

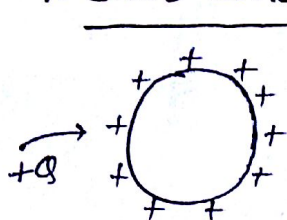
CAPACITOR

* C does not depend on Q & V .

The capacity of conductor to store charge on its shape & size.

* Dielectric strength of medium $\rightarrow$ It is max electric field after which insulation of medium get punched. for air $= 3 \times 10^6 \text{ V/m}$

solid conducting sphere $\rightarrow$



$$V_{\text{sphere}} = \left(\frac{1}{4\pi\epsilon_0} \right) \frac{Q}{R}$$

$$V_{\text{sphere}} - V_{\text{infinite}} = \Delta V = \frac{Q}{(4\pi\epsilon_0)R}$$

$$Q = (4\pi\epsilon_0 R) \Delta V$$

$$C = 4\pi\epsilon_0 R$$

$$Q = C \Delta V$$

* In conductor

$$Q(\uparrow) = V(\uparrow)$$

means, charge (Q) $\propto$ pot. (V)

Unit $\rightarrow$ MKS $\rightarrow \frac{C}{\text{volt}}$, Farad (F)
CGS $\rightarrow$ stat farad
($1 \text{ F} = 9 \times 10^{11} \text{ stat.F}$)

Imp #

$C_{\text{earth}} = ?$

* Theoretical value $\rightarrow$ By considering it conductor of $6370(6400) \text{ km}$

$$C = 711 \mu\text{F}$$

* Practically $\rightarrow$ earth can accept unlimited charge so its capacitance is infinite (∞)

* capacitance of any earth conductor is infinite.

EX $\rightarrow$



capacitance = C
take limited charge



capacitance = ∞
take unlimited charge

$$\left\{ \text{b/c } C = \frac{Q}{V} = \frac{Q}{0} = \infty \right\}$$

potential energy stored in charge conductor $\rightarrow$
Some external work is done during charging of conductor which is stored in form of pot. energy.

$$U = \frac{1}{2} C V^2 = \frac{1}{2} Q V = \frac{Q^2}{2C}$$

* p.e $\oplus$ only in charge conductor but capacitance $\oplus$ in charge or, uncharge.

Energy Spent by Battery

$$\therefore \text{Emf} = V$$

$$1 \text{ charge} \rightarrow \text{Work} = V$$

$$q \rightarrow \text{Work} = qV$$

$$\text{Work by battery} = \Delta U + \text{Heat}$$

$$qV = \left(\frac{1}{2}qV - 0\right) + \text{Heat}$$

$$\text{Heat} = \frac{1}{2}qV$$

NOTE → During charging half of energy spent for battery is stored in the form of P.E of conductor & remaining half wasted in form of Heat in connected wire.

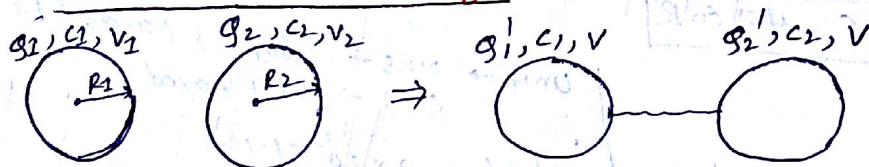
EX → For a conductor

$$\text{If charge} = q = PE = U$$

$$\text{then charge} = q/2 = PE = ?$$

$$C = \text{const.}, U = \frac{q^2}{2C} \propto q^2, U' = \frac{U}{4}$$

** Re-distribution of charge: →



* $C_{\text{system}} = C_1 + C_2 + C$ ← It is capacitance of conductor wire it is negligible.

$$C_{\text{system}} = C_1 + C_2$$

* Total charge of system is re-distributed is ratio of capacitance.

$$q \propto CV$$

$$q \propto C \propto R$$

* $V_{\text{common}} = ?$

$$q_1 + q_2 = q_1' + q_2'$$

$$C_1 V_1 + C_2 V_2 = C_1 V + C_2 V$$

$$V_{\text{common}} = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$$

$$V_{\text{common}} = \frac{q_1 + q_2}{C_1 + C_2}$$

$$V_{\text{system}} = \frac{q_{\text{system}}}{C_{\text{system}}}$$

* Energy loss (ΔU)

$$U_{\text{Initial}} = \frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2$$

$$U_{\text{Final}} = \frac{1}{2} C_1 V^2 + \frac{1}{2} C_2 V^2$$

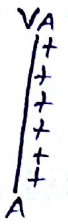
$$U_{\text{Initial}} - U_{\text{Final}}$$

$$\Delta U = U_{\text{Final}} - U_{\text{Initial}}$$

$$\Delta U = \frac{C_1 C_2}{2(C_1 + C_2)} (V_1 - V_2)^2$$

Concept of capacitor : $\rightarrow$ A/R Generally, capacitors are formed by using 2 conductors having equal & opposite charge on them. The advantage of using opp. charge is to confine all the electrical energy in certain volume so that it can be easily extracted when required.

Case I



pot. of plate A

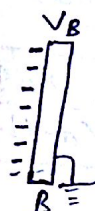
$$V_A' = V_A - V_B^- + V_B^+$$

$$\therefore |V_B^-| > |V_B^+| \quad \left(\text{Here } B^- \text{ more due to more impact of plate A} \right)$$

$$\Rightarrow V_A' < V_A$$

$$\therefore C = \frac{Q}{V(\downarrow)} = C(\uparrow)$$

Case II



$$V_A'' = V_A - V_B^-$$

$$V_A'' < V_A$$

$$\therefore C = \frac{Q}{V(\downarrow\downarrow)} = C(\uparrow\uparrow)$$

A/R

* $\rightarrow$ When a neutral or, opp. charge conductor is placed near a charge conductor then charge of 1st conductor remains unchanged while pot. changes significantly. So capacitance $\uparrow$

Type of capacitor $\rightarrow$

iii $\rightarrow$ Parallel Plate capacitor (PPC) $\rightarrow$

$$C = \frac{A\epsilon_0}{d}$$

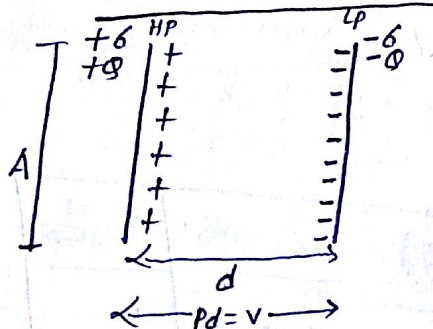
lii $\rightarrow$ Spherical capacitor $\rightarrow$

$$C = 4\pi\epsilon_0 \left(\frac{ab}{b-a} \right)$$

lii $\rightarrow$ Cylindrical capacitor $\rightarrow$

$$C = \frac{2\pi\epsilon_0 L}{\ln \frac{R_2}{R_1}}$$

iii $\rightarrow$ Parallel plate capacitor : $\rightarrow$



(a) $\rightarrow$ Electric field (E) $\rightarrow$

$$\left[\begin{array}{l} E_{\text{inside}} = \frac{\sigma}{\epsilon_0 \epsilon_r} = \frac{V}{d} \quad (\text{uniform}) \\ E_{\text{outside}} = 0 \end{array} \right]$$

