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Lecture No. 13

Work, Power, Energy

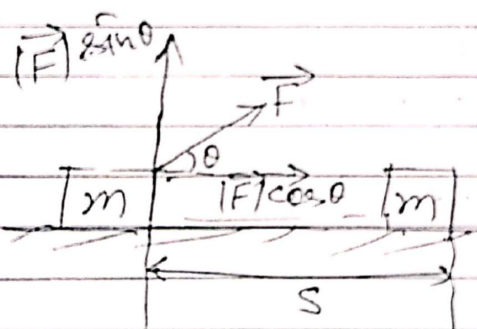
In everyday life, any activity that requires muscular or mental effort is defined as work.

But in science, the work is defined as dot product of Force and Displacement vectors.

$$W = \vec{F} \cdot \vec{S}$$

$$W = |\vec{F}| |\vec{S}| \cos \theta$$

θ is the angle b/w $\vec{F}$ and $\vec{S}$



$$W = |\vec{F}| |\vec{S}| \cos \theta$$

$$W = [|\vec{F}| \cos \theta] |\vec{S}|$$

component of $\vec{F}$ in the direction of $\vec{S}$

Unit :

$W = \text{Force (disp)}$

$W = \text{Newton (meter)}$

Unit $\downarrow$
 $[W] = \text{Joule} \leftarrow \text{SI unit}$

dim $[W] = M^1 L^2 T^{-2}$

Scalar Quantity

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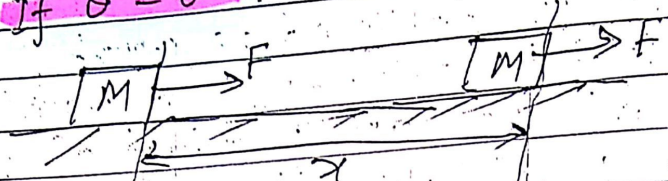
. ACUTE. ATOT

NOTE:

1. $W = |\vec{F}| |\vec{s}| \cos \theta$

$W = F s \cos \theta$

If $\theta = 0^\circ$



$W = F s \cos 0^\circ$

$W = F s$

2. If θ acute

$0^\circ < \theta < 90^\circ$

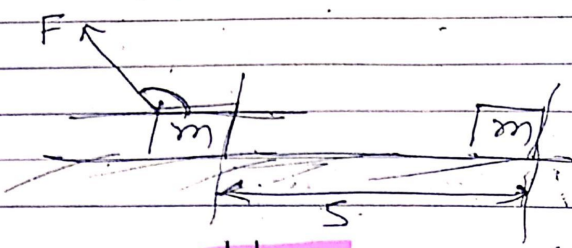
$\cos \theta \Rightarrow +ive$

$W = |\vec{F}| |\vec{s}| \cos \theta$

$W = +ive$

positive work

3. If θ is obtuse



$\theta \rightarrow obtuse$

$\cos \theta \rightarrow negative$

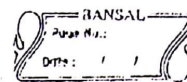
$W = |\vec{F}| |\vec{s}| \cos \theta$

$W \rightarrow negative$

$\Rightarrow$ Problems based on General definition of Work $\rightarrow$
(Work done by a constant force) $\rightarrow$

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Q.1. A body is displaced from the point (1,2) to the another point (2,3) under the action of constant Force $F = 2\hat{i} + 3\hat{j} + 2\hat{k}$. Calculate the work done.

$$(1,2) \quad \vec{r}_1 = 1\hat{i} + 2\hat{j}$$

$$(2,3) \quad \vec{r}_2 = 2\hat{i} + 3\hat{j}$$

यदि कितना $\vec{d} = \vec{r}_2 - \vec{r}_1$

$$\vec{d} = 1\hat{i} + 1\hat{j}$$

$$|\vec{F}| = 2\hat{i} + 3\hat{j} + 2\hat{k}$$

$$W = \vec{F} \cdot \vec{d}$$

$$= (2\hat{i} + 3\hat{j} + 2\hat{k}) \cdot (1\hat{i} + 1\hat{j})$$

$$W = (2)(1) + (3)(1) + (2)(0)$$

$$= 2 + 3 = 5 \text{ Joule}$$

Q.2. Three forces are acting on the particle which are given as

$$\vec{F}_1 = 1\hat{i} - 2\hat{j} + 2\hat{k} ; \vec{F}_2 = -3\hat{i} + 4\hat{j} - \hat{k} ;$$

$$\vec{F}_3 = \hat{i} - \hat{j}$$

and the body is displaced from the initial point (0,1,1) to the final point (-1,2,1), then calculate the work done.

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

$$= -1\hat{i} + 1\hat{j} + \hat{k}$$

$$\vec{r}_1 = 0\hat{i} + 1\hat{j} + 1\hat{k}$$

$$\vec{r}_2 = -1\hat{i} + 2\hat{j} + 1\hat{k}$$

$$\vec{d} = \vec{r}_2 - \vec{r}_1 = -1\hat{i} + 1\hat{j} + 0\hat{k}$$

$$W = \vec{F}_{\text{net}} \cdot \vec{d}$$

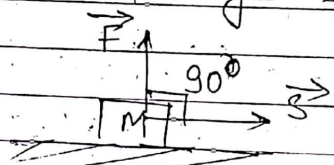
$$W = (-\hat{i} + \hat{j} + \hat{k}) \cdot (-\hat{i} + \hat{j})$$

$$= (-1)(-1) + (1)(1) + (1)(0)$$

$$W = 2 \text{ Joule}$$

→ Golden Key Points :-

If the applied force and displacement vector are in $\perp$ direction then work done by the force is always zero.



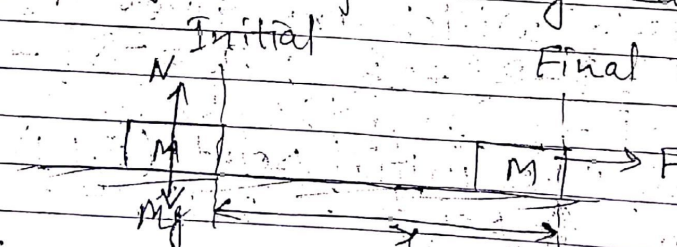
$$W = \vec{F} \cdot \vec{s}$$

$$W = |\vec{F}| |\vec{s}| \cos 90^\circ$$

$$W = 0$$

zero-work.

Q. 3. Calculate the work done by the gravity in the following case:



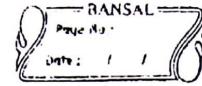
$$\text{Work done by Force} = Fx \cos 0^\circ = Fx$$

$$\text{Work done by Gravity} = [Mg][x] \cos 90^\circ = 0$$

$$\text{Work done by Normal} = 0$$

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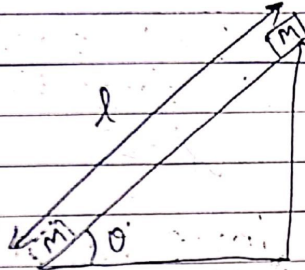
Statement

Work done by
on

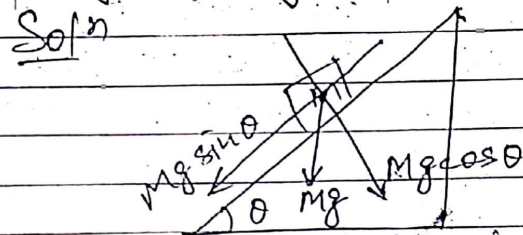
हमेशा ऐसा लिखना ही सही होता है

Q.4. Calculate the work done by gravity on block in the following case :

Soln



$W_{\text{gravity}} = ?$

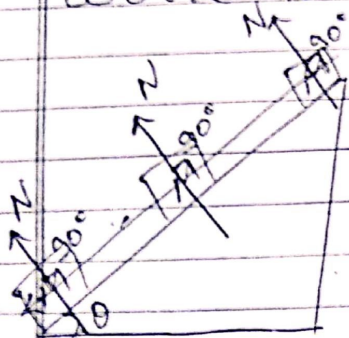


Force की ऐसी component जो displacement की direction

$$W = (Mg \sin \theta) l$$

$$W = Mgl \sin \theta$$

Q.5. In the above case calculate the work done by the normal force.



N and displacement

हमेशा 90° पर है।

$$W = 0$$

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Conceptual Points :-

$$W = \vec{F} \cdot \vec{S}$$

$$W = |\vec{F}| |\vec{S}| \cos \theta$$

$$W = [|\vec{F}| \cos \theta] |\vec{S}|$$

component of $\vec{F}$ in the direction of $\vec{S}$.

$$W = |\vec{F}| [|\vec{S}| \cos \theta]$$

displacement $\vec{S}$ in the direction of $\vec{F}$.

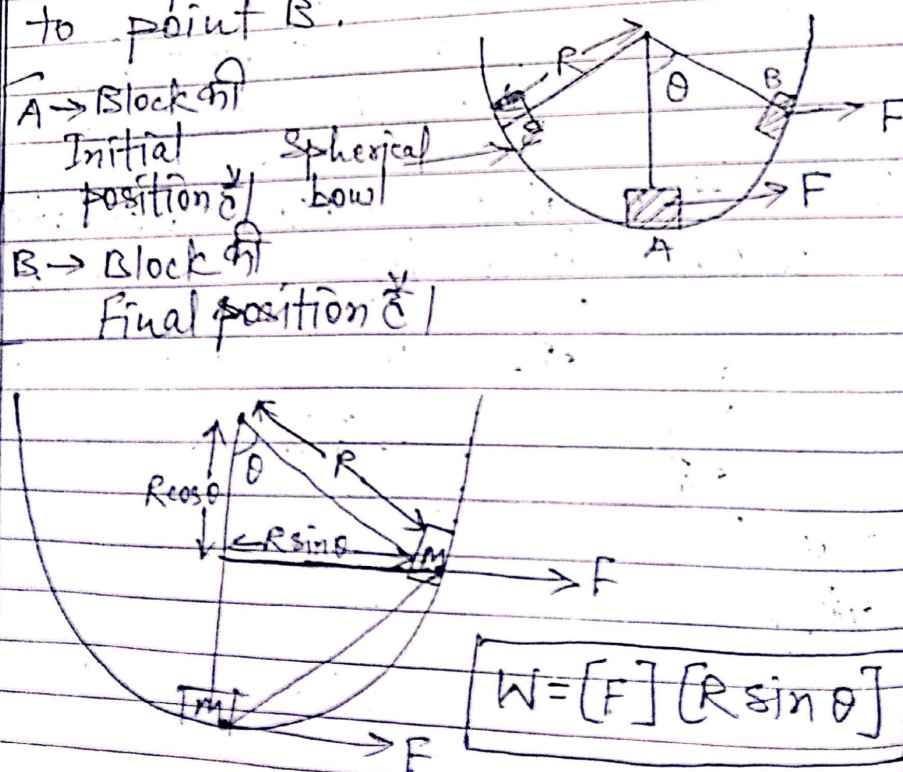
Q.6. Calculate the work done by the external force 'F', if the block is displaced from the point A to point B.

