

SIMPLE HARMONIC MOTION [S.H.M.]

11) Aperiodic motion → A particle not repeat path in a same time interval.
 * also called non-periodic motion.
 Eg → * A BUS or ~~vehicle~~ ^{vehicle} move in st. line
 * Motion of gas molecule.

12) Periodic motion → If particle repeat its path in a same time interval.

- EX → * Motion of pendulum
 * Motion around the nucleus
 * Motion of planet around the sun.
 * Motion of Earth w.r.t self axis.
 * Motion of swift.
 * Motion of lig in 'U' tube.
 * Motion of wooden plank at surface of liq.

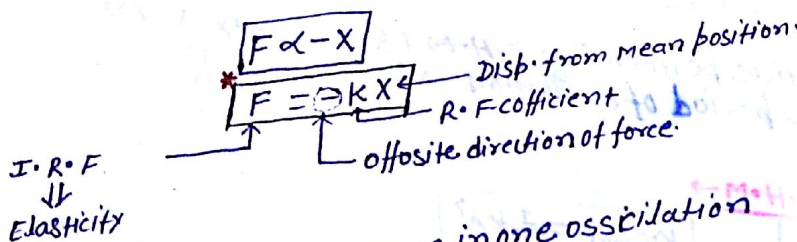
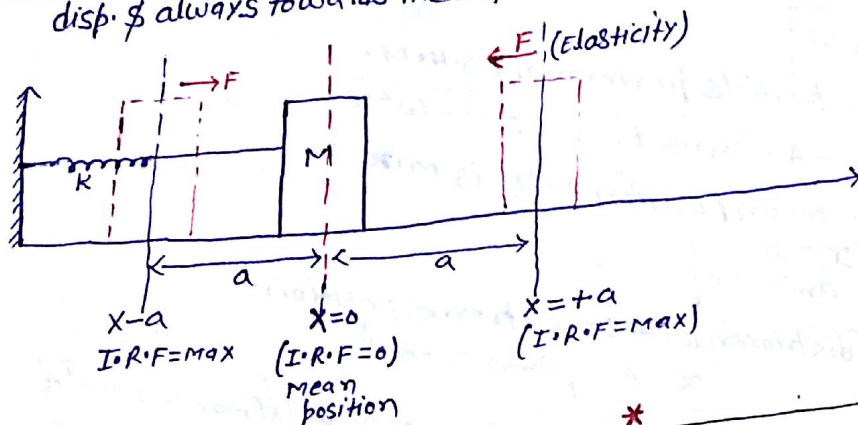
13) Oscillatory/vibrational motion → If particle repeat its path w.r.t same point & complete throw & flow motion in same time interval.

- Eg → * Motion of pendulum
 * Motion of swimmer
 * Motion of lig in 'U' tube.
 * Motion of medium particle when wave propagate in a medium.

AIIMS

NOTE → All oscillating motion is periodic but all periodic motion is not oscillatory.

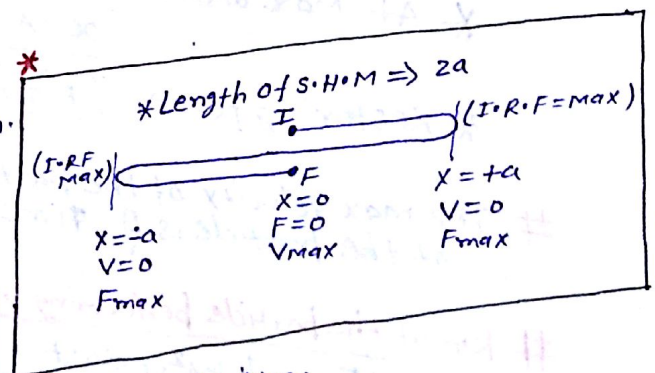
14) Simple harmonic motion → If a oscillatory motion is directly proportional to the disp. & always towards mean position.



Exam
 * Displacement and distance in one oscillation
 Resp. zero & 4a.

Amplitude → Max disp. of medium particle from mean position.

NOTE → * Elasticity Inertia is necessary for oscillatory motion.
 * S.H.M. occur in stable eqn.
 * One vibrational vibration of polyatomic gas molecule about its eqm. position → S.H.M.



Condition for S.H.M.

$$F = -kx$$

$$A = -\omega^2 x$$

$$\frac{K}{m} = \omega^2 \quad (\omega = \text{angular freq.})$$

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

$$\frac{d^2x}{dt^2} + \omega^2 x = 0 \quad (\text{Differential eqn of S.H.M.})$$

Velocity of the particle performing S.H.M. →

$$v = A\omega \sqrt{\frac{A^2 - x^2}{A^2}}$$

$$v = \omega \sqrt{A^2 - x^2}$$

$$v = \frac{2\pi}{T} \sqrt{A^2 - x^2}$$

$$v = 2\pi n \sqrt{A^2 - x^2}$$

x = disp. of the particle from mean position.
 T = Time period
 A = Max disp. or, Amplitude.
 ω = Angular freq.
 n = Linear freq.

