'MATTER WAVE'

Wave Associated with Moving particle.

Wave or, De-brogaly wave or, *probable wave.

De-brogali Hypothesis → Every particle ⊕nt in nature represent dual nature. (wave $ particle)

⇒ Wavelength of light $\lambda = \frac{h}{p} = \frac{h}{m_{eff}(c)}$

⇒ Wavelength of moving particle

$\lambda = \frac{h}{p} = \frac{h}{m_{eV}}$; * $m_R = \frac{m_0}{\sqrt{1-\frac{V^2}{c^2}}}$ (m_0 → Rest mass, V → velocity of particle, c → velo of light) ; $\lambda = \frac{h}{m_0 V}\sqrt{1-\frac{c^2}{V^2}}$; $V \neq 0$, $V \neq 0$

* $V = 0$ (Rest) ⇒ $\lambda = \infty$ (not define) * $V = c \Rightarrow \lambda = 0$
* $V <<< c \Rightarrow \left(\frac{V}{c}\right) <<< 1 \Rightarrow \lambda = \frac{h}{m_0 V}$ (neglect)
* 'V' in range of 'c' → $\lambda = \frac{h}{m_R V}$

K.E Transfer $= \frac{1}{2} m_0 V^2 = \frac{P}{2m_0} \Rightarrow P = \sqrt{2m_0 K.E_{Transfer}}$

$$\lambda = \frac{h}{P} = \frac{h}{\sqrt{2m_0 K.E_{Transfer}}}$$

Work done by pot. on charge particle → * $K.E = q\Delta V$ *

$$\lambda = \frac{h}{P} = \frac{h}{m_R V} = \frac{h}{\sqrt{2m_0 K.E}} = \frac{h}{\sqrt{2m_0 q\Delta V}}$$

*** # * If wavelength of wave (λ) = size of obstracle ⇒ Differaction take place.
* If wavelength of wave (λ) > size of obstracle ⇒ Reflection [In case of light shadow formed]
* If wavelength of wave(λ) < size of obstracle ⇒ rectilinear propagation.

NOTE → *De-brogali principle is applicable on micro as well as macro particle but practically it is proved only for micro particle.

2016 AMU * Wave nature of particle practically proved by diffrection principle in a devission Germer & G.B Thomson Exp. **

*** # Standard Result : → $\lambda = \frac{h}{p} = \frac{h}{m_R V} = \frac{h}{\sqrt{2m_0 K.E}} = \frac{h}{\sqrt{2m_0 q\Delta V}}$

|I| → Electron → $\lambda_e = \frac{12.27}{\sqrt{\Delta V(\text{volt})}} A^\circ = \frac{12.27}{\sqrt{K.E(eV)}}$

$\Delta V = \frac{(12.27)^2}{\lambda_e^2} = \frac{150}{\lambda_e^2}$; $\Delta V = \frac{150}{\lambda_e^2}$ volt (λ_e → A°) , $K.E_e = \frac{150}{\lambda_e^2}$ eV (λ_e in A°)

|II| → Proton → $\lambda_p = \frac{0.286}{\sqrt{\Delta V(\text{volt})}} A^\circ = \frac{0.286}{\sqrt{K.E(eV)}} A^\circ$

|III| → Deutron → $\lambda_D = \frac{0.202}{\sqrt{\Delta V(\text{volt})}} A^\circ = \frac{0.202}{\sqrt{K.E(eV)}} A^\circ$

|IV| → α-particle → $\lambda_\alpha = \frac{0.101}{\sqrt{\Delta V(volt)}} A^\circ = \frac{0.101}{\sqrt{\frac{K.E}{2}(ev)}}$

#

particle	charge	Mass
e^-	$-e$	$m_e = \frac{m_p}{1836}$
p	+e	m_p
d	+e	$2m_p$
α	+2e	$4m_p$

*** # Ratio of de-brogali wavelength associated with e^-, H^+, deutron & α-partide when it is move with.
|i| → same Momentum |ii| → same velocity |iii| → same K.E |iv| → same pot.

|i| → $\lambda = \frac{h}{p}$ = same ⇒ $\lambda_e : \lambda_p : \lambda_d : \lambda_\alpha = 1:1:1:1$

|ii| → $\lambda = \frac{h}{mv} \propto \frac{1}{m}$ ⇒ $\lambda_e : \lambda_p : \lambda_d : \lambda_\alpha = \frac{1}{m_e} : \frac{1}{m_p} : \frac{1}{m_d} : \frac{1}{m_\alpha}$