A → Block at

Initial position

B → Block at

Final position



$$W = [F] [R \sin \theta]$$

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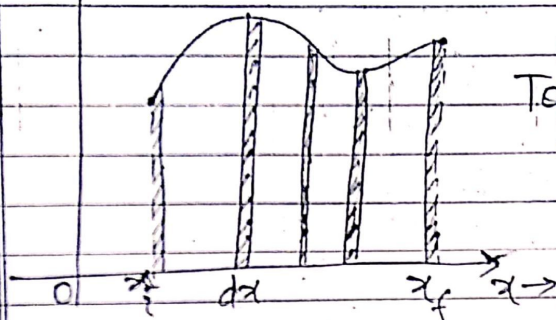
Work done by Variable Force :-

If the given Force is not constant then calculation of the work done

$F \uparrow$

$$\text{Strip area} = [F_x] [dx] = dW$$

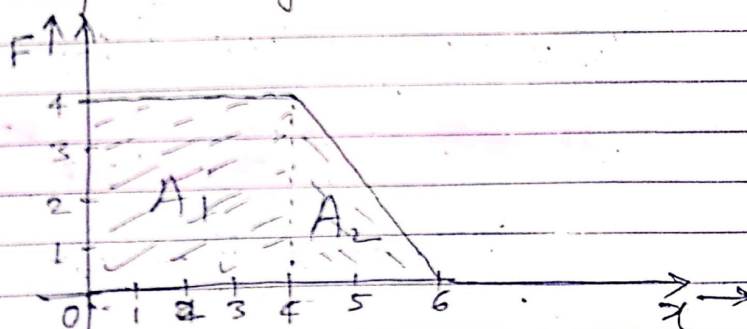
$$\text{Total Work done} = \int dW = \int_{x_i}^{x_f} F_x dx$$



Graph

$$\text{Area under the } F-x \text{ curve} = W = \int_{x_i}^{x_f} F dx$$

Q.7. Calculate the work done by the Force in the following case



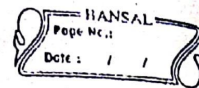
$$A_1 = 1 \times 6 = 4(4) = 16 \text{ J}$$

$$A_2 = \frac{1}{2} \times B \times H = \frac{1}{2} \times (6-4) \times 4 = 4 \text{ J}$$

$$\text{Work done} = A_1 + A_2 = 16 + 4 = 20 \text{ J}$$

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Q.8. The force acting on the particle is given as $F = 0.5x + 10$, then calculate the work done by the force if the body is displaced from $x = 0$ to $x = 2$ m.

Solⁿ

F ; x पर depend कर रहा है

F variable

$$W = \int_{x_i}^{x_f} F dx$$

$$x_i = 0$$

$$x_f = 2$$

$$W = \int_0^2 (0.5x + 10) dx$$

$$W = \int_0^2 0.5x dx + \int_0^2 10 dx$$

$$W = 0.5 \int_0^2 x dx + \int_0^2 dx$$

$$W = 0.5 \left[\frac{x^2}{2} \right]_0^2 + 10 [x]_0^2$$

upper limit - lower limit

$$W = 0.5 \left[\frac{2^2}{2} - 0 \right] + 10 [2 - 0]$$

$$W = 0.5 \left[\frac{4}{2} \right] + 20$$

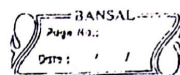
$$W = 1 + 20$$

$$W = 21 \text{ Joule}$$

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Lecture No. 14



→ Calculation of Work done if the force is variable

$$W = \int_{r_i}^{r_f} \vec{F} \cdot d\vec{r} \quad \text{हमेशा } d\vec{r} = dx \hat{i} + dy \hat{j}$$

Q.1. The force acting on the particle is given as $x\hat{i} + y^2\hat{j}$ then calculate the work done in moving the body from point A(1,2) to point B(-3,4).

$$W = \int_{(1,2)}^{(-3,4)} \vec{F} \cdot d\vec{r} \quad \text{खास बात ध्यान रखें}$$

$$d\vec{r} = dx \hat{i} + dy \hat{j}$$

$$\vec{F} = x\hat{i} + y^2\hat{j}$$

$$W = \int_{(1,2)}^{(-3,4)} [x\hat{i} + y^2\hat{j}] \cdot [dx \hat{i} + dy \hat{j}]$$

Imp. Step

$$W = \int_1^{-3} x dx + \int_2^4 y^2 dy$$

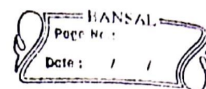
$$W = \left[\frac{x^2}{2} \right]_1^{-3} + \left[\frac{y^3}{3} \right]_2^4$$

$$W = \left[\frac{(-3)^2}{2} - \frac{1^2}{2} \right] + \left[\frac{4^3}{3} - \frac{2^3}{3} \right]$$

$$W = \frac{68}{3} \text{ Joule}$$

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Q.2. The force acting on the particle is given as $-6x^3 \hat{i}$ then calculate the work done in moving the body from $x = -1\text{m}$ to $x = -4\text{m}$.
 $\vec{F} = -6x^3 \hat{i}$

$$x = -1\text{m to } x = -4\text{m}$$

$$W = \int_{-1}^{-4} \vec{F} \cdot d\vec{s}$$

$$W = \int_{-1}^{-4} [-6x^3 \hat{i}] \cdot [dx \hat{i} + dy \hat{j}]$$

$$W = \int_{-1}^{-4} -6x^3 dx = -6 \int_{-1}^{-4} x^3 dx$$

$$W = -6 \left[\frac{x^4}{4} \right]_{-1}^{-4} = -6 \left[\frac{(-4)^4}{4} - \frac{(-1)^4}{4} \right]$$

$$W = -\frac{765}{2} \text{ J}$$

Golden key points regarding calculation of work done :-

1. If F is constant

$$W = \vec{F} \cdot \vec{s}$$

2. If curve b/w F and x is given

$$W = \text{Area under } F-x \text{ curve}$$

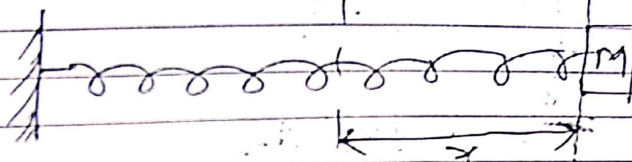
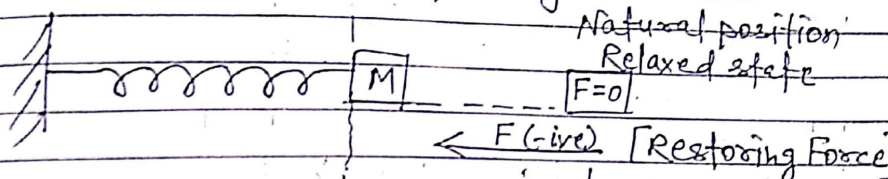
3. If F is variable

$$W = \int \vec{F} \cdot d\vec{s} \quad \left[d\vec{s} = dx \hat{i} + dy \hat{j} \right]$$

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⇒ Work done by spring force ⇒



$$\vec{F} = kx (-\hat{i})$$

$$\boxed{\vec{F} = -kx \hat{i}}$$

$k \rightarrow$ spring constant
 $x \rightarrow$ extension

$$W = \int_{x=0}^x \vec{F} \cdot d\vec{r}$$

$$W = \int_0^x (-kx \hat{i}) \cdot (dx \hat{i} + dy \hat{j})$$

$$W = \int_0^x -kx dx = -k \int_0^x x' dx$$

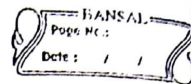
$$W = -k \left[\frac{x^2}{2} \right]_0^x = -k \left[\frac{x^2}{2} - 0 \right]$$

$$\boxed{W = -\frac{1}{2} kx^2}$$

↑
 Work done by spring

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Work Energy Theorem :-

$$W = \int_{x_i}^{x_f} F dx = \int_{x_i}^{x_f} ma dx$$

$$\boxed{a = v \cdot \frac{dv}{dx}}$$

$$W = \int_{v_i}^{v_f} m \left[v \cdot \frac{dv}{dx} \right] dx$$

$$W = \int_{v_i}^{v_f} m v' dv = m \int_{v_i}^{v_f} v' dv$$

$$W = m \left[\frac{v^2}{2} \right]_{v_i}^{v_f} = m \left[\frac{v_f^2}{2} - \frac{v_i^2}{2} \right]$$

$$W = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

$$\boxed{W = KE_f - KE_i}$$

$$\boxed{W = \Delta KE}$$

$$\boxed{W = KE_{\text{final}} - KE_{\text{initial}}}$$

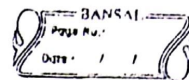
Points to remember :-

1. Work done by spring force

$$\boxed{W = -\frac{1}{2} k x^2}$$

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2. Work - Energy Theorem

$$W_{\text{Total}} = \Delta KE$$

$$W_{\text{Total}} = \text{Final KE} - \text{Initial KE}$$

Q. 1. The force $\vec{F} = 2\hat{i} + 3\hat{j}$ acting on a body having mass 5 kg and its initial position is given as $\vec{r}_1 = \hat{i} + 2\hat{j}$ and at that time its initial speed is 10 m/s. Under the action of this force body is displaced to the final position $\vec{r}_2 = 2\hat{i} + 3\hat{j}$, then calculate the final speed of the body.

$$\vec{F} = 2\hat{i} + 3\hat{j}$$

$$\vec{r}_1 = \hat{i} + 2\hat{j}$$

$$\vec{r}_2 = 2\hat{i} + 3\hat{j}$$

$$\vec{s} = \vec{r}_2 - \vec{r}_1 = 2\hat{i} + 3\hat{j} - \hat{i} - 2\hat{j}$$

$$|\vec{s}| = \hat{i} + \hat{j}$$

$$W = \vec{F} \cdot \vec{s} = (2\hat{i} + 3\hat{j}) \cdot (\hat{i} + \hat{j})$$

$$= 2\hat{i} + 3\hat{j} = 5 \text{ Joule}$$

$$\vec{F} = 2\hat{i} + 3\hat{j}$$

$$KE_i = \frac{1}{2} m (10)^2 = \frac{1}{2} \times 5 \times 10^2 = 250 \text{ J}$$

$$KE_f = \frac{1}{2} m v^2 = \frac{1}{2} \times 5 \times v^2$$

$$W = \Delta KE = KE_f - KE_i$$

$$5 = \frac{5}{2} v^2 - 250 \Rightarrow \frac{5}{2} v^2 = 255 \Rightarrow v^2 = \frac{255 \times 2}{5}$$

$$\Rightarrow v^2 = 102 \Rightarrow v = \sqrt{102} = 10.1 \text{ m/s}$$

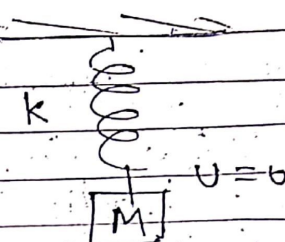
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Q.2. A block is attached to the spring as shown in the figure then calculate the max^m extension in the spring.