Imp. * velocity of the particle at extreme position or at max disp. is equal to zero or min.

$$x = A = 90^\circ \quad v = \omega \sqrt{A^2 - x^2} \quad v_{\min} = 0$$

* At mean position or, at an equilibrium position the velocity of the particle will be maximum.

$$x = 0$$

$$v = \omega \sqrt{A^2 - 0^2}$$

$$v_{\max} = A\omega = A(2\pi n) = A \times \left(\frac{2\pi}{T}\right)$$

* Average velocity or, mean velocity

$$\vec{v}_{\text{avg}} = 0$$

* Accn of the particle performing S.H.M.

$$a = -A\omega^2 \sin \omega t$$

$$a = -\omega^2 x$$

* At eqm or, mean position Accn is min

$$x = 0$$

$$a_{\min} = 0$$

* At max displacement or, extreme position.

$$x = A$$

$$a_{\max} = -A\omega^2$$

* Restoring force

$$F = ma$$

$$F = -m\omega^2 x$$

$$F_{\max} = -m\omega^2 A$$

The max velocity of the particle performing S.H.M is ' α ' & Max Accn of the particle is β . Time period of oscillation → $T = 2\pi \left(\frac{\alpha}{\beta}\right)$

K.E of the particle performing S.H.M →

$$E_k = \frac{1}{2} k A^2 \cos^2 \omega t$$

$$E_k = E_{k\max} \cdot \cos^2 \omega t$$

$$E_{k\max} = \frac{1}{2} k A^2$$

Relation b/w K.E & disp →

$$E_k = \frac{1}{2} k (A^2 - x^2)$$

* K.E of the particle is max at Mean position or, equilibrium position → $E_{k\max} = \frac{1}{2} k A^2$

* At max disp or, at extreme position

$$x = A \quad E_{k\min} = 0$$

Avg. K.E or, mean K.E →

$$\langle K.E \rangle = \frac{E_{Kmax} + E_{Kmin}}{2}$$

$$\langle K.E \rangle = \frac{\frac{1}{2}KA^2 + 0}{2}$$

$$\langle K.E \rangle = \frac{1}{4}KA^2$$

$$\langle K.E \rangle = \frac{1}{2} \times \frac{1}{2}KA^2$$

$$\langle K.E \rangle = \frac{1}{2}E_{Kmax}$$

$$\langle K.E \rangle = \frac{K.E_{max} + K.E_{min}}{2}$$

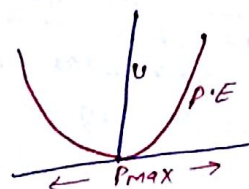
$$\langle K.E \rangle = \frac{1}{2}KA^2$$

$$\boxed{\langle K.E \rangle = \frac{1}{2}K.E_{max}}$$

P.E of Particle performing S.H.M →

$$\boxed{U = \frac{1}{2}Kx^2}$$

$$\boxed{U = \frac{1}{2}Kx^2}$$



Avg. P.E or, mean K.E →

$$\langle P.E \rangle = \frac{U_{Kmax} + U_{Kmin}}{2}$$

$$\langle P.E \rangle = \frac{1}{2}U_{Kmax}$$

relation b/w P.E & time →

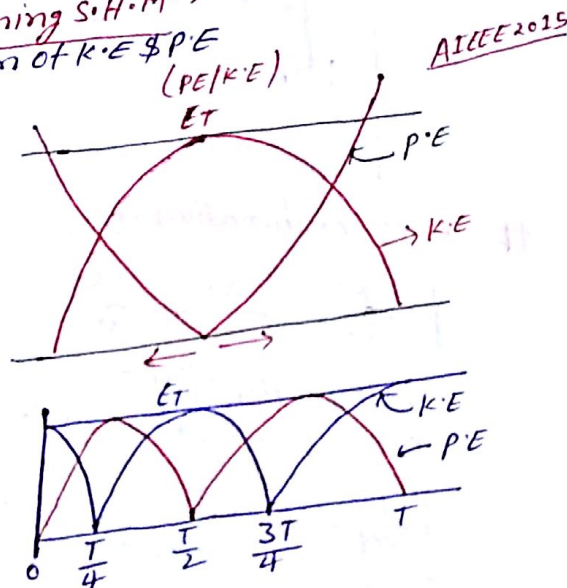
$$\boxed{U = \frac{1}{2}Kx^2}$$

$$\boxed{U = \frac{1}{2}KA^2 \sin^2 \omega t}$$

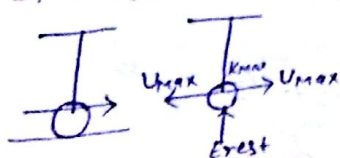
Total E in S.H.M remain

Total energy of the particle performing S.H.M →
Conserved it is equal to sum of K.E & P.E

$$\begin{aligned} E_T &= \frac{1}{2}KA^2 \\ E_T &= \frac{1}{2}m\omega^2 A^2 \\ E_T &\propto m \\ E_T &\propto \omega^2 \propto n^2 \propto \frac{1}{T^2} \propto A^2 \\ E_T &\propto A^2 \end{aligned}$$



NOTE → If Energy at rest is given in question.