$= 1840 : 1 : \frac{1}{2} : \frac{1}{4}$

$= 1800 \times 4 : 4 : 2 : 1$

$\lambda_e > \lambda_p > \lambda_d > \lambda_\alpha$

|iii| → K.E ⇒ same ⇒ $\lambda = \frac{h}{\sqrt{2mK.E}} \propto \frac{1}{\sqrt{m}}$

$\lambda_e : \lambda_p : \lambda_d : \lambda_\alpha = \sqrt{1840 \times 4} : \sqrt{4} : \sqrt{2} : \sqrt{2}$

|iv| → ΔV ⇒ same ⇒ $\lambda = \frac{h}{\sqrt{2m_0 q \Delta V}} \propto \frac{1}{m}$

NOTE → In same condition de-brogali wavelength of moving e^- is max. & in a neutral particle max for photon.

** Standard Result for Neutral particle

|I| → Neutron: → $\lambda_n = \frac{h}{p} = \frac{h}{m_R v} = \frac{h}{\sqrt{2m_0 K.E}} = \frac{0.286}{\sqrt{K.E(ev)}} A^\circ$

|II| → ** Thermal Neutron: → In a fussion Rxn energy of the neutron is very high that's why moderator is required for fission, moderated neutron.
* It behave as a gas molecule. → $K.E = \frac{f}{2}KT$

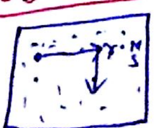
f=3 $K.E = \frac{3}{2}KT \Rightarrow \lambda = \frac{h}{\sqrt{2m(\frac{3}{2}KT)}}$

* $K = \frac{R}{N_A} = 1.38 \times 10^{-23} J/K$ (Boltzmann const.)

$\lambda = \frac{h}{\sqrt{3mKT}} = \frac{28.2}{\sqrt{T(K)}} A^\circ$

|III| → Photon → * $\lambda = \frac{h}{p} = \frac{h}{2mev(c)} = \frac{hc}{E_{ph}} = \frac{12400}{E_{ph}(ev)} A^\circ$

|IV| → Gas Molecule →



$\propto \frac{1}{\sqrt{MOT}}$

$\lambda = \frac{h}{3\sqrt{\frac{3KT}{m}}}$

$\lambda = \sqrt{\frac{h}{\sqrt{3mKT}}}$

$V_{R.M.S} = \sqrt{\frac{3KT}{m}}$

$\lambda = \frac{h}{\sqrt{3(\frac{M_W}{N_A})K.T}}$

comparison of Electron & photon →

	Electron	photon
[I] → Rest mass →	$m_0 = 9.1\times10^{-31} kg = \frac{m_p}{1840}$	$(m_0)_{ph} = 0$
[II] → R.M.S →	$E_0 = m_0c^2 = 0.511 mev$	$(E_0)_{ph} = 0$
[III] → K.E →	$K.E = \frac{1}{2}mv^2$ $(v<<c)$ → $K.E = \frac{hv}{2\lambda_e}$	
[IV] → T.E →	$T.E = \frac{hc^2}{\lambda_e v}$	$T.E_{ph} = K.E_{ph} = \frac{hc}{\lambda}$

[I] → condition Ist → Electron & photon move with same de-brogli wavelength. *

$\lambda_e = \lambda_{ph} = \lambda$, $K.E_{ph} = \frac{hc}{\lambda_{ph}}$, $K.E_{e^-} = \frac{hc}{2\lambda_{e^-}}$, $\frac{K.E_{e^-}}{K.E_{ph}} = \frac{hv/2\lambda}{hc/\lambda} = \frac{v}{2c}$

$v<<c \Rightarrow$ $K.E_{e^-} << K.E_{ph}$

[II] → condition 2nd →

$\lambda_{e^-} = \lambda_{ph} \Rightarrow$ compair its total energy.

$T.E_{ph} = \frac{hc}{\lambda_{ph}}$, $T.E_e = \frac{hc^2}{\lambda_e v}$, * $\frac{T.E_{e^-}}{T.E_{ph}} = \frac{hc^2/hv}{hc/\lambda} = \frac{c}{v} > 1$

* $T.E_e > T.E_{ph}$

III → condition 3rd ⇒ Eelectron & photon move with same K.E compair its de-brogali wavelength.