(b) $\rightarrow$ Pot. energy (U) $\rightarrow U = \frac{1}{2} CV^2 = \frac{1}{2} QV = \frac{Q^2}{2C}$

(c) $\rightarrow$ Energy density / Electrostatic pressure (u) $\rightarrow$ energy store in unit vol.

$$u = \frac{\text{Energy}}{\text{vol.}}$$

$$U = \frac{1}{2} \frac{\epsilon_0}{a} \left(\frac{V}{d} \right)^2 = \frac{1}{2} \epsilon_0 E^2$$

$$* U = \left\{ \frac{1}{2} \epsilon_0 \left(\frac{\sigma}{\epsilon_0} \right)^2 = \frac{\sigma^2}{2\epsilon_0} \right\}$$

1d) $\rightarrow$ Attraction force b/w // plate: $\rightarrow$

$$= \frac{\sigma^2}{2\epsilon_0} \quad \left| \quad F = \left(\frac{\sigma^2}{2\epsilon_0} \right) A \right. *$$

$$= \left(\frac{\sigma^2}{2\epsilon_0} \right) A$$

1e) $\rightarrow$ capacitance (C) $\rightarrow$

|Air| $\left\{ C_{air} = \frac{\epsilon_0 A}{d} \right\}$

 $C_{med} = \frac{\epsilon_0 \epsilon_r A}{d}$

$\therefore C_{med} = \epsilon_r C_{air}$

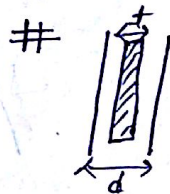
$\therefore \epsilon_r > 1$

$\Rightarrow C_{med} > C_{air}$

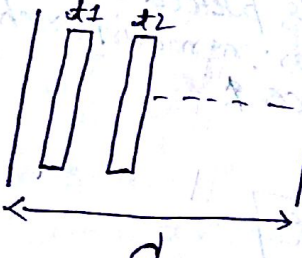
If $C_{air} = C_{med}$

$$\frac{\epsilon_0 A}{d_{air}} = \frac{\epsilon_0 \epsilon_r A}{d_{med}}$$

$$d_{air} = \frac{d_{med}}{\epsilon_r}$$

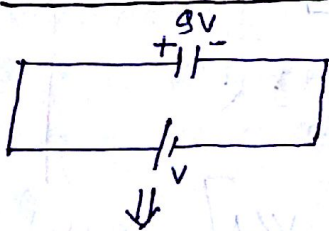


$$C_{partial} = \frac{\epsilon_0 A}{(d-t) + t/\epsilon_r}$$

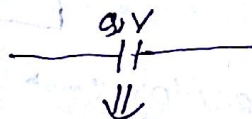
 $C = \frac{\epsilon_0 A}{(d-t_1-t_2) + \left(\frac{t_1}{\epsilon_{r1}} + \frac{t_2}{\epsilon_{r2}} \right)}$

* capacitance is independent of position of slab with respect to plate.

Variation in charge PPC $\rightarrow$



$V_c = \text{const}$
(battre connected)



$Q_c = \text{const}$
battre disconnected

Case-I $\rightarrow$ Battery^{dis}connected ($Q_c = \text{const}$) $\rightarrow$

Variation	$C = \frac{\epsilon_0 \epsilon_r A}{d}$	$V_C = \frac{Q}{C}$	 $Q_C = \text{const}$	$E = \frac{Q}{A \epsilon_0 \epsilon_r}$	$V_C \propto \frac{Q^2}{2C}$
$\epsilon_r (\uparrow)$	$\uparrow$	$\downarrow$	$\leftrightarrow$	$\downarrow$	$\downarrow$
$A (\uparrow)$	$\uparrow$	$\downarrow$	$\leftrightarrow$	$\downarrow$	$\downarrow$
$d (\uparrow)$	$\downarrow$	$\uparrow$	$\leftrightarrow$	$\leftrightarrow$	$\uparrow$

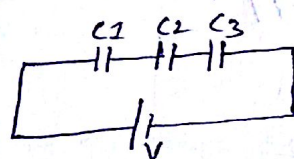
Case II $\Rightarrow$ Battery connected ($V_c = \text{const}$)

Variation	$C = \frac{\epsilon_0 \epsilon_r A}{d}$	$V_c = \text{const}$	$Q \propto C$ $Q_c = CV$	$E = \frac{V}{d}$	$U_c = \frac{1}{2} CV^2$
$\epsilon_r \uparrow$	$\uparrow$	$\longleftrightarrow$	$\uparrow$	$\longleftrightarrow$	$\uparrow$
$A \uparrow$	$\uparrow$	$\longleftrightarrow$	$\uparrow$	$\longleftrightarrow$	$\uparrow$
$d \uparrow$	$\downarrow$	$\longleftrightarrow$	$\downarrow$	$\downarrow$	$\uparrow$

* If nothing is given in question then consider battery disconnected.

Grouping of capacitor $\Rightarrow$

Series



$\rightarrow Q = \text{same}$

$$\rightarrow \frac{1}{C_{\text{net}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

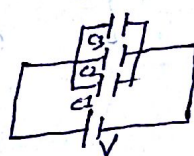
$$\rightarrow C_{\text{net}} = \frac{C_1 \times C_2}{C_1 + C_2}$$

$\rightarrow$ voltage distribution

$$V \propto \frac{1}{C}$$

$$V_1 : V_2 : V_3 = \frac{1}{C_1} : \frac{1}{C_2} : \frac{1}{C_3}$$

parallel



$\rightarrow V = \text{same}$

$$\rightarrow C_{\text{net}} = C_1 + C_2 + C_3$$

$\rightarrow$ charge distribution

$$Q = CV$$

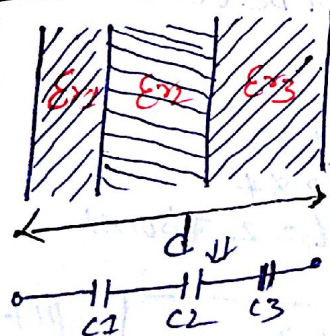
$$Q \propto C$$

$$Q_1 : Q_2 : Q_3 = C_1 : C_2 : C_3$$

Distribution in PPC by dielectric medium $\Rightarrow$

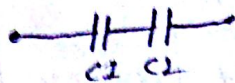
$$\left| \begin{array}{c} \text{Air} \\ \text{PPC} \end{array} \right| \quad C = \frac{\epsilon_0 A}{d}$$

|a| $\rightarrow$ Division in distance $\rightarrow$



[Each section is capacitor & all capacitor are in series]