Concept

At max^m extension
final velocity = 0



$$W_{\text{total}} = \Delta KE$$

max Extension

$$W_{\text{gravity}} + W_{\text{spring}} = \text{Final KE of spring} - \text{Initial KE}$$

$$mg[x] = \frac{1}{2} k x^2 = 0 - 0$$

$$mgx = \frac{1}{2} k x^2$$

$$2mg = kx$$

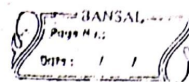
$$x = \frac{2mg}{k}$$

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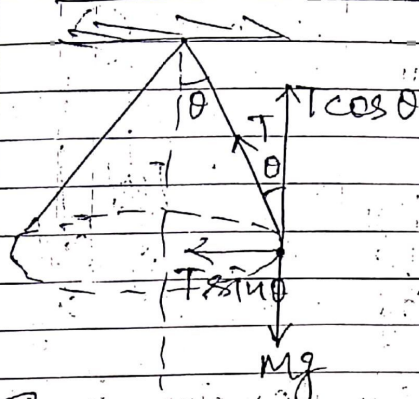
Lecture No. 15



Circular Motion

(Extra-Tips)

→ Conical Pendulum -



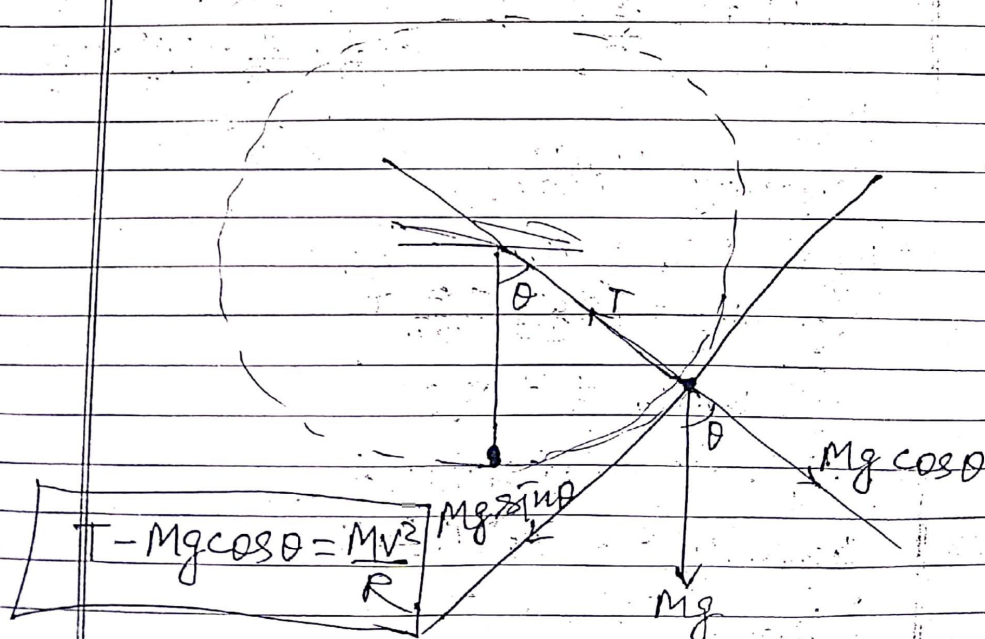
Step-① Identify the Plane of circular motion

Step ② component in radial and tangential direction if any of it

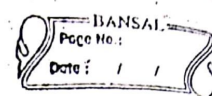
$$T \cos \theta = Mg$$

$$T \sin \theta = \frac{Mv^2}{R}$$

→ Simple Pendulum



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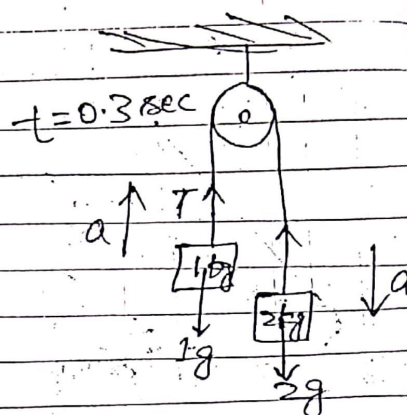
Q.1. Calculate the following parts if the system is released at $t=0$.

(a) Work done by the gravity on 1 kg block.

(b) Work done by gravity on 2 kg block.

(c) Work done by tension in the string of 1 kg block.

(d) Work done by tension in the string of 2 kg block.



$$2g - T = 2a$$

$$T - 1g = 1a$$

$$g = 3a$$

$$a = \frac{g}{3}$$

$$s = ut + \frac{1}{2}at^2$$

$$u = 0; a = \frac{g}{3}; t = 0.3$$

$$s = 0 + \frac{1}{2} \left(\frac{g}{3} \right) (0.3)^2$$

$$= \frac{1}{2} \cdot \frac{g}{3} \times \frac{0.3}{10} \times \frac{0.3}{10}$$

2 kg

$$W_{\text{gravity}} = +2g \left[\frac{3}{20} \right] \quad S = \frac{3}{20} \text{ m.}$$

2 kg $\Rightarrow$ कितना चला = $\frac{3}{20}$ m

$$W_{\text{gravity}} = [2g] \left[\frac{3}{20} \right]$$

$$W_{\text{gravity}} = +3 \text{ J}$$

1 kg $\Rightarrow$ कितना चला = $\frac{3}{20}$ m

$$W_{\text{gravity}} = -[1g] \left[\frac{3}{20} \right]$$

$$W_{\text{gravity}} = -1.5 \text{ J}$$

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1 kg

Tension $\Rightarrow$ Work done

$$W_T = + [T] \left[\frac{z}{20} \right]$$

2 kg

Tension $\Rightarrow$ Work done

$$W_T = - [T] \left[\frac{z}{20} \right]$$

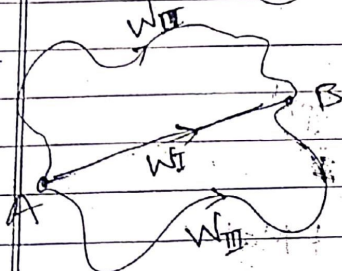
NOTE: Work done by tension for whole system is zero.

Conservative and Non-conservative force

$\rightarrow$ Conservative Force $\rightarrow$

Those forces are said to be conservative if the work done by the forces does not depend on path. Example of conservative forces are — (i) Force of gravity, (ii) Spring force.

1.



$$W_I = W_{II} = W_{III}$$

Conservative Force :-

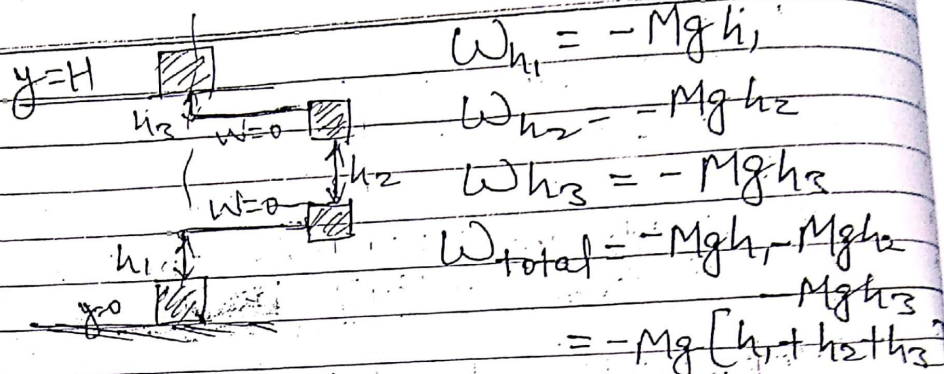
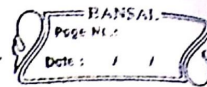


$$W_{\text{gravity}} = -MgH$$

$$[W_g = -MgH]$$

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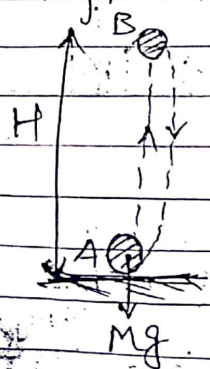
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$$[W_H = -Mgh]$$

1. अगर किसी Force के द्वारा किया गया Work path पर depend नहीं करे
 ↓
 तो Force conservative Force कहलाता है।

2. If the force is conservative then work done by these forces in a closed path is always zero (If the beginning point and the ending point are same then such a path is defined as closed path).



$$A \rightarrow B$$

$$W_{gravity} = -MgH$$

$$B \rightarrow A$$

$$W_{gravity} = +MgH$$

Initial point $\leftrightarrow$ Final point [same]

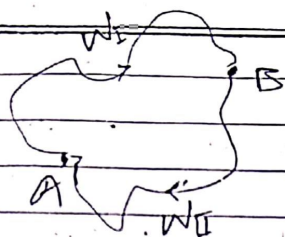
$$W_{total} = W_{A \rightarrow B} + W_{B \rightarrow A}$$

$$= -MgH + MgH$$

$$W_{total} = 0$$

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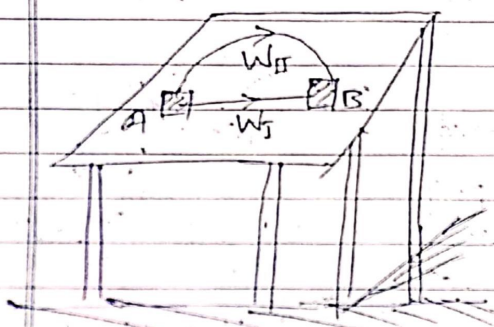
Force $\rightarrow$ conservative

$$W_I + W_{II} = 0$$

2. Non-conservative Force :-

If the force is not conservative then such forces are known as non-conservative forces.

Eg: Frictional force is the eg. of non-conservative force.



$$W_I \neq W_{II}$$

$W_{II} \Rightarrow$ ऊर्जा का Work
 $\downarrow$
against friction

$\Rightarrow$ Golden key points regarding conservative and non-conservative forces:-

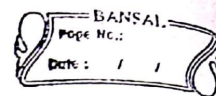
[इसको याद कर जना! तो - - -]

- ① Conservative force में Work done path पर depend नहीं करता है।
- ② Force of Gravity and spring Force are the examples of conservative Forces.

③ अगर किया गया Work path पर

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depend करेंगी

Force $\Rightarrow$ Nonconservative होगी /

④ frictional force is the example of Nonconservative force.