$$\boxed{E_T = E_{rest} + U_{max}}$$

* Fraction of P.E in T.E → $\frac{\frac{1}{2}Kx^2}{\frac{1}{2}KA^2}$

* A particle having mass 'm' moving with a velo. 'v' strike from spring & compress it. cal. compression in spring → $\boxed{x = v \sqrt{\frac{m}{k}}}$

* A block is placed at a spring. If block is separated from the platform which is performing S.H.M then the condition $\rightarrow$

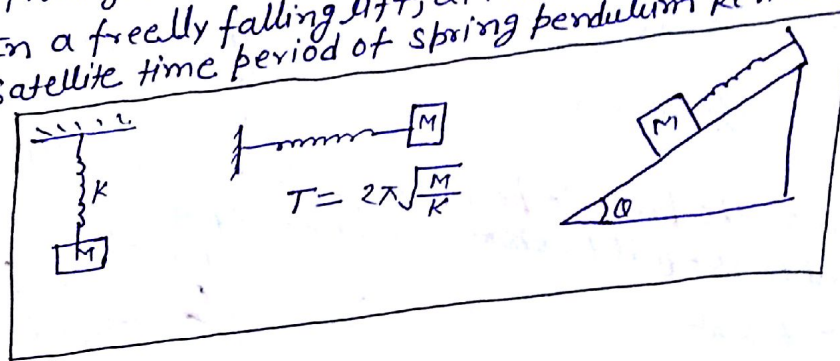
$$\begin{cases} \omega^2 A = g \\ \frac{k}{m} A = g \end{cases}$$

$$E_T = K_{\max} = U_{\max} = \frac{1}{2} k A^2$$

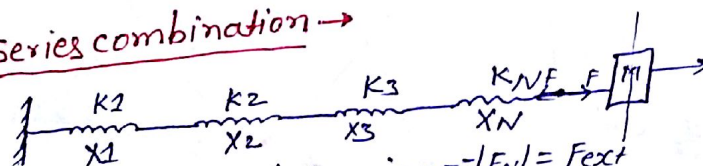
Spring pendulum

- * Spring is used to store elastic P.E.
- * When spring is compressed or extended then work is done & this work is stored in the form elastic P.E.
- * The force const. of a spring is inversely proportional to the natural length.
- * If a block of mass 'M' is suspended from a spring & it is slightly pulled & released it will perform S.H.M in this case.

- * Restoring force is $\propto$ to the disp.
- * Time period of spring pendulum only depends on the mass of suspended body & on force const. & it is independent from gravitation Accn. & Extension in spring.
- * In a freely falling lift, at the centre of Earth & in artificial satellite time period of spring pendulum remain unchanged.



Series combination $\rightarrow$



$$* |F_1| = |F_2| = |F_3| = \dots = |F_N| = F_{\text{ext}}$$

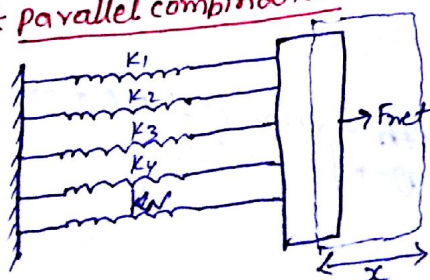
$$* X = X_1 + X_2 + \dots + X_N$$

$$\frac{1}{K_{\text{eq}}} = \frac{1}{K_1} + \frac{1}{K_2} + \dots + \frac{1}{K_N} \quad (\text{Spring const. of combination})$$

$$T = 2\pi \sqrt{\frac{M}{K_{\text{eq}}}}$$

$$T = 2\pi \sqrt{M \left(\frac{1}{K_1} + \frac{1}{K_2} + \dots + \frac{1}{K_N} \right)} \quad \text{Time period of combination.}$$

Parallel combination



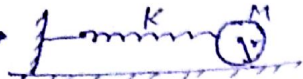
$$K_{\text{eq}} = K_1 + K_2 + K_3 + \dots + K_N$$

$$T = 2\pi \sqrt{\frac{M}{K_{\text{eq}}}}$$

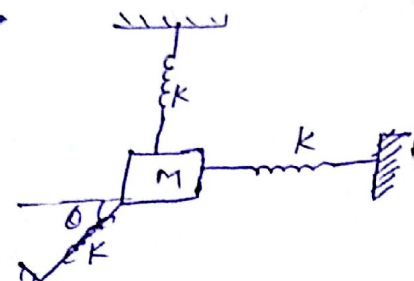
$$T = 2\pi \sqrt{\frac{M}{K_1 + K_2 + \dots + K_N}}$$

Standard Result for Spring : →

1a) →  $T = 2\pi \sqrt{\frac{\mu}{k}}$ $\mu = \frac{M_1 M_2}{M_1 + M_2}$ $\mu = \text{Reduce mass.}$

1b) →  $T = 2\pi \sqrt{\frac{M}{k} \left(1 + \frac{K^2}{I}\right)}$ ($k = \text{spring const}$
 $K = \text{radius of gyration}$)

1c) →  $T = 2\pi \sqrt{\frac{M + \frac{I}{K^2}}{k}}$

1d) →  $T = \sqrt{\frac{M}{k(1 - \cos^2 \theta)}}$

* Astromot use spring pendulum becoz do not depend on $\ln \& L$

Two different spring oscillate with same mass. It is time period resp. $T_1 \& T_2$ calculate time period of combination when—

iii) → It will connect in series & oscillate same mean.