EX →



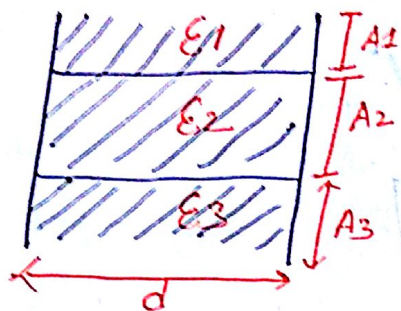
$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

$$C_{eq} = \frac{2\epsilon_1 \epsilon_2}{\epsilon_1 + \epsilon_2}$$

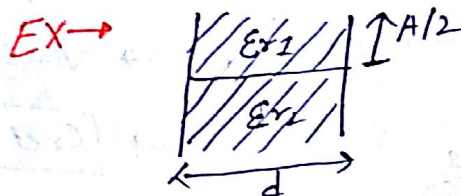
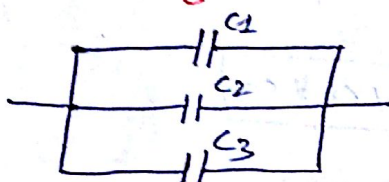
$$C_{eq} = \frac{\epsilon_0 A}{\left(d - \frac{t_1}{\epsilon_1} + \frac{t_2}{\epsilon_2} \dots\right) + \left(\frac{t_1}{\epsilon_1} + \frac{t_2}{\epsilon_2}\right)}$$

* Equivalent dielectric const $(\epsilon_r)_{eq} = \frac{2\epsilon_1 \epsilon_2}{\epsilon_1 + \epsilon_2}$

|b| → Division in Area: →



[Each section is a capacitor and all capacitor are in ||.]



$$C_{eq} = C_1 + C_2 = \left(\frac{\epsilon_1 + \epsilon_2}{2}\right) C$$

$$(\epsilon_r)_{eq} = \frac{\epsilon_1 + \epsilon_2}{2}$$

* $\frac{x \& y}{A \cdot M} \Rightarrow \frac{x+y}{2}$, $G.M = \sqrt{xy}$, $H.M = \frac{2xy}{x+y}$

Multiple capacitor →

→ * Given plate arrangement converted into equivalent circuit using point potential method.

ii) → No. of all plate is done

iii) → For making capacitor plate should be conductive there pot. should be diff.

EX →

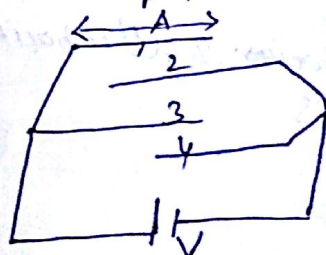
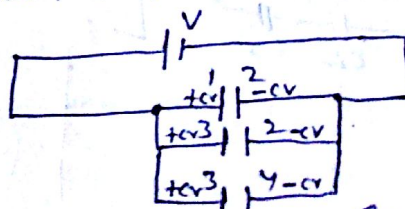


plate = 4

conductor = 2

separate potential = 2 = point



iii) → charge on each plate

$$Q_1 = +CV$$

$$Q_2 = -CV - CV = -2CV$$

$$Q_3 = +CV + CV = +2CV$$

$$Q_4 = -C$$

$$ii) \rightarrow C_{eq} = 3C = 3\left(\frac{\epsilon_0 A}{d}\right)$$

* $\rightarrow$ In case of n -parallel plate, if alternate plate are connected with each other then max $(n-1)$ capacitor are there in parallel grouping so, $C_{max} = (n-1)C$

Spherical capacitor $\Rightarrow$

Case I $\rightarrow$ If ^{inner} sphere is charged & outer is earthed $\rightarrow$



$$V_{small} = \frac{kQ}{r} - \frac{kQ}{R}$$

$$V_{big} = \frac{kQ}{R} - \frac{kQ}{R} = 0$$

pot. difference

$$V = kQ \left(\frac{1}{r} - \frac{1}{R} \right), \quad V = \frac{kQ}{4\pi\epsilon_0} \left(\frac{R-r}{Rr} \right)$$

$$\frac{Q}{V} = \frac{4\pi\epsilon_0 Rr}{R-r}$$

$$C_I = \frac{4\pi\epsilon_0 Rr}{R-r}$$

* If r = fixed $R \rightarrow \uparrow$ then capacitance = ?

$$C = \frac{4\pi\epsilon_0 r}{1 - \frac{r}{R}} \quad (\text{divided by } R)$$

$C = \downarrow$ bcoz $\frac{r}{R} \downarrow$ so, $1 - \frac{r}{R} \uparrow$ something by $\frac{1}{\text{denominator}} (\downarrow)$

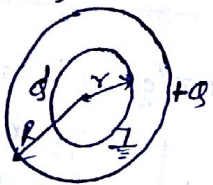
* If r = Fixed, $R \downarrow$ then capacitance $\rightarrow \uparrow$

* If $R = \infty$ then capacitance?

$$C = \frac{4\pi\epsilon_0 \epsilon_r r}{1 - \frac{r}{R}} \quad (C = 4\pi\epsilon_0 \epsilon_r r)$$

Each individual conductor is also a capacitor considering that its other conductor is at infinite.

Case II $\rightarrow$ If outer sphere is charged & inner is earthed.
AS inner sphere is earthed the same charge Q' is shifted from earth to this sphere for making its pot zero.



$$V_{big} = \frac{kQ}{R} + \frac{kQ'}{R}$$

$$V_{small} = 0$$

$$\therefore \text{P.D.} \quad V = \frac{kQ}{R} + \frac{k}{R} \left(-\frac{r}{R} Q \right)$$

$$V = \frac{kQ}{R} \left(1 - \frac{r}{R} \right)$$

$$V = \frac{Q}{4\pi\epsilon_0 \epsilon_r R} \left(\frac{R-r}{R} \right)$$

$$C_{II} = 4\pi\epsilon_0 \epsilon_r R \left(\frac{R}{R-r} \right)$$

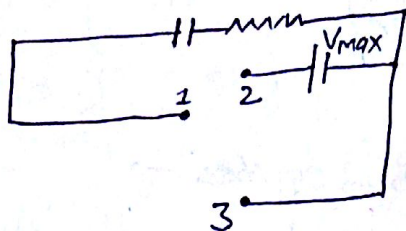
$$C_{II} = \frac{4\pi\epsilon_0 \epsilon_r Rr}{R-r} + 4\pi\epsilon_0 \epsilon_r R$$

$$C_{II} = C_I + 4\pi\epsilon_0 \epsilon_r R$$

$$C_{II} > C_I$$

* If nothing is given in question then we use case I

charging & discharging of capacitor →



case I → charging →

(1-2)

$t = 0$	$t = \uparrow$	$t = \infty$ (full charging)
$V_c = 0$	$V_c = \uparrow$	$V_c = V_{max}$ (max)
$Q_c = 0$	$Q_c = \uparrow$	$Q_c = CV_{max}$ (max)
$U_c = 0$	$U_c = \uparrow$	$U_c = \frac{1}{2} CV_{max}^2$ (max)
$I_{ckt} = \frac{V_{max}}{R}$ (max)	$I_{ckt} \downarrow$	$I_{ckt} = 0$

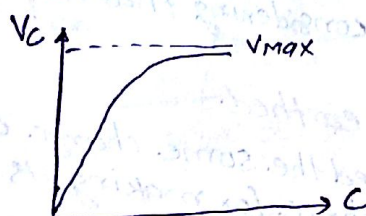
* Here, R is used to safe the capacitor from starting excess current.