$\Rightarrow$ Differentiation :-

$$1. \frac{d}{dx} x^3 \quad 2. \frac{d}{dx} 5x^5 \quad 3. \frac{d}{dx} (50)$$

$$\Rightarrow 3x^2 \quad \Rightarrow 5[5x^4] \quad \Rightarrow 0$$

$$1. \frac{\partial}{\partial x} (x^2 y) \Big|_{y=\text{const}} \quad 2. \frac{\partial}{\partial x} (x^3 y z^2) \Big|_{y, z=\text{const}}$$

[Partial diff. $x^2 y$ का x के respect में]

$$\Rightarrow y \frac{\partial}{\partial x} x^2 \Rightarrow y [2x^1]$$

$$\Rightarrow y z^2 \left[\frac{\partial}{\partial x} x^3 \right] \Rightarrow y z^2 [3x^2]$$

$$3. \frac{\partial}{\partial z} (x^2 y z^5) \Big|_{x, y=\text{const}} \quad 4. \frac{\partial}{\partial y} (x^2 y) \Big|_{x=\text{const}}$$

$$\Rightarrow x^2 y \left[\frac{\partial}{\partial z} z^5 \right] \Rightarrow x^2 y [5z^4]$$

$$\Rightarrow x^2 \left[\frac{\partial}{\partial y} y \right] \Rightarrow x^2 [1]$$

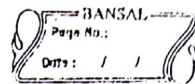
$$Q.1. \frac{\partial}{\partial z} (5x^2) \Big|_{x, y=\text{const}} \quad Q.2. \frac{\partial}{\partial y} (x^2 z) \Big|_{x, z=\text{const}}$$

$$\Rightarrow 0 \quad \Rightarrow 0$$

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→ dell (easy)



Q.3. $\frac{\partial}{\partial y} [4x^2y^3] \mid x, z = \text{const}$

$$\Rightarrow 4x^2 \left[\frac{\partial}{\partial y} y^3 \right] \Rightarrow 4x^2 [3y^2] \\ \Rightarrow 12x^2y^2$$

⇒ NOTE: Condition to check whether the given force is conservative or not

✓ ⇒ $\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x} \quad \mid x, y, z$

$\frac{\partial F_y}{\partial z} = \frac{\partial F_z}{\partial y}$

$\frac{\partial F_x}{\partial z} = \frac{\partial F_z}{\partial x}$

$\left(\frac{\partial F_y}{\partial z} = \frac{\partial F_z}{\partial y} \right)$

$\Rightarrow \frac{\partial}{\partial z} [y] = \frac{\partial}{\partial y} [0]$

$\Rightarrow 0 = 0$

Q.① $\vec{F} = \hat{i} + y\hat{j}$

Solⁿ $\vec{F} = F_x\hat{i} + F_y\hat{j}$

$F_x = 1 \quad F_y = y$

Condition - ①

$\frac{\partial}{\partial y} F_x = \frac{\partial}{\partial x} F_y$

$\Rightarrow \frac{\partial}{\partial y} [1] = \frac{\partial}{\partial x} [y]$

$\Rightarrow 0 = 0$

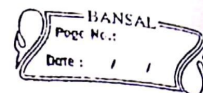
$\frac{\partial F_x}{\partial z} = \frac{\partial F_z}{\partial x}$

$\Rightarrow \frac{\partial}{\partial z} [1] = \frac{\partial}{\partial x} [0]$

$\Rightarrow 0 = 0$

Conservative Force

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Q.2. Check whether the given force is conservative or not.

$$\vec{F} = (3y)\hat{i} + (4x)\hat{j}$$

S.T.R. - (01)

$$\vec{F} = F_x\hat{i} + F_y\hat{j}$$

$$\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x}$$

$$\checkmark F_x = 3y$$

$$\frac{\partial}{\partial y} [3y] = \frac{\partial}{\partial x} [4x]$$

$$\checkmark F_y = 4x$$

$$3(1) = 4(1)$$

$$\checkmark F_z = 0$$

$$3 \neq 4$$

Non-Conservative

Q.1.

The force acting on the particle is given as $\vec{F} = x^2\hat{i} + y^2\hat{j}$ then calculate the work done if the particle is taken across the path which is given as

$$\vec{F} = x^2\hat{i} + y^2\hat{j}$$

$$\vec{F} = F_x\hat{i} + F_y\hat{j}$$

$$F_x = x^2$$

$$F_y = y^2$$

$$F_z = 0$$

S.T.R. - (01)

S.T.R. - (02)

$$\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x}$$

$$\frac{\partial F_x}{\partial z} = \frac{\partial F_z}{\partial y}$$

$$\frac{\partial}{\partial y} [x^2] = \frac{\partial}{\partial x} [y^2]$$

$$\frac{\partial}{\partial z} [y^2] = \frac{\partial}{\partial y} [0]$$

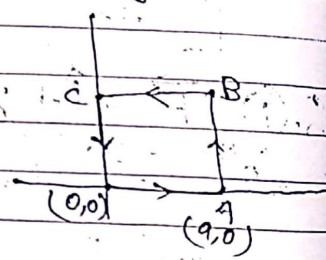
$$0 = 0$$

$$0 = 0$$

S.T.R. - (03)

$$\frac{\partial F_z}{\partial x} = \frac{\partial F_x}{\partial z} \Rightarrow \frac{\partial (0)}{\partial x} = \frac{\partial [x^2]}{\partial z} \Rightarrow 0 = 0$$

$\therefore$ Conservative Force



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NOTE: Since the above force is conservative and acc. to the definition of the conservative Work done in a closed path is always zero

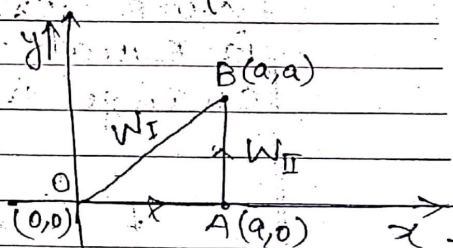
Q.2. The force acting on the particle is given as $\vec{F} = \hat{i} + y\hat{j}$ and the particle is displaced from point O to the final point B through two given path as shown in the figure.

(A) $W_I > W_{II}$

(B) $W_{II} > W_I$

(C) $W_I = W_{II}$

(D) पता नहीं



$$\vec{F} = \hat{i} + y\hat{j}$$

$$\vec{F} = F_x\hat{i} + F_y\hat{j}$$

$$F_x = 1$$

$$\text{STT-01}$$

$$F_y = y$$

$$\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x}$$

$$F_z = 0$$

$$\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x}$$

$$\frac{\partial}{\partial y}[1] = \frac{\partial}{\partial x}[y]$$

$$0 = 0$$

$$\text{STT-02}$$

$$\frac{\partial F_x}{\partial z} = \frac{\partial F_z}{\partial x}$$

$$\frac{\partial}{\partial z}[y] = \frac{\partial}{\partial x}[0]$$

$$0 = 0$$

$$\text{STT-03}$$

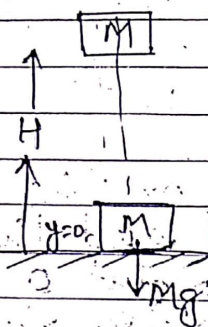
$$\frac{\partial F_z}{\partial x} = \frac{\partial F_x}{\partial z} \Rightarrow \frac{\partial}{\partial x}[0] = \frac{\partial}{\partial z}[1] \Rightarrow 0 = 0$$

NOTE: Since the above force is conservative and accor. to definition of conservative, Work done does not depends on path.

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Force and Potential Energy



$$W_{\text{gravity}} = -MgH$$

$$U = +MgH$$

Work done by the conservative force is equal to the negative of the change in potential energy.

$$W = -\Delta U$$

force is acting along
x-axis (one dimension)

$$W = -\Delta U$$

$$F(\Delta x) = -\Delta U$$

$$F = \frac{-\Delta U}{\Delta x}$$

$$\text{If } \Delta x \rightarrow 0$$

$$F = -\frac{dU}{dx}$$

$$W = -\Delta U$$

$$\int \vec{F} \cdot d\vec{r} = -\Delta U$$

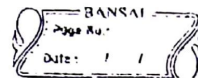
$$\Delta U = - \int_i^f \vec{F} \cdot d\vec{r}$$

$$U_f - U_i = - \int_i^f \vec{F} \cdot d\vec{r}$$

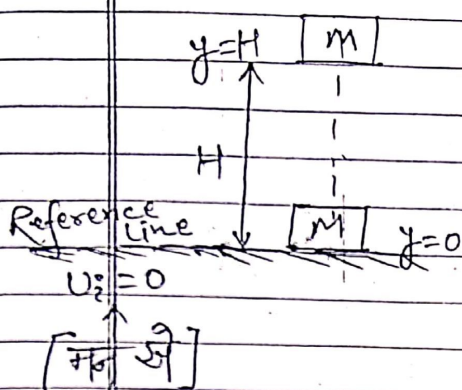
NOTE: Potential energy is defined only for conservative forces because work done by the conservative forces does not depend on path. And it depends only on initial and final state.

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→ Gravitational PE [Force of Gravity की PE]



$$U_f - U_i = - \int \vec{F} \cdot d\vec{r}$$

$$U_f - U_i = - \int_0^H mg(-\hat{j}) \cdot (dx\hat{i} + dy\hat{j})$$

$$U_f - U_i = + \int_0^H mg dy$$

$$U_f - U_i = mg \int_0^H dy$$

$$U_f - U_i = mg [y]_0^H$$

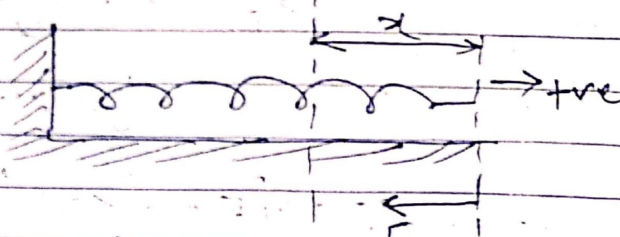
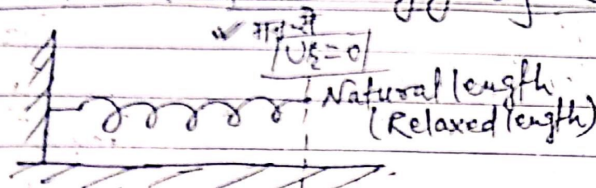
$$U_f - U_i = mg [H - 0]$$

$$U_f - U_i = mgh$$

$$U_i = 0$$

$$U_f = Mgh$$

→ Potential Energy of Spring



$$\vec{F} = kx(-\hat{i})$$

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$$U_f - U_i = - \int \vec{F} \cdot d\vec{r}$$

$$U_f - U_i = - \int kx(-\hat{i}) \cdot (dx\hat{i} + dy\hat{j})$$

$$U_f - U_i = + \int_0^x kx' dx'$$

$$U_f - U_i = k \left[\frac{x^2}{2} \right]_0^x$$

$$U_f - U_i = k \left[\frac{x^2}{2} - 0 \right]$$

$$U_f - U_i = \frac{1}{2} kx^2$$

$$\boxed{U_f = \frac{1}{2} kx^2}$$

NOTE: Always remember that concept of PE is defined only for conservative forces.

$$F = - \frac{dU}{dx}$$

Ex: $U = \frac{1}{2} kx^2$

$$F = ?$$

अगर U की function पढ़ा है तो

Sol: $F = - \frac{dU}{dx}$

F की calculation.

$$\Rightarrow F = - \frac{d}{dx} \left[\frac{1}{2} kx^2 \right]$$

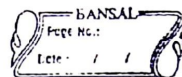
$$\Rightarrow F = - \frac{1}{2} k \frac{d}{dx} x^2$$

$$\Rightarrow F = - \frac{1}{2} k [2x]$$

$$\Rightarrow \boxed{F = -kx}$$

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Ex: $U = x^2 + 3x + 3$

$F = ?$

पता है $F = -\frac{dU}{dx}$

$F = -\frac{d}{dx} [x^2 + 3x + 3]$

$F = -\left[\frac{d}{dx} x^2 + \frac{d}{dx} 3x + \frac{d}{dx} 3\right]$

$F = -[2x + 3(1) + 0]$

$F = -[2x + 3]$

NOTE: Always remember that concept of PE is applicable only for conservative forces.