iii) → " " " " " " " "

iii) → $T_{\text{series}} = \sqrt{T_1^2 + T_2^2}$

iii) → $T_{\text{parallel}} = \frac{T_1 T_2}{\sqrt{T_1^2 + T_2^2}} = \frac{T_1 T_2}{T_s} \Rightarrow T_p T_s = T_1 T_2$

SIMPLE PENDULUM

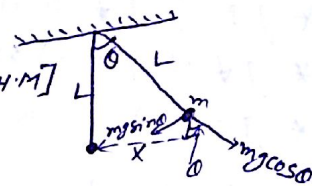
$F = -mg \sin \theta$

* $\theta \leq 5^\circ (\text{small}) \Rightarrow \text{SHM} [\theta > 5^\circ \Rightarrow \text{oscillation not S.H.M}]$

$F_R = -\left(\frac{mg}{L}\right)x$

$F = \frac{mg}{L} \Rightarrow \frac{m}{k} = \frac{L}{g}$

$T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{L}{g}}$



NOTE → Time period of simple pendulum depend on length of pendulum & gravitation Accⁿ. it is independent from mass of oscillating body.
 $L = \text{Length of pendulum}$
 * If point mass oscillate = length of spring.
 * If physical object is suspended = distance b/w point of suspension & centre of mass of oscillating body.

Affecting Factor : →

1a) → Effect of Length →

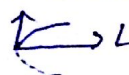
$T = 2\pi \sqrt{\frac{L}{g}}$

$g = \text{const} \Rightarrow T \propto \sqrt{L}$

* $L \uparrow \Rightarrow T \uparrow \Rightarrow n \downarrow (\text{slow})$

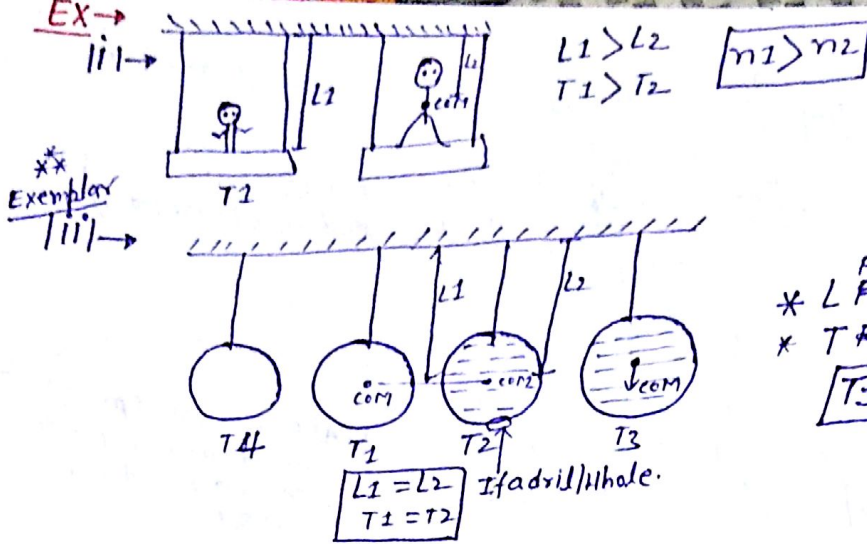
* $L \downarrow \Rightarrow T \downarrow \Rightarrow n \uparrow (\text{Fast})$

Graph $T \propto \sqrt{L}$ →



* $T \propto \sqrt{L}$





* L First ↑ then ↓
 * T First ↑ then ↓
 $T_3 > T_1 = T_2 = T_4$

#

* change in 1 sec = $\frac{1}{2} \times \Delta T$
 * change in 't' sec = $(\frac{1}{2} \times \Delta T) t_{\text{sec}}$
 * change per day = $(\frac{1}{2} \times \Delta T) 86400 \frac{\text{sec}}{\text{day}}$
 * $1 \text{ yr} = \pi \times 10^7 \text{ sec}$

1b1 → Effect of gravity : → $T = 2\pi \sqrt{\frac{L}{g}}$

* $L \Rightarrow \text{same} \Rightarrow T = \frac{1}{\sqrt{g}}$

* $g \uparrow \Rightarrow T \downarrow \Rightarrow n \uparrow$ (Fast)

* $g \downarrow \Rightarrow T \uparrow \Rightarrow n \downarrow$ (Slow)

* $g = 0 \Rightarrow T = \infty$ — $\begin{cases} \rightarrow \text{At the centre of Earth.} \\ \rightarrow \text{In a freely falling lift.} \\ \rightarrow \text{In a Artificial satellite.} \end{cases}$

Ex → * At height above 'h' from Earth surface.

iii → $T_s = 2\pi \sqrt{\frac{L}{g_s}}$

$T_h = 2\pi \sqrt{\frac{L}{g_s/(1+h/R)^2}}$

$T_h = T_s (1+h/R)$

% change = $\frac{T_h - T_s}{T_s} = \frac{h}{R} \times 100\%$

iii → * At depth 'd' from earth surface →

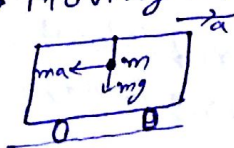
$g_d = g_s (1 - \frac{d}{R})$

$T_d = \frac{T_s}{\sqrt{1 - d/R}}$

* $d \uparrow \Rightarrow g_d \Rightarrow T_d \uparrow \Rightarrow n \downarrow$

iii → * In a moving ~~vehicle~~ vehicle

1a1 → Moving in a Horizontal plane.