By KVL

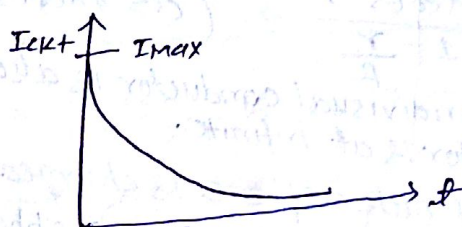
$$V_c = V_{max} (1 - e^{-t/RC})$$

$$I_{ckt} = I_{max} e^{-t/RC}$$

Graph b/w V_c & t



Graph b/w I_{ckt} & t



$$e = 2.73$$

$$\frac{1}{e} = 0.37$$

$$e^0 = 1$$

$$e^\infty = \infty$$

$$e^{-\infty} = 0$$

$$\log_{10} 2 = 0.301$$

$$\log_{10} 3 = 0.473$$

$$\log_{10} 5 = 0.699$$

$$\log_e x = 2.303 \log$$

$$\log e^2 = 2.303 \log 10^2 = 2.303 \times 0.301 = 0.693$$

$$\log_{10} x = 1, \log_{10} 10 = 1$$

Time cost (τ) → It is time in which approx 63% working take place.

$$V_c = V_{max} (1 - e^{-t/RC})$$

$$\text{If } t = RC$$

$$V_c = V_{max} (1 - e^{-1})$$

$$V_c = 0.63 V_{max}$$

$$\tau = RC$$

$$I_{mb} +$$

$$t = 5\tau = 99.3\%$$

$$t = \tau = 63\%$$

* Heat produced = $\frac{1}{2} CV^2$ [not depend on Resistance]

* If $R \uparrow \Rightarrow$ Heat $\leftrightarrow$
 timing $\uparrow$
 $I_{ckt} \downarrow$

* If $R \downarrow \Rightarrow$ Heat $\leftrightarrow$
 timing $\downarrow$
 $I_{ckt} \uparrow$

* Time for 99% charging $\rightarrow 0.921 \text{ msec}$

Trick
 $1\tau = 200 \mu\text{sec}$
 $5\tau = 1000 \mu\text{sec}$

$t = 5\tau = 99.9\%$ Work done so $t = \text{less than } 1000 \mu\text{sec}$

* time for 50% charging $\rightarrow t = 200 \times 10^{-6} \times 0.693 = t = 138.6 \mu\text{sec}$

case II $\Rightarrow$ Discharging $\rightarrow$

$t = 0$

$V_C = V_{\text{max}}$

$q_C = q_{\text{max}}$

$U_C = \frac{1}{2} CV_{\text{max}}^2$

$I_{ckt} = \frac{V_{\text{max}}}{R}$

$t = \uparrow$

$V_C = \downarrow$

$q_C = \downarrow$

$U_C = \downarrow$

$I_{ckt} = \downarrow$

$$\therefore V_C = V_{\text{max}} e^{-t/RC}$$

$$I_{ckt} = -I_{\text{max}} e^{-t/RC}$$

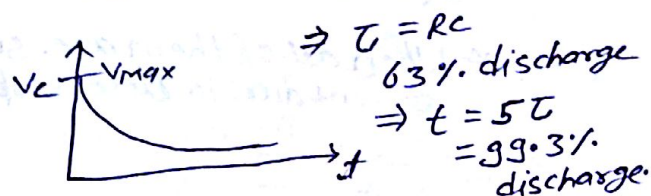
$t = \infty$ (Full discharge)

$V_C = 0$

$q_C = 0$

$U_C = 0$

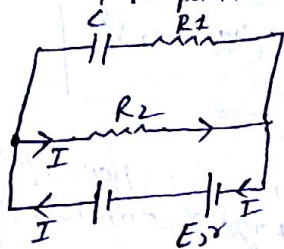
$I_{ckt} = 0$



Trick * $\tau = 1 \text{ sec}$ 63% discharge ho jayega, so 50% discharge less than 1 sec.

RC CKT $\rightarrow$ In steady state condⁿ capacitor are fully charge so current in capacitor branch is zero.

EX $\rightarrow$



In stable condⁿ find charge on capacitor

$$I = \frac{E}{R_2 + r}$$

$$\therefore V_{R_2} = I \times R_2 = \frac{R_2}{R_2 + r} E$$

$$\therefore V_{R_1} = \text{const} \times R_1 = 0 \times R_1 = 0$$

$$V_{C1} = V_{R_2} = \frac{R_2}{R_2 + r} E$$

$$Q = CV_C = \frac{R_2}{R_2 + r} CE$$

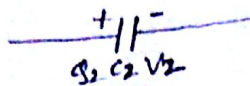
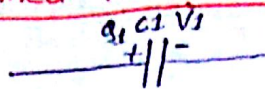
* If $R_2(\uparrow) \Rightarrow Q = ?$

$$Q = \frac{R_2}{R_2 + r} CE$$

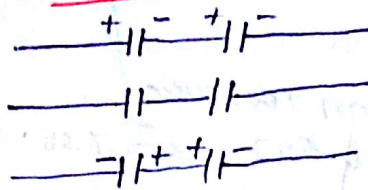
Trick
 $\frac{R_2}{R_2 + r}$ ratio का +1 = का, something upon का (योग)

NOTE $\rightarrow$ * At $(t=0)$, $I = I_{\text{max}}$ 'C' is like short CKT.
 * At $(t=\infty)$, $I = 0$ 'C' is like open CKT.

To connect two charge PPC →

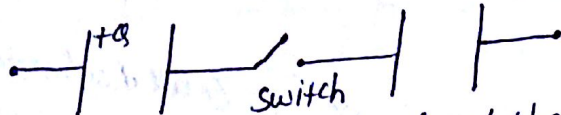


case-I → Series -



!! No close loop so no charge flow so no change in detail of capacitor.

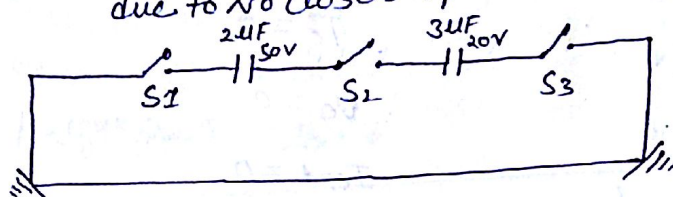
IIT
**
EX →



After switch on after long time what will charge on uncharged capacitor.

Ans → When switch is closed then no charge flow in uncharged capacitor due to no close loop.

IIT
EX →

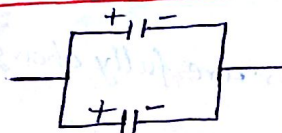


* If any one or two switch are made on then there is no close loop no charge, no current flow so, the details of charge & voltage remains same.