⇒ Force and Energy in 3-D :-

$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$

$F_x \rightarrow x$ component $\vec{F}$ का

$F_y \rightarrow y$ component $\vec{F}$ का

$F_z \rightarrow z$ component $\vec{F}$ का

$F = -\frac{dU}{dx}$

$F_x = -\frac{\partial U}{\partial x}$ $F_y = -\frac{\partial U}{\partial y}$ $F_z = -\frac{\partial U}{\partial z}$

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$$\left. \begin{aligned} F_x &= -\frac{\partial U}{\partial x} \quad [y, z = \text{const}] \\ F_y &= -\frac{\partial U}{\partial y} \quad [x, z = \text{const}] \\ F_z &= -\frac{\partial U}{\partial z} \quad [x, y = \text{const}] \end{aligned} \right\}$$

Ex: $U = x^2y + xy^2$
 $\vec{F} = ?$

Sol: पहले $F_x = -\frac{\partial U}{\partial x} \quad | \text{ मान से } y, z = \text{const}$

$$F_x = -\frac{\partial U}{\partial x} [x^2y + xy^2]$$

$$F_x = -\left[\frac{\partial}{\partial x} x^2y + \frac{\partial}{\partial x} xy^2\right]$$

$$F_x = -[y(2x) + y^2(1)]$$

$$F_x = -[2xy + y^2]$$

$$F_y = -\frac{\partial U}{\partial y} \quad | x = \text{const}$$

$$F_y = -\frac{\partial}{\partial y} [x^2y + xy^2]$$

$$F_y = -\left[\frac{\partial}{\partial y} x^2y + \frac{\partial}{\partial y} xy^2\right]$$

$$F_y = -[x^2(1) + x(2y)]$$

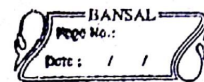
$$F_y = -[x^2 + 2xy]$$

$$\vec{F} = F_x \hat{i} + F_y \hat{j}$$

$$\vec{F} = -[2xy + y^2] \hat{i} - [x^2 + 2xy] \hat{j}$$

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Golden Key Points :-

① $W = -\Delta U$

② हमेशा ध्यान रखें

$U \rightarrow$ defined $\rightarrow$ conservative force

③ $W_{\text{conservative}} = -\Delta U$

ज्यादा अच्छा format

④ $F = -\frac{dU}{dx}$ कब लगाना है जब U पता है and $F = ?$

⑤ $U_f - U_i = - \int_i^f \vec{F} \cdot d\vec{r}$

⑥ $U_f - U_i = Mgh$

⑦ $U_f - U_i = \frac{1}{2} kx^2$

Natural length $U_i = 0$

$U_f = \frac{1}{2} kx^2$

⑧ $F_x = -\frac{\partial U}{\partial x}$

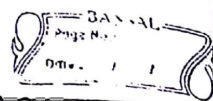
$F_y = -\frac{\partial U}{\partial y}$

$F_z = -\frac{\partial U}{\partial z}$

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Lecture No. 18



→ Practice on Partial Differentiation

$$Q.1) \frac{\partial}{\partial x} [x^2 - 3x + 5]$$

$$\Rightarrow \frac{\partial}{\partial x} x^2 - \frac{\partial}{\partial x} 3x + \frac{\partial}{\partial x} 5$$

$$\Rightarrow 2x - 3(1) + 0$$

$$\begin{aligned} Q.2) \frac{\partial}{\partial x} [x^2 y z] &\Rightarrow y z \cdot \frac{\partial}{\partial x} x^2 \\ &\Rightarrow y z [2x] \end{aligned}$$

$$Q.3) \frac{\partial}{\partial y} [5xz^2] \Rightarrow 0$$

$\nwarrow [x, z = \text{const}]$

$$Q.4) \frac{\partial}{\partial y} [y^3 z^2] \Rightarrow z^2 \left[\frac{\partial}{\partial y} y^3 \right] = z^2 (3y^2)$$

* Law of conservation of Energy

$$W_{\text{total}} = \Delta KE$$

↓

$$W_{\text{conservative}} + W_{\text{Non-conservative}} = \Delta KE$$

↓

$$-\Delta U + W_{nc} = \Delta KE$$

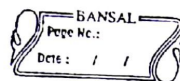
$$W_{nc} = \Delta KE + \Delta U$$

$$\boxed{W_{nc} = \Delta KE + \Delta PE}$$

↑
work done by
non-conservative
force

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If No non-conservative Force is present
(friction नहीं है)

$$W_{nc} = 0$$

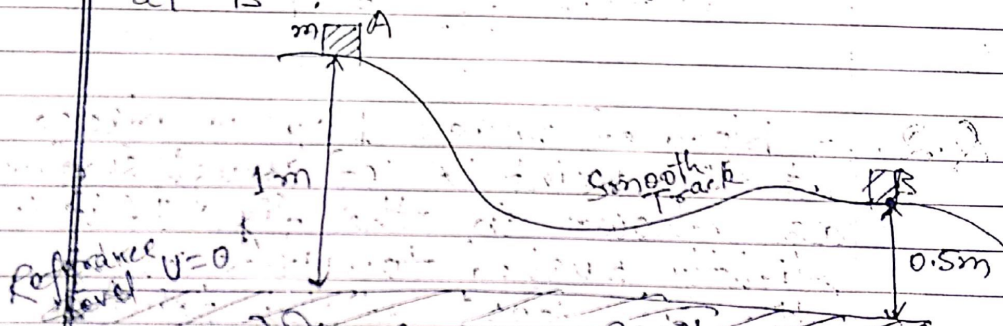
$$0 = \Delta KE + \Delta PE$$

$$0 = KE_f - KE_i + PE_f - PE_i$$

$$\boxed{KE_i + PE_i = KE_f + PE_f}$$

यहाँ हम जानते हैं कि friction नहीं है।
(Non-conservative force नहीं है)

Q.1 In the following fig if the block is slightly displaced from point 'A' then find out the speed of the block at 'B'.



क्योंकि friction नहीं है।

$$KE_A + PE_A = KE_B + PE_B$$

$$0 + mg[1] = \frac{1}{2}mv^2 + mg(0.5)$$

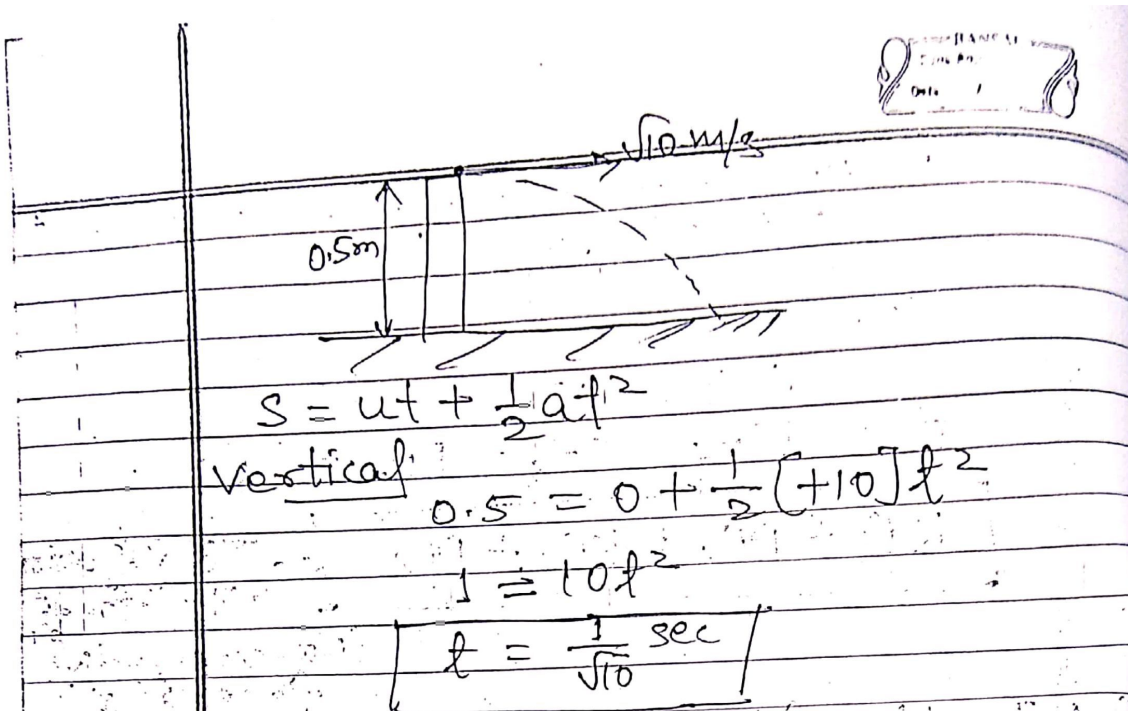
$$\boxed{v = \sqrt{10} \text{ m/sec}}$$

$$\Rightarrow 10 = \frac{1}{2}(v^2 + 10) \Rightarrow v^2 = 20 - 10$$

$$\Rightarrow v^2 = 10 \Rightarrow v = \sqrt{10} \text{ m/s}$$

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$$s = ut + \frac{1}{2}at^2$$

vertical $0.5 = 0 + \frac{1}{2} (+10) t^2$

$$1 = 10t^2$$

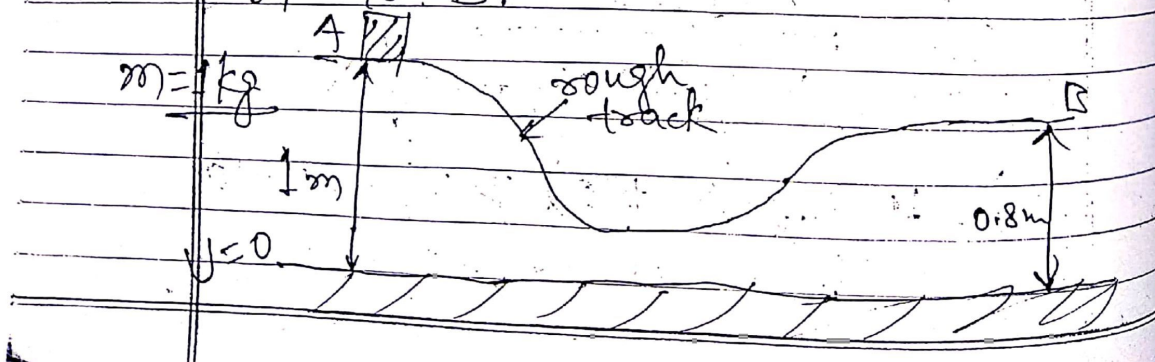
$$t = \frac{1}{\sqrt{10}} \text{ sec}$$

$$s = ut + \frac{1}{2}at^2$$

(H) $R = \sqrt{10}t + 0$

$$R = \sqrt{10} \left[\frac{1}{\sqrt{10}} \right] \Rightarrow R = 100\text{m}$$

Q.2. A block of mass 1 kg is placed at the point 'A' of rough surface as shown in the fig. If it is slightly pushed towards right then it stops at the point 'B' of the track, then calculate the work done by the frictional force during its motion from 'A' to 'B'.