$g_{\text{eff}} = \sqrt{g^2 + a^2}$

$T = 2\pi \sqrt{\frac{L}{g^2 + a^2}}$

|b| → Move in a circular path

$$g_{\text{eff}} = \sqrt{g^2 + \frac{v^4}{R^2}}$$

$$T = 2\pi \sqrt{\frac{L}{\sqrt{g^2 + \frac{v^4}{R^2}}}}$$

|c| → Move in a incline plane

$$T = 2\pi \sqrt{\frac{L}{g \cos \theta}}$$

Ex → In moving Lift →

|a| → Moving upward

$$g_{\text{eff}} = g + a$$

$$T_{\text{eff}} = 2\pi \sqrt{\frac{L}{g+a}}$$

|b| → Moving downward

$$g_{\text{eff}} = g - a$$

$$T_{\text{eff}} = 2\pi \sqrt{\frac{L}{g-a}}$$

$$\begin{aligned} * \text{ If } v \uparrow (\text{Accd}) &\Rightarrow a = +ve \\ * \text{ If } v \downarrow (\text{Retard}) &\Rightarrow a = -ve \end{aligned}$$

|c| → Freely falling condn

$$a = g \Rightarrow g_{\text{eff}} = g - g = 0$$

$$T_{\text{eff}} = \infty$$

|d| → If chain is break → Weightlessness condn

$$a = g$$

|e| → If lift move with const Accd_n upward or, downward.

$$v = \text{const}$$

$$a = 0$$

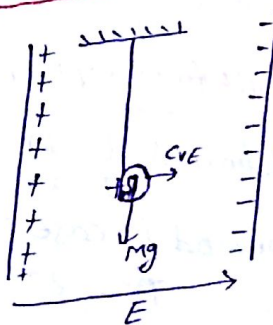
$$T = 2\pi \sqrt{\frac{L}{g}}$$

Ex → oscillate in @nce of Electric field →

|a| → Horizontal electric field

$$g_{\text{eff}} = m \sqrt{g^2 + \left(\frac{qVE}{m}\right)^2}$$

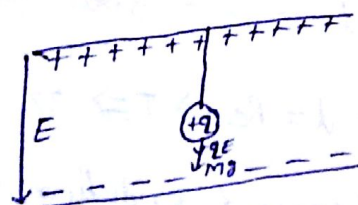
$$T_{\text{eff}} = 2\pi \sqrt{\frac{L}{g^2 + \left(\frac{qE}{m}\right)^2}}$$



|b| → Downward electric field

$$g_{\text{eff}} = g + \frac{qVE}{m}$$

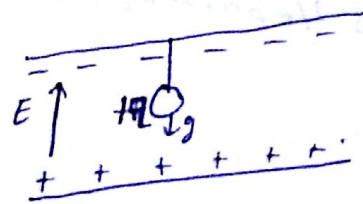
$$T_{\text{eff}} = 2\pi \sqrt{\frac{L}{g + \frac{qVE}{m}}}$$



|c| → upward electric field

$$T = 2\pi \sqrt{\frac{L}{g - \frac{qE}{m}}}$$

$$g_{\text{eff}} = g - \frac{qVE}{m}$$



Ex → In a non-viscous liq →

$$W_{\text{eff}} = mg \left(1 - \frac{d_l}{d_b}\right)$$

$$g_{\text{eff}} = g \left(1 - \frac{d_l}{d_b}\right)$$

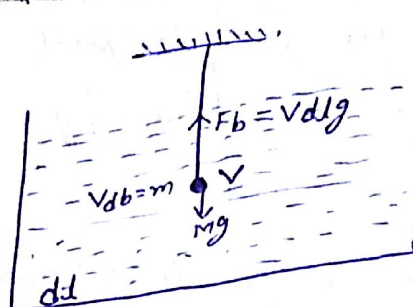
$$T_{\text{eff}} = 2\pi \sqrt{\frac{L}{g \left(1 - \frac{d_l}{d_b}\right)}}$$

$$T_L = \frac{T_{\text{air}}}{\sqrt{1 - \frac{d_l}{d_b}}}$$

→ density of liq
→ density of body

$$\ast \text{ Specific density} = \frac{d_{\text{body}}}{d_{\text{H}_2\text{O}}} = 6$$

$$T_{\text{H}_2\text{O}} = \frac{T_{\text{air}}}{\sqrt{1 - \frac{1}{6}}}$$



** Two simple pendulum of time period T_1 & T_2 start vibrating at same phase then min time after which they are again in same phase ($T_1 > T_2$) → $T = \frac{T_1 T_2}{T_1 - T_2}$

* If $(n+1)$ → no. of oscillation completed by smaller pendulum then, (n) → will be no. of oscillation completed by larger pendulum. $T = (n+1)T_2 = nT_1$