* When all of them are switch are made then there is charge in charge current due to close loop is formed.

case II → parallel -

1a) → To connect same polarity ends →

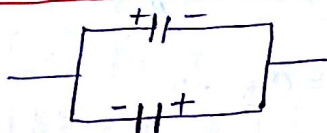


* If voltage are different then charge sharing take place to obtain same voltage.

$$V_{\text{common}} = \frac{V_1 C_1 + V_2 C_2}{C_1 + C_2}$$

$$\Delta U = \frac{C_1 C_2}{2(C_1 + C_2)} (V_1 - V_2)^2$$

1b) → To connect opposite polarity plate →



* charge sharing does take place to obtain common voltage & common polarity.

$$V_2 \rightarrow -V_2$$

$$V_{\text{common}} = \frac{C_1 V_1 - C_2 V_2}{C_1 + C_2}$$

$$\Delta U = \frac{C_1 C_2}{2(C_1 + C_2)} (V_1 + V_2)^2$$

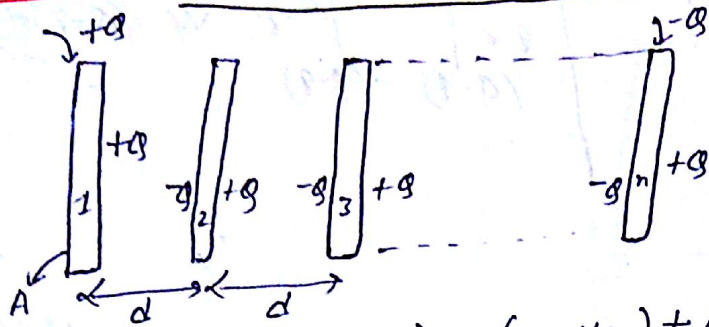
(plus)

* Nothing is said in q. then we assume same polarity.

* NOTE → If $C_2 = xC$, the $V_2 = C$
 $V_2 = V/x$ b/c $q = \text{constant}$ $q = CV$
 $V \propto \frac{1}{C}$

Different cases of capacitor

case-I $\Rightarrow$ 'n' no. of plates are placed: -



$$V_1 - V_n = (V_1 - V_2) + (V_2 - V_3) + (V_3 - V_4) + \dots + (V_{n-1} - V_n)$$

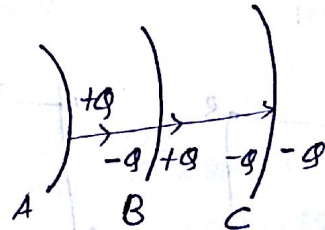
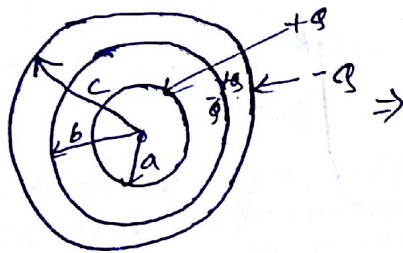
$$\Delta V = \frac{Qd}{A\epsilon_0} + \frac{Qd}{A\epsilon_0} + \dots + \frac{Qd}{A\epsilon_0}$$

$$\Delta V = \frac{Qd(n-1)}{A\epsilon_0}$$

$$Q = \left[\frac{A\epsilon_0}{(n-1)d} \right] \Delta V$$

$$C = \frac{A\epsilon_0}{(n-1)d}$$

case-II $\Rightarrow$



$$V_A = \frac{kQ}{a} - \frac{kQ}{c}$$

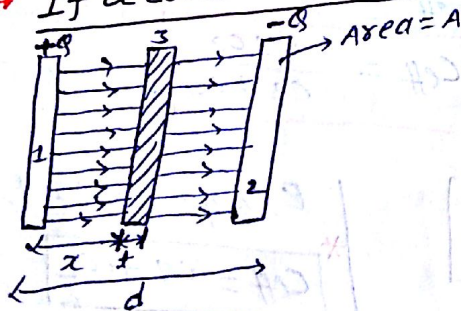
$$V_C = \frac{kQ}{c} - \frac{kQ}{c} = 0$$

$$\Delta V = V_A - V_C = kQ \left[\frac{c-a}{ac} \right]$$

$$Q = \frac{(4\pi\epsilon_0)ac}{(c-a)} \Delta V$$

$$C = \frac{(4\pi\epsilon_0)ac}{c-a}$$

case-III $\Rightarrow$ If a conducting slab with thickness 't' is introduced. $\rightarrow$

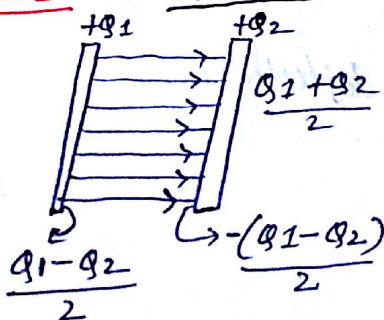


$$V_1 - V_2 = (V_1 - V_3) + (V_3 - V_2) = \frac{Qx}{A\epsilon_0} + \frac{Q(\alpha - t - x)}{A\epsilon_0}$$

$$\Delta V = \frac{Q(d-t)}{A\epsilon_0}$$

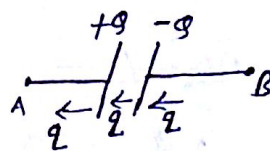
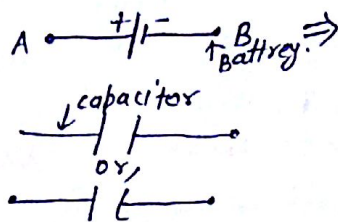
$$C = \frac{A\epsilon_0}{d-t}$$

case-IV $\Rightarrow$ Actual charge on capacitor $\rightarrow$

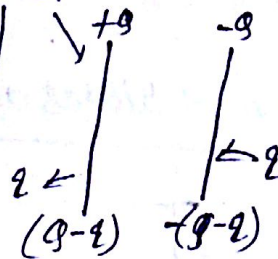


$$\text{charge on capacitor} = \frac{Q_1 - Q_2}{2}$$

Battery & capacitor →

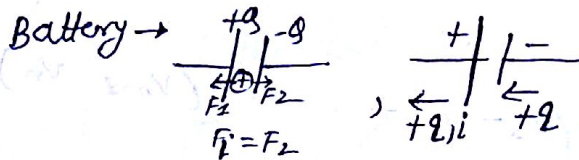


capacitor



$$V_i = \frac{Qd}{A\epsilon_0}$$

$$V_f = \frac{(Q-Q)d}{A\epsilon_0}$$



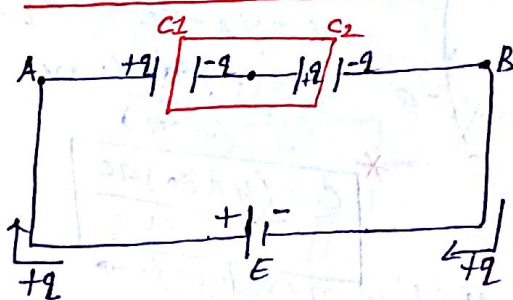
* ⊕ve charge will leave from ⊕ve terminal & must enter the ⊖ve terminal of the battery.