AIPMT 2016 * $K.E_{ph} = \frac{hc}{\lambda_{ph}}$ $= \frac{h^2}{2m\lambda^2}$

* $\lambda_e = \frac{h}{\sqrt{2mK.E_{e^-}}}$

* $\frac{\lambda_{e^-}}{\lambda_{ph}} = \frac{h/\sqrt{2K.E}}{hc/E}$

* $\frac{\lambda_e}{\lambda_{ph}} = \frac{1}{c}\sqrt{\frac{E}{2m_0}}$

**

* $\sqrt{E} = c\sqrt{2m_0} \Rightarrow \lambda_{e^-} = \lambda_{ph}$
* $\sqrt{E} = c\sqrt{2m_0} \Rightarrow \lambda_{e} > \lambda_{ph}$
* $\sqrt{E} = c\sqrt{2m_0} \Rightarrow \lambda_{e} < \lambda_{ph}$

$\sqrt{E} = c\sqrt{2m_0}$

$E = 2(m_0c^2) = 2\times0.51$

$E = 1.02 MeV$

**

* $E = 1.02$ MeV $\Rightarrow \lambda_{e^-} = \lambda_{ph}$
* $E > 1.02$ MeV $\Rightarrow \lambda_{e^-} > \lambda_{ph}$
* $E < 1.02$ MeV $\Rightarrow \lambda_{e^-} < \lambda_{ph}$

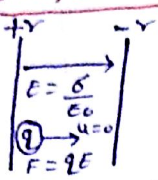
NOTE → * If e^- & photon move $\bar{c}$ same de-brogali wavelength then $K.E_{ph} > K.E_{e^-}$ · but Total energy of photon is less than from e^-

* If e^- & photon move $\bar{c}$ same K.E then de-brogali wavelength depend on magnitude of energy.

*** # Special case →

Case [I] → Motion of charged particle in presence of electric field.

[A] → Uniform Electric field →

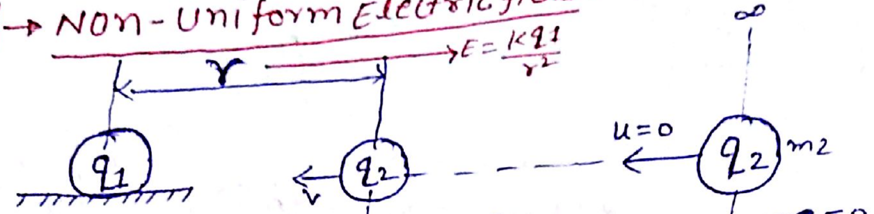


$$V=\frac{qEt}{m}$$

* $$P=mv=qEt$$

* $$\lambda=\frac{h}{P}=\frac{h}{qET}$$

[B] → Non-Uniform Electric field: →



$K.E_r=\frac{1}{2}mv^2$

$P.E_r=\frac{K(q_1)(q_2)}{r}$

$K.E_\infty=0$

$P.E_\infty=0$

* $$K.E_r=\frac{Kq_1q_2}{r}$$

** $$\lambda=\frac{h}{\sqrt{2m\left(\frac{Kq_1q_2}{r}\right)}}$$

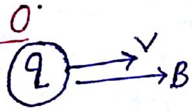
NOTE → In presence of electric field, de-broglie wavelength of charged particle is change at every instant.

Case [II] → Motion of charge particle in presence of Mag. field →

Lorentz force → $F=qVB\sin\theta$

* $V=0\Rightarrow F=0\Rightarrow$ Rest $\Rightarrow\lambda=\infty$

[A] → $\theta=0°$

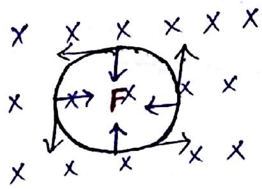


$F=0\Rightarrow a=0\Rightarrow V=const\Rightarrow\lambda=\frac{h}{mv}=const.$

[B] → $\theta=180°$

$F=0,\ a=0,\ V=const,\quad \lambda=\frac{h}{mv}=const$

[C] $\theta=90°$



$\vec{F}\perp\vec{V}\Rightarrow W=0=\Delta K.E$

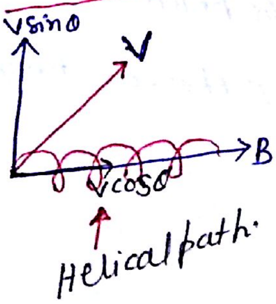
* K.E = const, * Speed = const.