** Standard Result for simple pendulum: →

i) → second pendulum ⇒ $L = 1\text{m}$ at surface of earth
 $T = 2\text{sec} = \text{const.}$

ii) → Time period in case of long length pendulum.
 $T = 2\pi \sqrt{\frac{1}{g} \left(\frac{1}{1} + \frac{1}{R_e} \right)}$

$$\text{iii) } L = \infty \Rightarrow T \Rightarrow \infty = 2\pi \sqrt{\frac{R_e}{g}} = \frac{84.6 \text{ min}}{\sqrt{2}} = 1.414 \text{ hr.}$$

$$\text{iii) } L = R_e \Rightarrow T \Rightarrow 2\pi \sqrt{\frac{R_e}{2g}} \Rightarrow \frac{84.6 \text{ min}}{\sqrt{2}} = 0.707 \times 84.6 \text{ min.}$$

* Length of second pendulum at surface of Moon. Where gravitation Accⁿ is $1/6$ part of surface of Earth → $L_m = \frac{L_e}{6} = \frac{1}{6} \text{m}$

Compound pendulum

l = distance of suspension from centre of oscillation (C.O.)

r = radius of gyration about C.O.

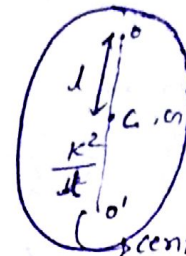
$$T = 2\pi \sqrt{\frac{I}{mgl}}$$

$$T = 2\pi \sqrt{\frac{l + \frac{k^2}{l}}{g}}$$

If ' l ' is replaced by $\frac{k^2}{l}$

$\therefore$ then ($T \rightarrow$ same)

$\therefore T$ is same at distance $l + \frac{k^2}{l}$ from C.O.



centre of oscillation.

Linear S.H.M

i) $F = -kx$

ii) $k = m\omega^2$

$T = 2\pi \sqrt{\frac{m}{k}}$

iii) $\frac{d^2x}{dt^2} + \omega^2 x = 0$

Angular S.H.M

i) $\tau = -C\theta$

ii) $C = I\omega^2$

$T = 2\pi \sqrt{\frac{I}{C}}$

iii) $\frac{d^2\theta}{dt^2} + \omega^2 \theta = 0$

For small angle disp.

$\sin \theta \approx \theta$

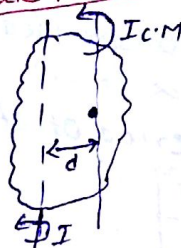
$\tau = -(mgl)\theta$

$\tau = -C\theta$

$C = mgl$

$I = M \cdot O \cdot I$
 m = mass of body
 g = gravity
 l = effective length.

Parallel axis theorem



$I = I_C + Md^2$

$I_C = Mk'^2$

k' = radius of gyration

a) Ring $I_C = MR^2 = Mk'^2 \Rightarrow k' = R$

b) Disc $I_C = \frac{MR^2}{2} = Mk'^2 \Rightarrow k' = \frac{R}{\sqrt{2}}$

c) Solid sphere $I_C = \frac{2}{5}MR^2 = Mk'^2 \Rightarrow k' = \sqrt{\frac{2}{5}}R$

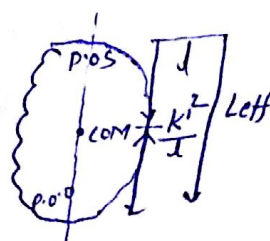
d) Hollow sphere $I_C = \frac{2}{3}MR^2 = Mk'^2 \Rightarrow k' = \sqrt{\frac{2}{3}}R$

e) Rod $I_C = \frac{ML^2}{12} = Mk'^2 \Rightarrow k' = \frac{L}{2\sqrt{3}}$

$T = 2\pi \sqrt{\frac{I_{eff}}{g}}$

$I_{eff} = \frac{k'^2}{l}$

$T_{P.O.O} = T_{O.S}$



k'^2 = radius of gyration.

Disc of Mass 'm' Radius 'R' suspended in a vertical w.r.t horizontal axis. calculate time period of its oscillation. when it is suspended from →

i) → Circumference.

$$* K' = \frac{R}{\sqrt{2}} \Rightarrow \frac{K'}{1} = \frac{R^2/2}{R} = R/2$$

$$* L_{eff} = \frac{K'^2}{1} + 1 = \frac{3R}{2}$$

$$* T_{eff} = 2\pi \sqrt{\frac{L_{eff}}{g}} = 2\pi \sqrt{\frac{3R}{2g}}$$

ii) → At distance R/2 from COM

$$* K' = \frac{R}{\sqrt{2}} = \frac{K' R}{1} = \frac{(R/\sqrt{2})^2}{R/2} = R$$

$$* L_{eff} = \frac{K'^2}{1} + 1 = \frac{3R}{2}$$

$$* T = 2\pi \sqrt{\frac{3R}{2g}}$$

iii) → At distance 3R/2 from circumference.

$$L^2 - \left(\frac{T^2 g}{4\pi^2}\right) L + K'^2 = 0 \quad \frac{T^2 g}{4\pi^2} = \frac{K'^2 + L^2}{1}$$