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~~$$KE_A + PE_A = KE_B + PE_B$$~~

↑
यह नहीं लगेंगा।

$$W_{nc} = \Delta KE + \Delta PE$$

$$W_{friction} = [KE_f - KE_i] + [PE_f - PE_i]$$

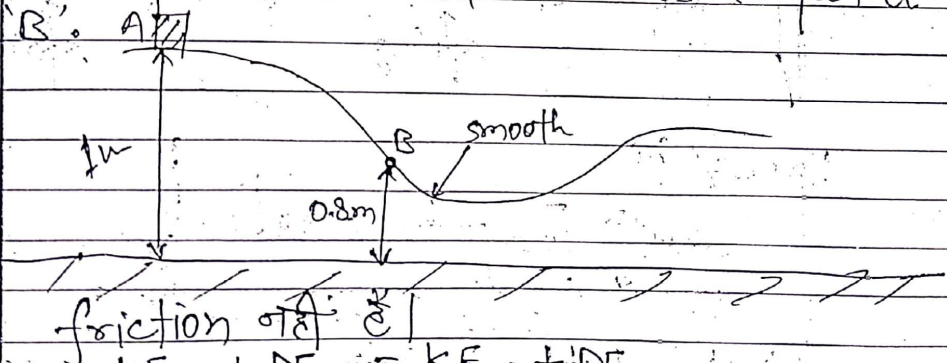
$$= [0 - 0] + [Mg(0.8) - Mg(1)]$$

$$W_{friction} = -Mg(0.2)$$

$$= -1(10)(0.2)$$

$$W_{friction} = -2J$$

Q.3. If the block is slightly pushed toward right from point A then find out its speed when it reaches to point B.



$$KE_A + PE_A = KE_B + PE_B$$

$$0 + Mg(1) = \frac{1}{2}mv^2 + Mg(0.8)$$

$$v = \frac{2m}{.8sec}$$

$$\frac{1}{2}v^2 = 10 - \frac{8 \times 10}{10} \Rightarrow \frac{1}{2}v^2 = 2 \Rightarrow v^2 = 4$$

$$\Rightarrow v = 2 \text{ m/s}$$

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⇒ Work done by the elastic Force

$$W_{\text{total}} = \Delta KE$$

$$W_c + W_{nc} = \Delta KE$$

↓

$$-\Delta U_{\text{gravity}} - \Delta U_{\text{spring}} + W_{nc} = \Delta KE$$

$$W_{nc} = \Delta KE + \Delta U_{\text{gravity}} + \Delta U_{\text{spring}}$$

Since friction is 0

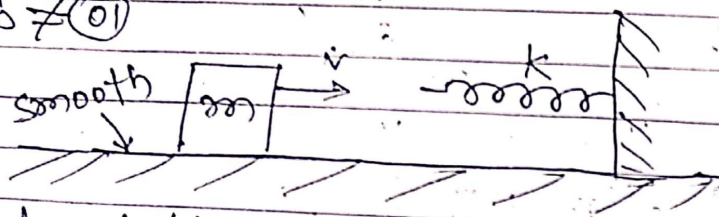
$$W_{nc} = 0$$

$$0 = \Delta KE + \Delta U_{\text{gravity}} + \Delta U_{\text{spring}}$$

$$0 = KE_f - KE_i + PE_f - PE_i + PE_{\text{spring}_f} - PE_{\text{spring}_i}$$

$$KE_i + PE_i + PE_{\text{spring}_i} = KE_f + PE_f + PE_{\text{spring}_f}$$

Prob # (1)



Find out the maximum compression of spring.

Sol

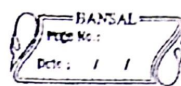
maximum compression

↓

Block ki final velocity = 0

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$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

$$\frac{mv^2}{k} = x^2 \Rightarrow x = \sqrt{\frac{m \cdot v^2}{k}}$$

OR

$$KE_i + PE_i + PE_{\text{spring}} = KE_f + PE_f + PE_{\text{spring}}$$

$$\frac{1}{2}mv^2 + 0 + 0 = 0 + 0 + \frac{1}{2}kx^2$$

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

JEE 2011

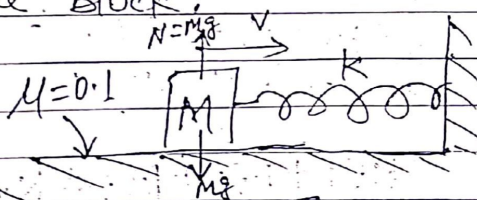
Q.4. A block of mass 0.18 kg is attached to the spring of force constant 2 N/m. Initially the block is at rest and the spring is unstretched. An impulse is given to the block as shown in the fig and the block slides a distance of 0.06 m and comes to rest. Calculate the initial velocity of the block.

$$M = 0.18 \text{ kg}$$

$$k = 2 \text{ N/m}$$

$$\mu = 0.1$$

$$S = 0.06 \text{ m}$$



$$KE_i + PE_i + PE_{\text{spring}} = KE_f + PE_f + PE_{\text{spring}}$$

अब ही ही! friction है।

$$W_{\text{fric}} = \Delta KE + \Delta PE + \Delta PE_{\text{spring}}$$

$$-[\mu N][0.06] = [0 - \frac{1}{2}MV^2] + [0 - 0] + [\frac{1}{2}kx^2 - 0]$$

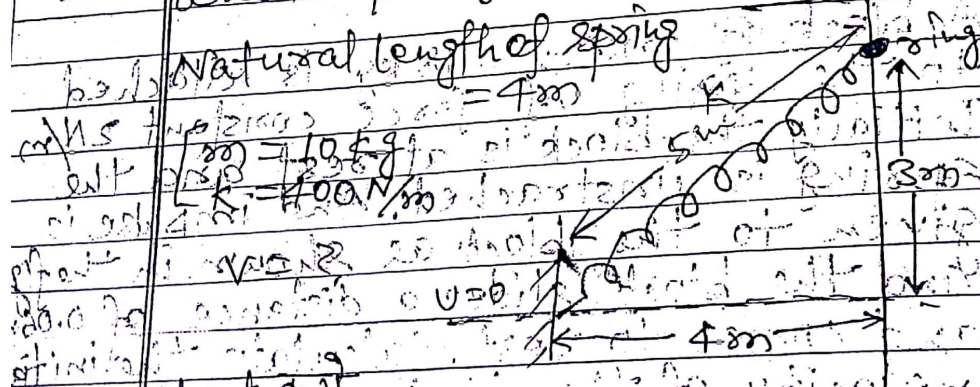
$$1.08 = 0.09 m^2/s^2$$

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$$\begin{aligned}
 -0.06 \text{ N} &= \frac{1}{2} k x^2 - \frac{1}{2} M V^2 \\
 -0.06 \times 0.1 \times 1.8 &= \frac{1}{2} \times 2 \times (0.06)^2 - \frac{1}{2} \times 0.18 \times V^2 \\
 -0.0108 &= 0.0036 - 0.09 V^2 \\
 \Rightarrow V^2 &= \frac{0.0144}{0.09} \Rightarrow V = \frac{0.12}{0.3} = 0.4 \\
 \Rightarrow V &= 0.4 \text{ m/sec} \\
 V &= \frac{N}{10} \Rightarrow N = ? \\
 N &= 4
 \end{aligned}$$

Q.5. Find out the velocity of the ring when spring becomes horizontal



check for

Spring stretched है या नहीं

$$x = 5 - 4 = 1\text{m}$$

$$KE_i + PE_i + PE_s = KE_f + PE_f + PE_s$$

$$0 + Mg(3) + \frac{1}{2} k (1)^2 = \frac{1}{2} M V^2 + 0 + 0$$

$$10 \times 10 \times 3 + \frac{1}{2} \times 400 = \frac{1}{2} \times 10 \times V^2$$

$$500 = 5V^2 \Rightarrow V = \sqrt{100}$$

$$\Rightarrow V = 10 \text{ m/s}$$

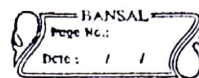
$$V = 10 \text{ m/s}$$

Natural length पर है

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Ex # 62



Q. 6 Find out the velocity of the ring when spring become horizontal.

$$k = \frac{4mg}{R}$$

natural length = $2R$

Smooth surface

$$v = ?$$

$$\text{H.C.} = \sqrt{\left(\frac{3R}{2}\right)^2 + (2R)^2}$$

$$= \frac{5R}{2}$$

$x = \text{H.C.} - \text{natural length}$

$$x = \frac{R}{2}$$

$$KE_i + PE_i + PE_s = KE_f + PE_f + PE_s$$

$$0 + Mg\left[\frac{3R}{2}\right] + \frac{1}{2}k\left[\frac{R}{2}\right]^2 = \frac{1}{2}mv^2 + 0 + 0$$

$$Mg\left[\frac{3R}{2}\right] + \frac{1}{2}\left[\frac{4mg}{R}\right]\left[\frac{R}{2}\right]^2 = \frac{1}{2}mv^2$$

$$v = \sqrt{4gR}$$

$$v = 2\sqrt{gR}$$

$$3Rg + \frac{4g}{R} \times \frac{R^2}{4} = v^2$$

$$\Rightarrow 3Rg + Rg = v^2$$

$$\Rightarrow 4Rg = v^2$$

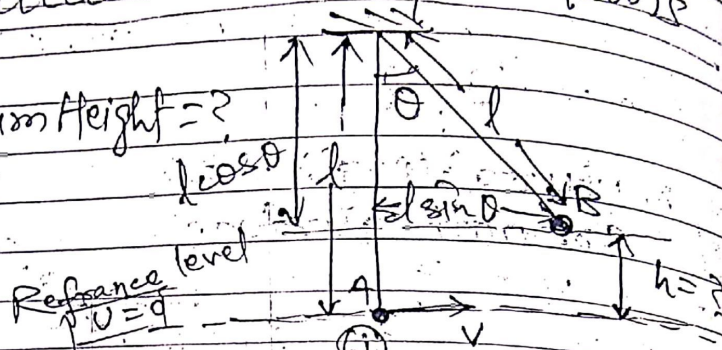
$$\Rightarrow v = 2\sqrt{Rg}$$

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Pendulum based questions

Ex # 01

maximum Height = ?



$$\boxed{\frac{1}{2}mv^2 = mgh}$$

OR

$$KE_i + PE_i = KE_f + PE_f$$

$$\frac{1}{2}MV^2 + 0 = 0 + Mgh$$

Highest

$$\frac{1}{2}MV^2 = Mgh$$

Home Work

$$V^2 = 2gh \Rightarrow$$

$$\boxed{V = \sqrt{2gh}}$$

H.C. Verma

Q. 36, 37, 40, 44, 47, 48, 51

Q.B. sheet

Q. 31, 32,

Ex # 2, Q. 6

Ex # 3, Q. 15

$$h = l - l \cos \theta$$

$$V = \sqrt{2g(l - l \cos \theta)}$$

$$\boxed{V = \sqrt{2gl(1 - \cos \theta)}}$$

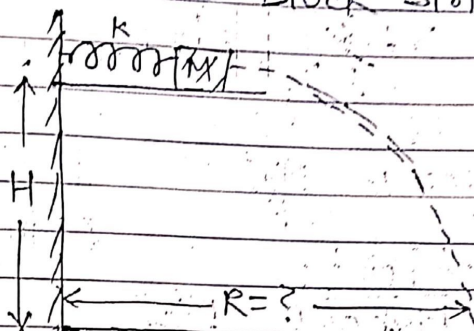
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Lecture No. 19

Questions based on Law of conservation of Energy.