* Inside the battery charge is taken from lower pot. plate to higher pot. plate with the help of battery mechanism.

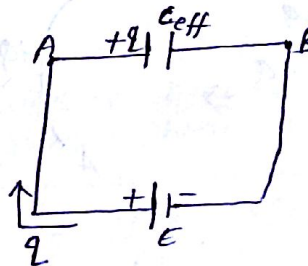
• Work done by battery = charge flow (Q) × pot difference b/w terminal. (Emf of battery)

Series & Parallel combination

Series combination



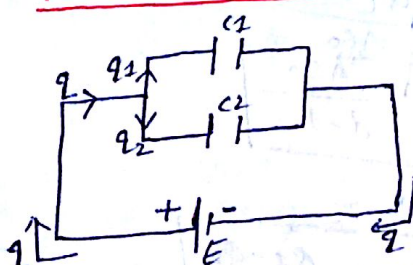
⇒



$$V_A - V_B = \frac{Q}{C_1} + \frac{Q}{C_2} = \frac{Q}{C_{eff}} = E \quad , \quad \frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$* \quad C_{eff} = \frac{C_1 C_2}{C_1 + C_2}$$

Parallel combination



$$* \quad \frac{Q_1}{Q_2} = \frac{C_1}{C_2}$$

$$Q = Q_1 + Q_2$$

$$E \times C_{eff} = E \times C_1 + E \times C_2$$

$$* \quad C_{eff} = C_1 + C_2$$

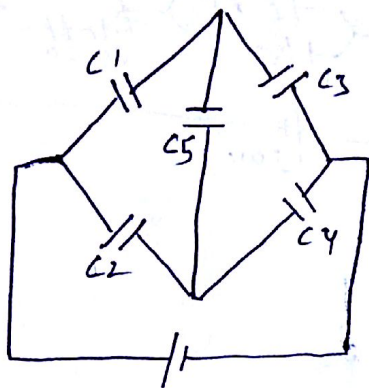
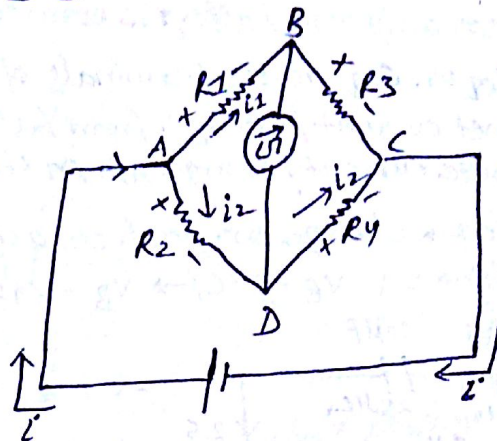
NOTE Wheatstone Bridge

When galvanometer should null (zero) deflection

$$\left. \begin{aligned} V_A - V_B &= +i_1 R_1 \\ V_A - V_D &= +i_2 R_2 \end{aligned} \right\} V_B - V_D$$

$$i_1 R_1 = R_2 i_2 \text{ --- (1) } , \quad i_3 R_3 = i_2 R_4 \text{ --- (2)}$$

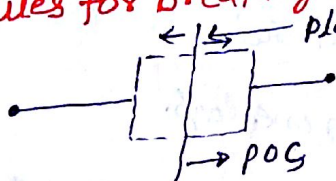
$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$



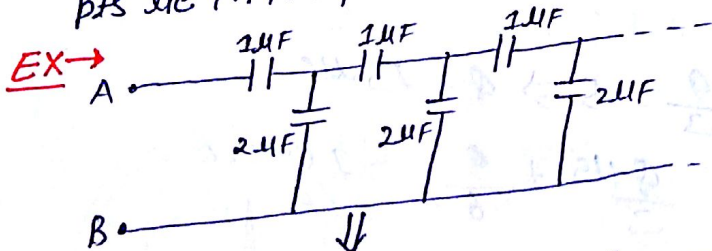
$$\frac{C_1}{C_2} = \frac{C_3}{C_4}$$

Balanced Wheat stone Bridge

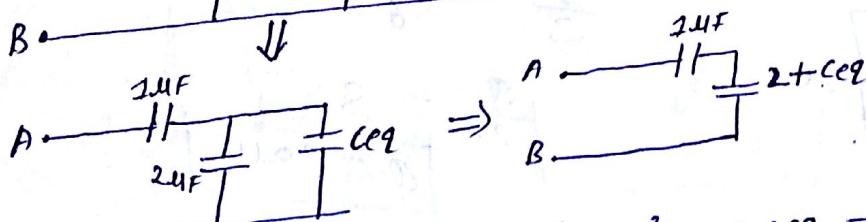
Rules for Breaking or, folding of ckt.



- * We can join any no. of points which are laying on plane of symmetry. They all are at same potential.
- * We can break a point in ckt in any no. of points if after breaking all the pts lie in the plane of symmetry.



$$C_{eq} = \frac{-2 \pm \sqrt{4+8}}{2} = (-1 + \sqrt{3}) \mu F$$



$$\frac{1}{C_{eq}} = \frac{1}{2+C_{eq}} + \frac{1}{1}$$

$$C_{eq} = \frac{1 \times (2+C_{eq})}{1+2+C_{eq}}$$

$$C_{eq}^2 = 3C_{eq} = C_{eq} + 2$$

$$C_{eq}^2 - 2C_{eq} - 2 = 0$$

General ckt solving -

Case-I $\Rightarrow$ When only one battery is connected -

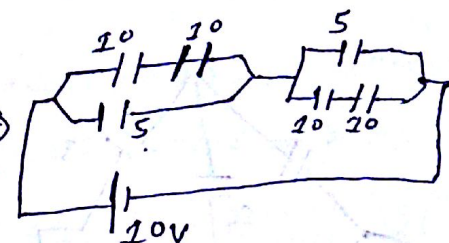
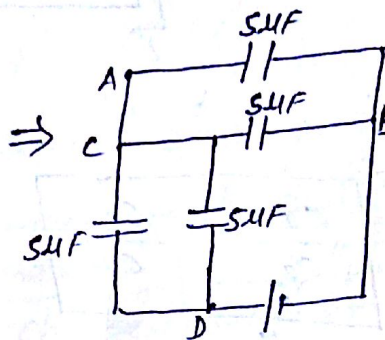
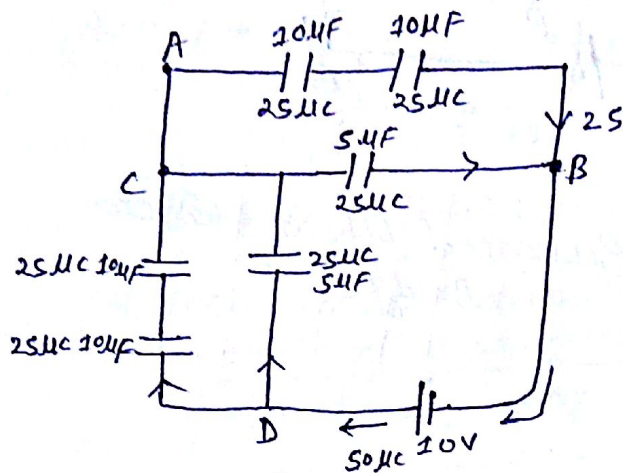
i) $\rightarrow$ Find R_{eq} or, C_{eq} across terminals of battery.

ii) $\rightarrow$ Find net current / charge from battery as $i_{net} = \frac{V}{R_{eq}}$ or, $Q_{total} = C_{eq} \times V$

iii) $\rightarrow$ Distribute current / charge A/C in the ckt.