$F=qVB\sin 90°=\frac{mv^2}{r}$

$P=mv=qBr$

* $$\lambda=\frac{h}{P}=\frac{h}{qBr}=const.$$

[D] → $\theta\neq 90°, 0°, 180°$

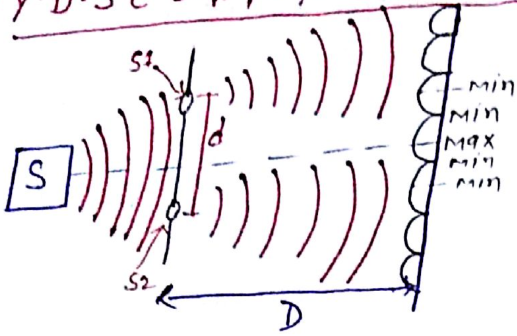


* $$\lambda_{\parallel}=\frac{h}{mv\cos\theta}=const.$$

$$\lambda_{\perp}=\frac{h}{mv\sin\theta}=const.$$

$$\lambda=\frac{h}{mv}=const.$$

Case |III| → Y·D·S·C Exp perform with Matter Wave



* $\beta = \frac{\lambda D}{d} = \frac{h}{\sqrt{2m_0 q \Delta V}} \frac{D}{d} \propto \frac{1}{\sqrt{\Delta V}}$

* $\Delta V \uparrow \Rightarrow \beta \downarrow$
$\Delta V \downarrow \Rightarrow \beta \uparrow$

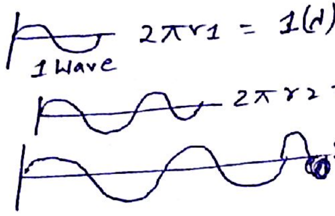
AIR AFIMS BHU 2011 AMU

Explanation of Rutherford Drawback & Bohr quantisation condn →

* |i| → Electron Rotate around the nucleus in a stable path that's why its Matter Waves is also move in a same circular orbit. When Wave are bounded it produce standing wave that's why energy of Rotating e⁻ remain same & e⁻ Rotate in stable circular orbit. A/R

|ii| → Length of its path is complete multiple of wavelength. $2\pi r = n\lambda$

* $n = 1 \Rightarrow$ (1 wave) $2\pi r_1 = 1(\lambda)$
* $n = 2 \Rightarrow$ $2\pi r_2 = 2(\lambda)$
* $n = 3 \Rightarrow$ $2\pi r_3 = 3(\lambda)$.



$2\pi r = n\lambda$
$2\pi r = n\left(\frac{h}{mv}\right)$
* $J = mvr = \frac{nh}{2\pi}$

Devison – Germer Exp → practically prove Wave nature of particle.

*Construction & Working → Electron gun based on thermeonic emmison principle. Electron beam incident on 'Ni' crystal & scatered it is collected by ionising chamber & practically calculate Max. Intensity condition.

* Thermionic emission → Emission of e⁻ takes place [redacted] by the thermal energy.
* Photoelectric emmision → Electron emission takes place with energy of photon.
* Field emission → Electron emission take place with external electrifield. ($10^8 Vm^{-1}$) Electron gun pot.

Max intensity is formed at 50° deviation & at 54 volt electron gun pot.

Exp. Result →

|i| → Max intensity $\boxed{\Delta V = 54 Volt}$ *
$\theta = 50° \Rightarrow \phi = 90° - \frac{50}{2} = \boxed{65}$

AIEEE/AIPMT
|ii| → * $D\sin\theta = n\lambda$
$2d\sin\theta = n\lambda$

λ = Wavelength, n = order of differaction = 1, 2, 3 - - -

$D\sin\theta = n\lambda$
$D_{Ni} = 2.15 A°$
$\lambda_{max} = \frac{D\sin\theta}{n_{min}}$ = 1
$\lambda_{max} = D\sin\theta$ *
$= \boxed{2.15 \sin 50}$

* $\lambda_{max} = 1.66 A°$
* $\lambda_{DB} = \frac{12.27}{\sqrt{\Delta V (volt)}} A°$
$= \frac{12.27}{\sqrt{54}}$
* $\lambda_{DB} = 1.65 A°$

λ practical value $\simeq$ λ_{DB} value