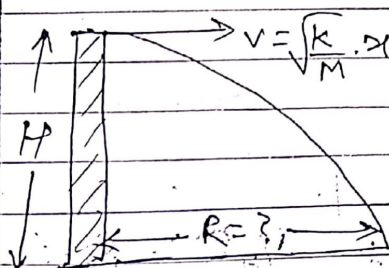
Q.1. Initially the spring is compressed by a distance x with the help of block as shown in the figure. Find out the range where the block strike the ground



$$\frac{1}{2} K x^2 \rightarrow \frac{1}{2} M v^2$$

$$\frac{1}{2} K x^2 = \frac{1}{2} M v^2 \rightarrow \frac{K x^2}{M} = v^2$$

$$\Rightarrow v = \sqrt{\frac{K}{M} \cdot x}$$



$$s = ut + \frac{1}{2} a t^2$$

vertical

$$H = 0 + \frac{1}{2} g t^2$$

$$t = \sqrt{\frac{2H}{g}}$$

$$s = ut + \frac{1}{2} a t^2$$

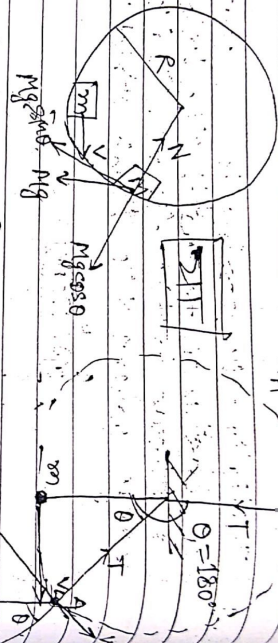
$$(H) R = vt + 0$$

$$R = \sqrt{\frac{K}{M} \cdot x} \cdot \sqrt{\frac{2H}{g}}$$

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→ Vertical Circular Motion :-



$$N - Mg \cos \theta = \frac{MV^2}{R}$$

Highest Point :-

$$T - Mg \cos \theta = \frac{MV^2}{R}$$

$$\theta \rightarrow 180^\circ$$

$$V \rightarrow V_H$$

$$T - Mg \cos 180^\circ = \frac{MV_H^2}{R}$$

$$T - Mg(-1) = \frac{MV_H^2}{R}$$

$$T + Mg = \frac{MV_H^2}{R}$$

$$T = \frac{MV_H^2}{R} - Mg$$

At highest point

$$T > 0$$

$$\frac{MV_H^2}{R} - Mg > 0$$

$$\frac{MV_H^2}{R} > Mg$$

$$V_H \geq \sqrt{Rg}$$

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$v_H = \sqrt{Rg}$

R

R

$v = 0$

$v_L = ?$
(velocity at lower point)

$$KE_L + PE_L = KE_H + PE_H$$

$$\frac{1}{2} M v_L^2 + 0 = \frac{1}{2} M [\sqrt{Rg}]^2 + Mg(2R)$$

$$\frac{1}{2} v_L^2 = \frac{1}{2} Rg + 2Rg$$

$$v_L^2 = \frac{5}{2} Rg$$

$$(v_L)_{\min} = \sqrt{\frac{5}{2} Rg}$$

Summary :-

$v_H = \sqrt{Rg}$

R

$v_{\min} = \sqrt{\frac{5}{2} Rg}$

at min velocity $\frac{5}{2}$ at lower most point पर दी जाए so that particle complete its circular path.

Important Point :-

What min^m velo is required at the lower most point so that particle just reaches at the middle point.

L → Lowest point
M → Middle point
 v_L → velocity at lowest point.

$v = 0$

$v_L = ?$

R

करना क्या है किता v_L दो so that particle JUST reaches at point M.

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KE_L + PE_L = PE_M + KE_M

$\frac{1}{2} M V_L^2 + 0 = M g R + 0$

$\frac{1}{2} M V_L^2 = M g R$

$V_L^2 = 2 g R$

$V_L = \sqrt{2 g R}$

Golden Key Points :-

1. $V < \sqrt{2 g R}$

↓

particle oscillatory motion

$L \rightarrow M \rightarrow L \rightarrow M$

$V = \sqrt{2 g R}$

↓

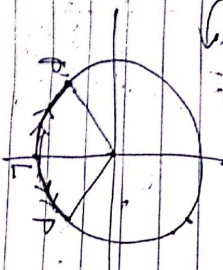
L पर गायस होते समय कभी नहीं रुकता है

$V < \sqrt{2 g R}$

↓

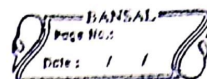
$L \rightarrow P \rightarrow L \rightarrow P$

oscillatory



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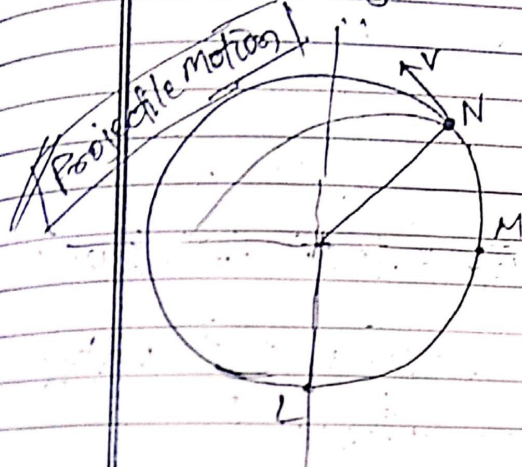


2. ~~$\sqrt{5gL} < v < \sqrt{2gL}$~~

क्या particle complete circle कर पायेगा ?

↓
नहीं

$\sqrt{2gL} < v < \sqrt{5gL}$



3. $v = \sqrt{5gL}$

↓
particle just complete its circular path

उस समय

Highest point पर $\sqrt{Rg}$ होगी।

4. $v > \sqrt{5gL}$

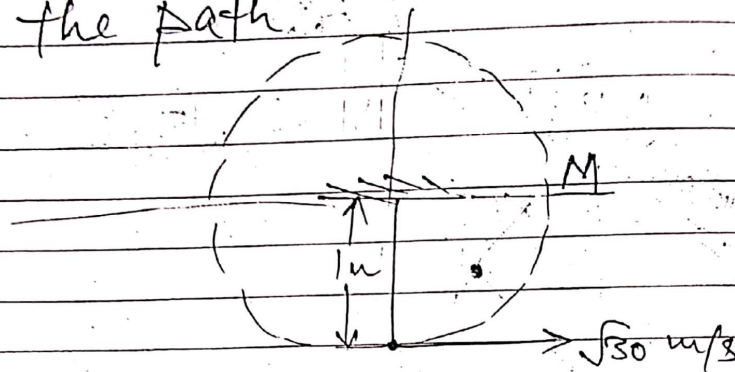
↓

Particle continue its circular path

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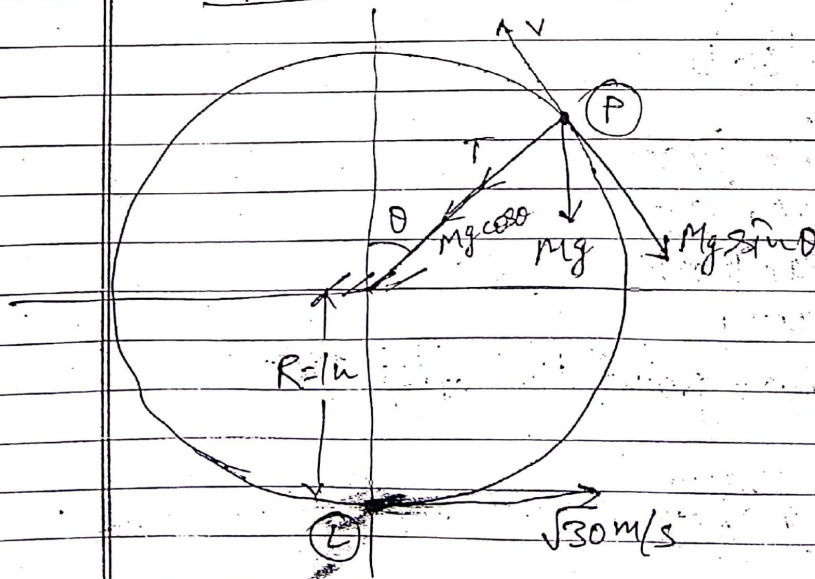
Q.2. The length of the string is 1m and the horizontal velocity at the lower point is $\sqrt{30}$ m/s. Calculate the angle made with the vertical where it leave the circular path and also calculate the velocity of the path.



पूरा चक्कर $\Rightarrow \sqrt{5Rg} = \sqrt{5(1)(10)} = \sqrt{50}$ m/s

M पर पहुँचने के $\Rightarrow \sqrt{2Rg} = \sqrt{2(1)(10)} = \sqrt{20}$ m/s

[M तो छोड़ करेगा]



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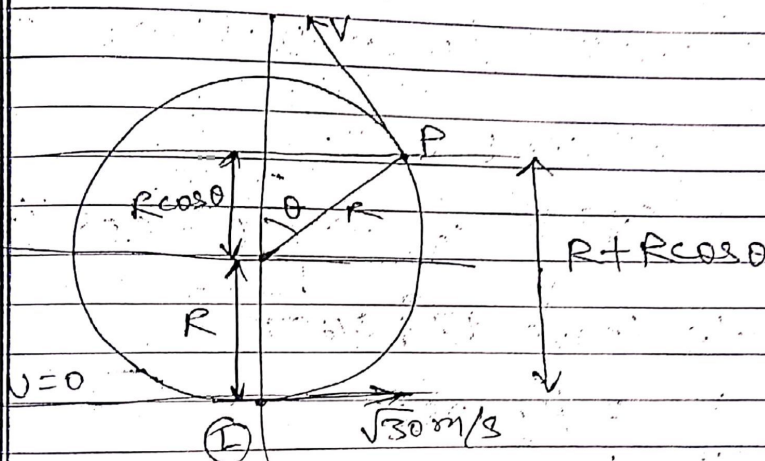
Concept # (01)

$$T + Mg \cos \theta = \frac{MV^2}{R}$$

Point A $\theta = 0$ $T = 0$

$$Mg \cos \theta = \frac{MV^2}{R}$$

$$V^2 = Rg \cos \theta$$



Concept # (02)

Law of conservation of energy

$$KE_L + PE_L = KE_P + PE_P$$

$$\frac{1}{2} M (\sqrt{30})^2 + 0 = \frac{1}{2} MV^2 + Mg[R + R \cos \theta]$$

$$15 = \frac{V^2}{2} + gR(1 + \cos \theta)$$

$$15 - gR(1 + \cos \theta) = \frac{V^2}{2}$$

$$30 - 2gR(1 + \cos \theta) = V^2$$

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Compare

$$Rg \cos \theta = 30 - 2gR(1 + \cos \theta)$$

$$Rg \cos \theta + 2gR(1 + \cos \theta) = 30$$

$$g = 10 \text{ m/s}^2$$

$$R = 1 \text{ m}$$

$$10 \cos \theta + 20(1 + \cos \theta) = 30$$

$$10 \cos \theta + 20 \cos \theta = 30 - 20$$

$$30 \cos \theta = 10$$

$$\cos \theta = \frac{1}{3} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{3}\right)$$

Circle leave करते समय

$$v^2 = Rg \cos \theta$$

$$R = 1 \text{ m}$$

$$g = 10 \text{ m/s}^2$$

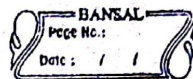
$$\cos \theta = \frac{1}{3} \leftarrow [\text{solve}]$$

तो

$$v^2 = 1(10)\left(\frac{1}{3}\right)$$

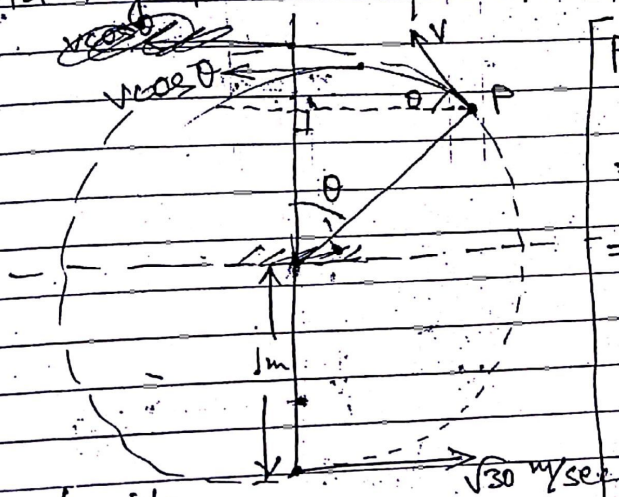
$$v = \sqrt{\frac{10}{3}} \text{ m/sec}$$

Lecture No. 20



Q.1.