EX $\rightarrow$ Find $\rightarrow$ |a| $\rightarrow$ charge on each capacitor

|b| $\rightarrow$ $V_A - V_B = ?$ |c| $\rightarrow$ $V_B - V_D = ?$



|a| $\rightarrow$ $C_{eq} = 5\mu F$, $Q_{total} = 10 \times 5 = 50\mu C$

|b| $\rightarrow$ $V_A - V_B = \frac{+25\mu C}{5\mu F} = +5V$

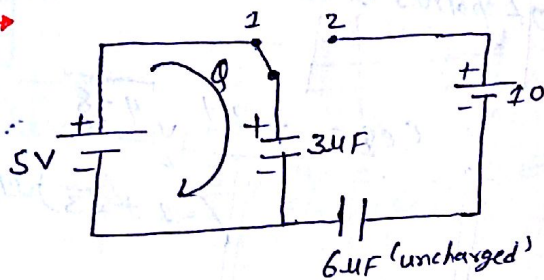
|c| $\rightarrow$ $V_B - V_D = -10V$

Case-II $\Rightarrow$ When more than one battery are connected -

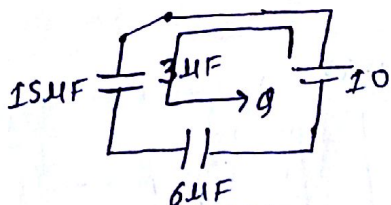
$\rightarrow$ Apply loop rule (KVL) in each loop of ckt.

$\rightarrow$ Only one current / charge will exist along one loop.

EX $\rightarrow$



Initially switch was at position 1 Find final charge on capacitor when switch is thrown to position 2.



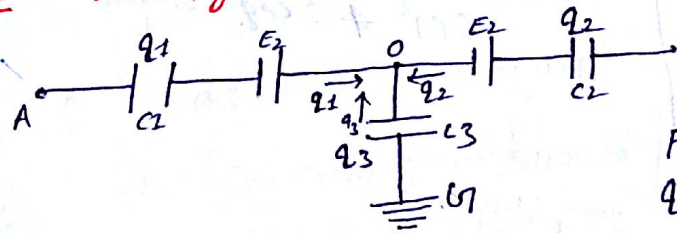
$$\frac{Q}{3} = 5 \Rightarrow Q = 15\mu C$$

$$\frac{Q+15}{3} + \frac{Q}{6} - 10 = 0$$

$$\frac{Q}{2} = 5$$

$$Q = 10\mu F$$

Case-III $\Rightarrow$ When given ckt is an open ckt.



Given $\Rightarrow V_A, V_B, C_1, C_2, C_3, E_1, E_2$

From KCL

$$Q_1 + Q_2 + Q_3 = 0 \quad \text{--- (1)}$$

$$V_A - V_0 = \frac{Q_1}{C_1} - E_1 \quad \text{--- (2)}$$

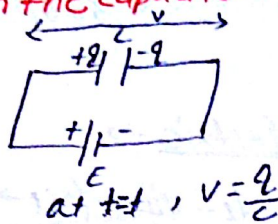
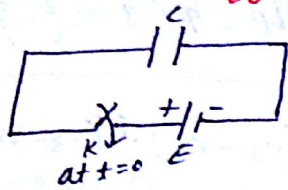
$$V_A - V_0 = \frac{+Q_2}{C_2} + E_2 \quad \text{--- (3)}$$

$$V_{in} - V_o = \frac{q_3}{C_3} \quad (4)$$

From (1) (2) (3) (4) $V = ?$

$$q_1 = ? , q_2 = ? , q_3 = ?$$

Potential energy stored in the capacitor:—



In next 'dt' time battery delivers dq charge to the capacitor
dW on capacitor

$$dW_{cap} = dU_{cap} = dq \cdot V$$

$$\int dU_{cap} = \int_0^Q \frac{q}{C} dq$$

$$* U_{cap} = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} C E^2 = \frac{1}{2} Q E$$

Total Work done by battery.

$$* W_{batt} = Q E = 2 U_{cap}$$

$$* \text{Heat generated} = H = W_{battery} - \uparrow \text{in PE of capacitor.}$$

A/R

NOTE

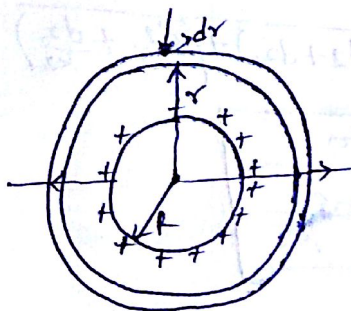
→ Energy density / Electrostatic pressure (U) →



$$\vec{E} = \frac{Q}{A \epsilon_0}$$

$$U = \frac{1}{2} \frac{Q^2}{C} , U = \frac{Q^2 d}{2 A \epsilon_0}$$

$$* U = \frac{U}{Ad} = \frac{Q^2}{2 A^2 \epsilon_0 d} = \frac{1}{2} \epsilon_0 E^2$$



$$C = 4 \pi \epsilon_0 R$$

$$U = \frac{Q^2}{2C} = \frac{Q^2}{8 \pi \epsilon_0 R}$$

$$* U_{self} = \frac{Q^2}{2C} = \frac{Q^2}{8 \pi \epsilon_0 R}$$

For linear isotropic dielectrics →

$$\vec{E}_{net} = \vec{E}_{ext} + \vec{E}_{in}$$

$$|\vec{E}_{net}| = |\vec{E}_{ext}| - |\vec{E}_{in}|$$

$$* |\vec{E}_{net}| = \frac{|\vec{E}_{ext}|}{k}$$

dielectric const.

