,dy = \int x\,dx + c$$

$$\frac{y^2}{2} = \frac{x^2}{2} + c$$

$$p = \frac{2y^2}{2xy} = \frac{y}{x}$$

$$\frac{dy}{dx} = \frac{y}{x}$$

$$\frac{1}{y}\frac{dy}{d} = \frac{dx}{x} \Rightarrow \log y = \log x + c.$$

$\frac{y^2}{2} - \frac{x^2}{2} - \frac{c^2}{2} = 0$

$\log y - \log x - \log c = 0$

$\log\left(\frac{y}{cx}\right) = 0$

$\frac{y}{cx} = e^0$

$y = cx \qquad y - cx = 0$

Hence general solution is

$(y^2 - x^2 + c^2)(y - cx) = 0$

* Equation solvable for y :-

(i) Convert the given eq^n in the ~~form~~

$$y = f\left(x, p, \frac{dp}{dx}\right) \quad -①$$

(ii) differenciate eq^n ① w.r.t x

$$\frac{dy}{dx} = f'(x, p)$$

replace $\frac{dy}{dx}$ with p

find the value of p and put in eq^n ①

Required General solution

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* Equation solvable for x :

i) convert the given eq^n in the form

$$x = f(y, p) \quad \text{---} ①$$

differenciate eq^n ① w.r.t y.

$$\frac{dx}{dy} = f\left(y, p, \frac{dp}{dy}\right)$$

replace $\frac{dx}{dy} = \frac{1}{p}$

$$\frac{1}{p} = f\left(y, p, \frac{dy}{dy}\right)$$

solve

Q. $y = px + p^3$

$$\frac{dy}{dx} = p.1 + x\frac{dp}{dx} + 3p^2\frac{dp}{dx}$$

$$p = \cancel{p} + x\frac{dp}{dx} + 3p^2\frac{dp}{dx}$$

$$(x+3p^2)\frac{dp}{dx} = 0$$

$$\frac{dp}{dx} = 0$$

$p = c$. Put in eq^n ①

$y = cx + c^3$

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Clairants

$y = px + f(p)$ — Replace by c

Solution $\rightarrow$ $y = cx + f(c)$

Q. $xp^2 + x = 2yp$

$xp^2 - 2yp + x = 0$

$p = \dfrac{+2y \pm \sqrt{4y^2 - 4x^2}}{2x}$

$y = \dfrac{xp^2 + x}{2p}$ — (i)

$\dfrac{dy}{dx} = \dfrac{1}{2}\left\{\dfrac{p.\{(x.2p\frac{dp}{dx} + p^2) + 1\} - (xp^2+x)\frac{dp}{dx}}{p^2}\right.$

$p = \dfrac{1}{2}\left\{\dfrac{2xp^2\frac{dp}{dx} + p^3 + p - xp^2\frac{dp}{dx} - x\frac{dp}{dx}}{p^2}\right\}$

$p^3 = xp^2\dfrac{dp}{dx} + \dfrac{p^3}{2} + \dfrac{p}{2} - \dfrac{xp^2}{2}\dfrac{dp}{dx} - \dfrac{x}{2}\dfrac{dp}{dx}$

$\dfrac{p^3}{2} = \dfrac{1}{2}xp^2\dfrac{dp}{dx} + \dfrac{p}{2} - \dfrac{x}{2}\dfrac{dp}{dx}$

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$$0 = \frac{1}{2} x \frac{dp}{dx}(p^2-1) - \frac{p}{2}(p^2-1)$$

$$\frac{(p^2-1)}{2}\left(x\frac{dp}{dx} - p\right) = 0$$

Now

$$x\frac{dp}{dx} - p = 0$$

$$\frac{dp}{p} - \frac{dx}{x} = 0$$

$$\int \frac{dp}{p} - \int \frac{dx}{x} = 0$$

$$\log p - \log x = \log c$$

$$\log \frac{p}{x} = \log c$$

$$\boxed{p = cx}$$

$$x(cx)^2 + x = 2y(cx)$$

$$c^2x^3 + x = 2ycx$$

$$c^2x^2 + 1 = 2yc \quad \text{Ans}$$

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Q. (i) $x(1+p^2) = 1$

$1+p^2 = \frac{1}{x}$

$p^2 = \frac{1}{x} - 1$

$p = \pm\sqrt{\frac{1-x}{x}}$

$\frac{dy}{dx} = \pm\sqrt{\frac{1-x}{x}}$

$\frac{dy}{dx} = \sqrt{\frac{1-x}{x}}$

$\int dy = \int \sqrt{\frac{1-x}{x}}\, dx$

$y =$

Put $x = \sin^2\theta$

$dx = 2\sin\theta\cos\theta\, d\theta$

$y = \int \frac{\cos\theta}{\sin\theta} \times 2\sin\theta \cos\theta\, d\theta$

$y = 2\int \cos^2\theta\, d\theta$

$y = 2\int \left(\frac{1+\cos 2\theta}{2}\right) d\theta$

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$y = \int (1 + \cos 2\theta) d\theta$