* corresponding to same time period 4-points is possible.

iv) → w.r.t COM

$$L = 0 \quad T = 2\pi \sqrt{\frac{L}{g}} = \infty$$

$$\frac{K'^2}{1} = \infty \quad L_{eff} = \frac{K'^2}{1} + 1 = \infty$$

NOTE → * If compound pendulum suspended from its C.O.M time period of pendulum is ∞ (max)

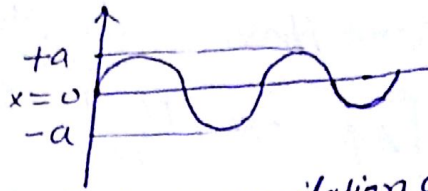
* When $K' = 1$, time period of pendulum is min.

$$T = 2\pi \sqrt{\frac{K'^2 + 1}{g}}$$

Types of oscillation

1a) → Free oscillation → If amplitude, energy, time period of oscillation remain same.

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$



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1b) → Damped oscillation → If Amplitude of oscillation decrease with time.

$$F_R = -k_1 x$$

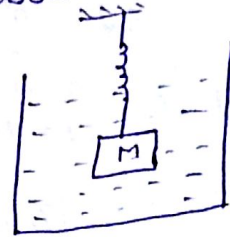
$$F_V = -6\pi\eta r v = -k_2 v$$

terminal velo.

viscous force

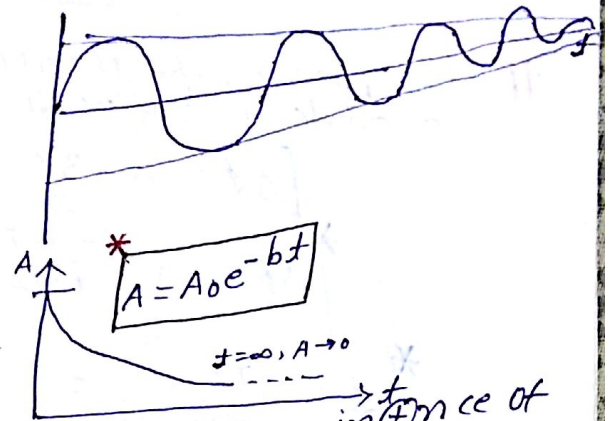
$$F_{net} = -k_1 x - k_2 v$$

$$\frac{d^2x}{dt^2} + \left(\frac{k_2}{M}\right) \frac{dx}{dt} + \left(\frac{k_1}{M}\right) x = 0$$



A_0 = Amplitude at $t=0$
 b = damping coefficient
 t = time

NOTE → In damping oscillation Amplitude ↓ exponentially w.r. to time.



1c) → Forced oscillation → If free oscillation perform in presence of external periodic force.

$$\frac{d^2x}{dt^2} + \left(\frac{k_2}{M}\right) \frac{dx}{dt} + \left(\frac{k_1}{M}\right) x = \frac{F_{ex}}{M}$$

$x=0$ to $x = A/2 = \frac{1}{2}$ or 30°
 $x=0$ to $x = \frac{A}{\sqrt{2}} = \frac{1}{\sqrt{2}}$ or 45°
 $x=0$ to $x = \frac{\sqrt{3}A}{2} = 60^\circ$
 $x=0$ to $x = A = 90^\circ$

$$\frac{\omega t}{\pi/6} = 30^\circ$$

$$\pi/4$$

$$\pi/3$$

$$\pi/2$$

$$\frac{t}{T/12} = \frac{2\pi}{T} \quad \text{AIPMT}$$

$$T/8$$

$$T/6$$

$$T/4$$

A particle execute S.H.M. its velocity at position x_1 & x_2 resp. v_1 & v_2 then Amplitude & time period →

$$a = \sqrt{\frac{v_1^2 x_2^2 - v_2^2 x_1^2}{v_1^2 - v_2^2}}$$

$$T = 2\pi \sqrt{\frac{x_2^2 - x_1^2}{v_2^2 - v_1^2}}$$

AIPMT-2015

particle execute S.H.M. with amplitude 'a' calculate position of particle
 where, i) → $v = \frac{v_{max}}{2} \Rightarrow x = \pm \sqrt{\frac{3}{2}} a$

ii) → $v = \frac{\sqrt{3}}{2} v_{max} \Rightarrow x = \pm \frac{a}{2}$

Two particle execute S.H.M With time period resp. T_1 & T_2 ($T_1 > T_2$).
If initially it will start With same phase. calculate time after
Which again its come in a same phase.