$$* \vec{E}_{in} = |\vec{E}_{ext}| - |\vec{E}_{net}|$$

$$* |\vec{E}_{in}| = |\vec{E}_{ext}| \left(1 - \frac{1}{k}\right)$$

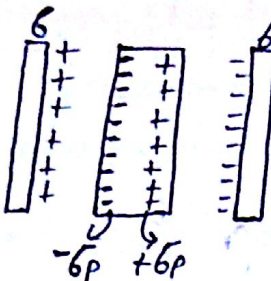
For conductor

$$* k = \infty$$

$$* 1 \leq k \leq \infty$$

$$* k = 1 \rightarrow \text{For vacuum}$$

NOTE →



$$\vec{E}_{\text{initially}} = \frac{\sigma}{\epsilon_0}$$

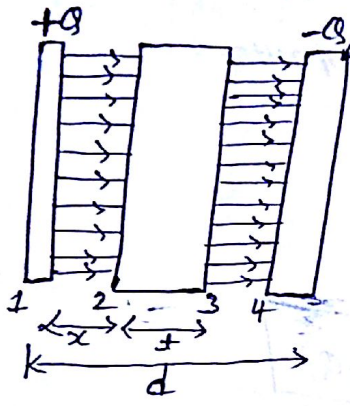
$$\vec{E}_{\text{initially}} = \frac{\sigma}{K\epsilon_0}$$

$$\frac{\sigma - \sigma_p}{\epsilon_0} = \frac{\sigma}{K\epsilon_0}$$

$$\sigma_p = \sigma \left(1 - \frac{1}{K}\right)$$

$$q_p = q \left(1 - \frac{1}{K}\right)$$

case IV → capacitor with dielectric →



$$V_2 - V_4 = (V_1 - V_2) + (V_2 - V_3) + (V_3 - V_4)$$

$$\Delta V = \frac{Qx}{A\epsilon_0} + \frac{Qt}{KA\epsilon_0} + \frac{Q(d-x-t)}{A\epsilon_0}$$

$$C = \frac{A\epsilon_0}{d - t + \frac{t}{K}}$$

NOTE →

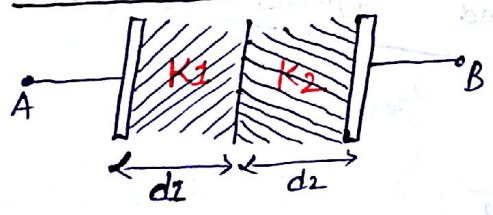
iii → If 'n' no. of plates with $(t_1, K_1), (t_2, K_2), (t_3, K_3), \dots, (t_n, K_n)$ are introduced,

$$C = \frac{A\epsilon_0}{d - (t_1 + t_2 + \dots + t_n) + \left(\frac{t_1}{K_1} + \frac{t_2}{K_2} + \dots + \frac{t_n}{K_n}\right)}$$

iii → If a conducting slab with thickness 't' is introduced.

$$C = \frac{A\epsilon_0}{d - t}$$

case V → capacitor filled with different dielectrics →



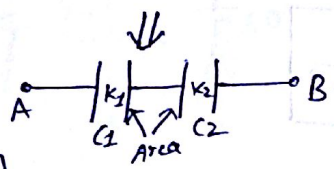
$$C = \frac{A\epsilon_0}{(d_1 + d_2) - (d_1 + d_2) + \left(\frac{d_1}{K_1} + \frac{d_2}{K_2}\right)}$$

$$C = \frac{A\epsilon_0}{\frac{d_1}{K_1} + \frac{d_2}{K_2}}$$

$$C_1 = \frac{K_1 A \epsilon_0}{d_1}$$

$$C_2 = \frac{K_2 A \epsilon_0}{d_2}$$

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$



case VI → Dielectric const is varying →

$$K = K_0 \left(1 + \frac{x}{L}\right)$$

$$dC = \frac{(L dx) \epsilon_0 K}{d}$$

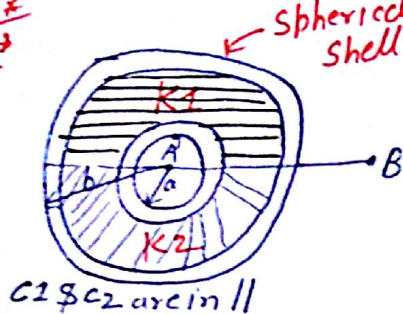
$$\int dx = \int_0^L \frac{K_0 \epsilon_0 L}{d} \left(1 + \frac{x}{L}\right) dx$$

$$K = K_0 \left(1 + \frac{x}{d}\right)$$

$$dC = \frac{L A K_0 \epsilon_0 \left(1 + \frac{x}{d}\right)}{dx}$$

$$\therefore \frac{1}{C_{eq}} = \int \frac{1}{dC} = \int_0^d \frac{dx}{\left(1 + \frac{x}{d}\right)} \times \frac{1}{L A K_0 \epsilon_0}$$

Ex 3



$$C_1 = \frac{2\pi\epsilon_0 abk_1}{b-a}$$

$$C_2 = \frac{2\pi\epsilon_0 abk_2}{b-a}$$

$$C_{eq} = C_1 + C_2 = \frac{2\pi\epsilon_0 ab}{b-a} (k_1 + k_2)$$

case-VII → When dielectric slab is inserted in charged capacitor →

|a| → When battery is disconnected →

$$[Q = \text{const}]^*$$

$$C_i = \frac{A\epsilon_0}{d}$$

$$+|-|$$

|ii| → capacitance (C) = KC

|iii| → potential = $\frac{V}{K}$

$$V' = V/K$$

$$C' = KC$$

$$+ \frac{q}{K} \frac{q}{K}$$

$$|iii| \rightarrow E = \frac{V}{d} \Rightarrow K' = \frac{E}{K}$$

$$|iv| \rightarrow W_{ext} = U' = \frac{U}{K}$$

$$W_{ext} = \Delta U = 0 \text{ ve.}$$

|b| → When battery remains connected →

$$[V = \text{const}]^*$$

|ii| → capacitance (C) $\Rightarrow C' = KC$

|iii| → charge (Q) = $Q' = KV$

|iii| → Electric field (E) $\Rightarrow \text{const.}$

|iv| → pot energy (U) $\Rightarrow U' = KU$

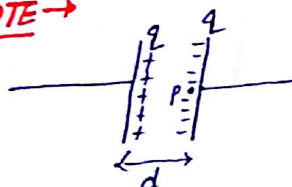
|v| → $W_{ext} = ?$

$$W_{ext} + W_{battery} = \Delta U$$

$$W_{ext} = (K-1) \frac{CV^2}{2} - (K-1) CV^2$$

$$* W_{ext} = -\frac{1}{2} CV^2 (K-1) < 0$$

NOTE →



Electric field at 'P' due to +q plate

$$* E_p = \frac{q}{2A\epsilon_0}$$

$$\text{Force on 2nd plate} \Rightarrow F = qE_p = \frac{q^2}{2A\epsilon_0}$$

case-VIII → Force acting on dielectric slab while insertion.

|a| → When battery is disconnected - (system have only capacitor)

$$F = - \frac{dU_{\text{capacitor}}}{dx}$$

$$* U_{\text{capacitor}} = \frac{1}{2} \frac{q^2}{C} = \frac{q^2}{2(Ax+b)}$$

$$* F = - \frac{dU}{dx}$$

|b| → When battery remain connected →

$$dU = dU_{\text{capacitor}} + dU_{\text{battery}}$$

$$* dU_{\text{battery}} = -dW_{\text{battery}}$$

$$\checkmark \text{Charge release from battery } (dq) = (dC)V$$

$$* dq = \frac{(K-1)\epsilon_0 V}{d} (dx)$$

$$F = - \frac{dU_{\text{system}}}{dx}$$

$$* F = \frac{(K-1)\epsilon_0 V^2}{2d} = \text{const.}$$