$y = \theta + \frac{\sin 2\theta}{2}$

$y = \theta + \sin\theta \cos\theta + c$

$y = (\sin^{-1}\sqrt{x} + \sqrt{x}\sqrt{1-x} + c)$

$= \pm(\sin^{-1}\sqrt{x} + \sqrt{x}\sqrt{1-x} + c)^2$

Q. $x^2p^2 + xyp - 6y^2 = 0$

$p = \frac{-xy \pm \sqrt{x^2y^2 - 4x^2.(-6y^2)}}{2x^2}$

$p = \frac{-xy \pm \sqrt{x^2y^2 + 24x^2y^2}}{2x^2}$

$p = \frac{-xy \pm 5xy}{2x^2}$

$p = \frac{\cancel{4}^{2}xy}{\cancel{2}x^2} = \frac{2y}{x}$ $\qquad$ $p = -\frac{6xy}{2x^2} = -\frac{3y}{x}$

$\frac{dy}{dx} = \frac{2y}{x}$ $\qquad$ $\frac{dy}{dx} = \frac{-3y}{x}$

$\frac{dy}{2y} = \frac{dx}{x}$ $\qquad$ $-\frac{dy}{3y} = \frac{dx}{x}$

$\frac{1}{2}\log y = \log x + \log c$ $\qquad$ $-\frac{1}{3}\log y = \log x + \log c$

$y = cx^2$ $\qquad$ $y = (cx)^{-3}$

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$y = cx^3$, $y = \frac{c}{x^3}$

$\left(y - cx^2\right)\left(y - \frac{c}{x^3}\right) = 0$

Q $\frac{dy}{dx} - \frac{dx}{dy} = \frac{x}{y} - \frac{y}{x}$

$p - \frac{1}{p} = \frac{x}{y} - \frac{y}{x}$

$\frac{p^2 - 1}{p} = \frac{x}{y} - \frac{y}{x}$

$\frac{p^2 - 1}{p} = \frac{x^2 - y^2}{xy}$

~~pla~~

$xyp^2 - xy = (x^2 - y^2)p$

$xyp^2 - (x^2 - y^2)p - xy = 0$

$p = \frac{(x^2 - y^2) \pm \sqrt{(x^2 - y^2) - 4(xy).(-xy)}}{2xy}$

$= \frac{(x^2 - y^2) \pm \sqrt{(x^2 - y^2) + 4x^2y^2}}{2xy}$

$= \frac{(x^2 - y^2) \pm \sqrt{(x^2 + y^2)^2}}{2xy}$

$$p = \frac{(x^2-y^2) \pm (x^2+y^2)}{2xy}$$

$$p = \frac{2x^2}{2xy} \qquad p = \frac{-2y^2}{2xy}$$

$$p = \frac{x}{y} \qquad p = \frac{-y}{x}$$

Q. $y = px + p^3$

Q. $x^2(y-px) = yp^2$ | $\cancel{\frac{x^2}{p^2}(y-px)} = y$

$x^2y - px^3 = yp^2$

$yp^2 + x^3p - x^2y = 0$

$$p = \frac{-x^3 \pm \sqrt{x^6 - 4\cdot y\cdot(-x^2y)}}{2y}$$

$$= \frac{-x^3 \pm \sqrt{x^6 + 4x^2y^2}}{2y}$$

END

Q.1 $x\frac{dy}{dx} + y = y^2 \log x$

2. $2y \sin x \, dy + (y^2 \cos x + 2x) dx = 0$

~~2. $2y \sin x \, dy + (y^2 \cos x + 2x) dx = 0$~~

3. $p^3(x + 2y) + 3p^2(x + y) + (y + 2x)p = 0$

4. $p^2 - 7p + 12 = 0$

5. $xp^2 - yp - y = 0$

6. $y = 2p + \sqrt{1+p^2}$

7. $p^3 - 2xyp + 4y^2 = 0$

8. $x = y \mp a \log p.$

9. $y^2 = a^2(1+p^2)$

10. $y = (1+p)x + p^2$

11. $e^{4x}(p-1) + e^{2y}p^2 = 0$