Same phase = even Multiple of π

$$\Delta\phi = 2\pi \left(\frac{T_1 - T_2}{T_1 T_2} \right) t = 0, 2\pi, 4\pi$$

$$\Delta\phi = 0, 2\pi, 4\pi, 6\pi \dots 2n\pi$$

$$t = 0, \frac{T_1 T_2}{T_1 - T_2}, \frac{2 T_1 T_2}{T_1 - T_2} \dots \frac{n T_1 T_2}{T_1 - T_2}$$

$$T_{\min} = \frac{T_1 T_2}{T_1 - T_2}$$

In upper concept If initially particle $\oplus$ not $\ominus$ in same phase
calculate time after Which it come in opposite phase.

$$\Delta\phi = \pi, 3\pi, 5\pi \dots (2n-1)\pi$$

$$n = 1, 2, 3 \dots$$

$$\Delta\phi = 2\pi \left(\frac{T_1 - T_2}{T_1 T_2} \right) t = \pi, 3\pi, 5\pi \dots (2n-1)\pi$$

$$t = \frac{T_1 T_2}{2(T_1 - T_2)} = \frac{3 T_1 T_2}{2(T_1 - T_2)} = \frac{5 T_1 T_2}{2(T_1 - T_2)} \dots \frac{(2n-1) T_1 T_2}{2(T_1 - T_2)}$$

NOTE → * Time or position is not define. Avg of K.E & P.E is $\frac{1}{4} k a^2$
* If freq of S.H.M is 'n' then freq of K.E & P.E is '2n',
* If freq of S.H.M is 'n' then conversion of freq
(Kinetic to potential or, pot. to Kinetic) is '4n'

A spring of Spring const. 'K' divide into unequal part ratio of
length is 1:n. Spring const. of individual part.

$$L_2 = n L_1$$

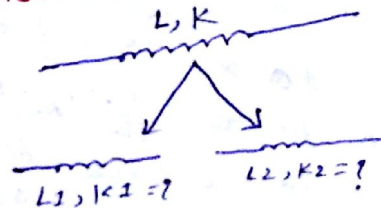
$$* L_1 = \frac{L}{n+1} \Rightarrow K \propto \frac{1}{L}$$

$$* \frac{k_1}{K} = \frac{L}{L_1} = \left(\frac{L}{\frac{L}{n+1}} \right)$$

$$k_1 = (n+1)K$$

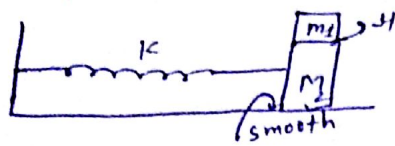
$$* L_2 = n L_1 = \frac{nL}{n+1} \Rightarrow K \propto \frac{1}{L} \Rightarrow \frac{k_2}{K} = \frac{L}{L_2} = \frac{L}{\frac{nL}{n+1}}$$

$$k_2 = \frac{(n+1)K}{n}$$



$$L_1, k_1 = ? \quad L_2, k_2 = ?$$

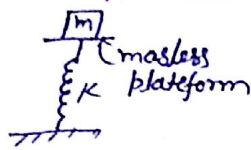
Find max. amplitude of oscillation, so that m_1 does not leave on m_2 .



$$a = \left(\frac{k}{m_1 + m_2} \right) = mg$$

$$a = \frac{\mu(m_1 + m_2)g}{k}$$

Max amplitude of oscillation so that block remain in contact.



during upward accelⁿ

$$N_1 = m(g + a)$$

during down ward accelⁿ

$$N_2 = m(g - a)$$

If $N_2 > 0 \Rightarrow$ block remain in contact.

$$\begin{aligned} g &> a \quad \rightarrow \text{Accel}^n \\ g &> a\omega^2 \quad \rightarrow \text{Amplitude} \end{aligned}$$

$$a_{\max} = \frac{mg}{k}$$

2016
IIPMER

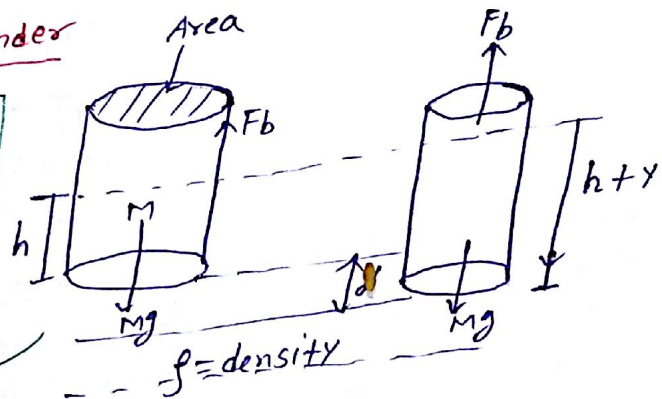
Time period of floating of cylinder

$$T = 2\pi \sqrt{\frac{h}{g}} \quad (l = h)$$

Time period of liq in U-tube.

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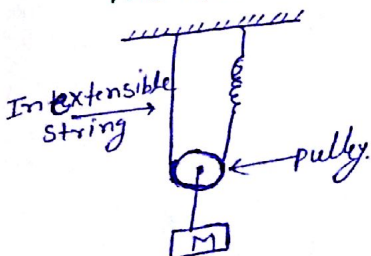
$$T = 2\pi \sqrt{\frac{m}{A\rho g}}$$



Motion of an oscillating liquid in U-tube $\rightarrow$ simple harmonic & time period Independent of density of liq.

**
exemplar

Time period of mass 'm' when displaced from equilibrium position & then released for the system shown in fig $\rightarrow$



$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{4K}{m}}} = 2\pi \sqrt{\frac{m}{4K}}$$

Exemplar

The length of second pendulum on the surface of Earth is 1m. then of second pendulum on the Moon $\rightarrow$ $\boxed{1/6 \text{ m}}$