The length of the string is 1m and the horizontal velocity of the bob at the lowest point is $\sqrt{30}$ m/s then calculate the min velocity of the body of the bob during its subsequent motion.



Point P 92

$$T = 0$$

$$\Rightarrow \cos \theta = \frac{1}{3}$$

$$\Rightarrow v = \sqrt{rg\left(\frac{1}{3}\right)}$$

$$= \sqrt{\frac{10}{3}} \frac{\text{m}}{\text{sec}}$$

last Q. H
Ans $\sqrt{\frac{10}{3}}$ m/sec

min. velocity

Highest point

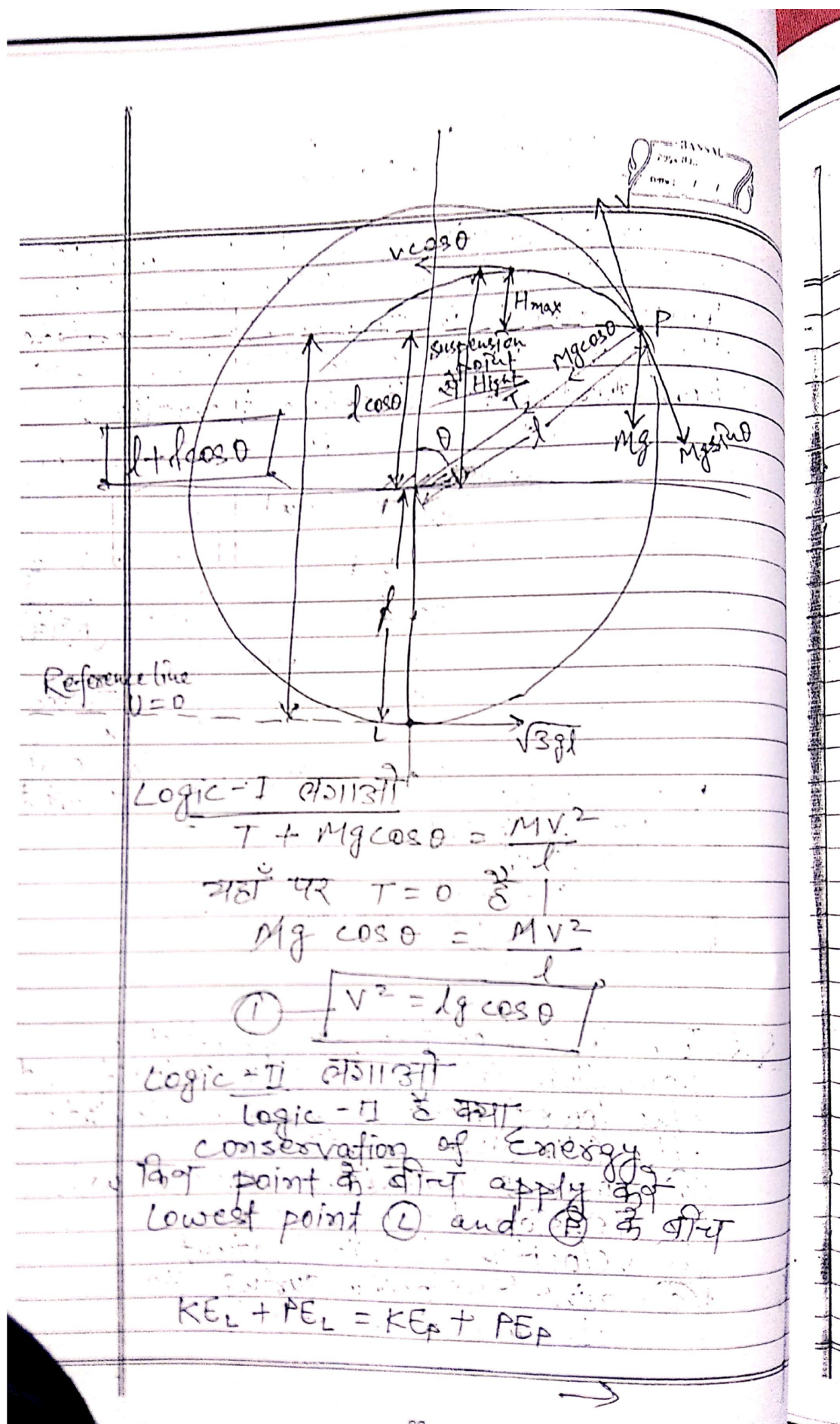
$$v \cos \theta \Rightarrow \frac{1}{3} \sqrt{\frac{10}{3}} \frac{\text{m}}{\text{sec}}$$

$$\sqrt{\frac{10}{3}} \left[\frac{1}{3} \right]$$

- Q.2. The length of the string of the pendulum is 'l' and the velocity at the lowest point is $\sqrt{3gl}$. Calculate the following parts -
- angle θ with the vertical where Tension become zero;
 - velocity at this instant
 - min velocity during its subsequent motion.

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$$\frac{1}{2} M [\sqrt{3gl}]^2 + 0 = \frac{1}{2} M V^2 + Mg[l + l \cos \theta]$$

$$\frac{1}{2} \cdot 3gl - g[l + l \cos \theta] = \frac{V^2}{2}$$

$$\underline{3gl - 2gl - 2gl \cos \theta = V^2}$$

Then, $3gl - 2gl - 2gl \cos \theta = lg \cos \theta$

$$gl = 3gl \cos \theta$$

$$\cos \theta = \frac{1}{3} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{3}\right)$$

Eqn ① $v^2 = gl \cos \theta$

$$v^2 = gl \left(\frac{1}{3}\right)$$

$$\underline{v = \sqrt{\frac{gl}{3}}}$$

Projectile motion करेगा।

We know

Highest point पर velocity minimum होती है।

$v \cos \theta$ ← Projectile chapter में concept दिया है।

$$\underline{\sqrt{\frac{gl}{3}} \cdot \left[\frac{1}{3}\right]} \text{ Ans}$$

$$H_{\max} = \frac{v^2 \sin^2 \theta}{2g}$$

$$H_{\max} = \frac{v^2}{2g} [1 - \cos^2 \theta]$$

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v and $\cos 30^\circ$ 2.59.

$$H_{\max} = \frac{1}{2g} \left[2l \left(\frac{1}{3} \right) \right] \left(1 - \frac{1}{3} \right)$$

$$H_{\max} = \frac{4l}{27}$$

Suspension point से कुल Height.

$$l \cos \theta + H_{\max}$$

↑ Ans

Ex# I → Q. 19

H.C. Verma → Q. 56, 57,

Q. 3. A block is placed on the top of the smooth sphere, now it is gently pushed towards right then calculate the following parts -

- (a) the angle made with the vertical where it leaves the sphere.
- (b) the velocity of the block when it leaves the sphere.

Logic - I लगीओ

circular motion याद करो

$$Mg \cos \theta - N = \frac{Mv^2}{R}$$

Block अपनी contact loose कर रहा है

$[N = 0]$ रखो

$$Mg \cos \theta = \frac{Mv^2}{R}$$

$$v^2 = Rg \cos \theta$$

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$\theta = ?$
 $V = ?$

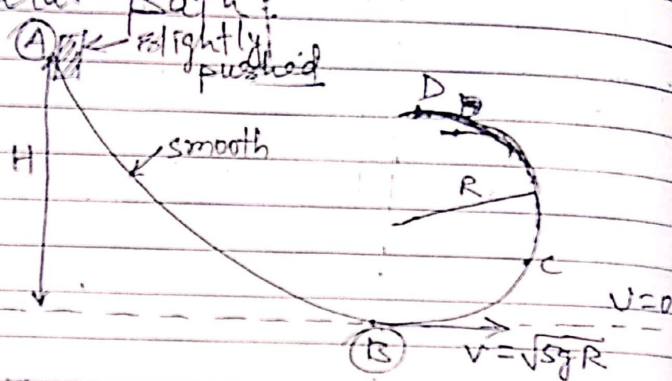
A Speed = 0
 B $V = ?$
 $V = 0$

Logic - II
 A and B के बीच
 conservation of Energy (होती है)
 $KE_A + PE_A = KE_B + PE_B$
 $0 + MgR = \frac{1}{2}Mv^2 + MgR \cos \theta$
 $gR - gR \cos \theta = \frac{v^2}{2}$
 $v^2 = 2gR - 2gR \cos \theta$
 $gR \cos \theta = 2gR - 2gR \cos \theta$
 $\cos \theta = \frac{2}{3}$ या $\theta = \cos^{-1}\left(\frac{2}{3}\right)$
 Speed equation $v^2 = Rg \cos \theta$
 $\Rightarrow v^2 = Rg \left(\frac{2}{3}\right)$
 $v = \sqrt{\frac{2}{3}Rg}$

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8.4. Calculate the min value of H so that if the block is slightly pushed from point A then it complete its vertical circular path.



$$KE_A + PE_A = KE_B + PE_B$$

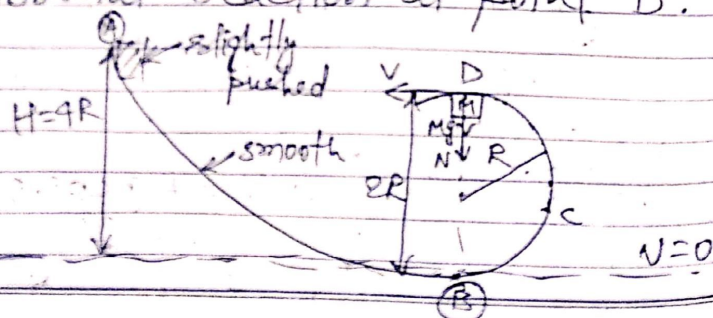
इस eqn को सही-सही Fill up करना ही imp. है।

$$0 + Mgh = \frac{1}{2} M [\sqrt{5gR}]^2 + 0$$

$$gH = \frac{1}{2} \cdot 5gR$$

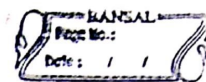
$$\boxed{H = \frac{5R}{2}}$$

→ If $H = 4R$, then calculate the normal reaction at point D.



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$$N + Mg = \frac{MV^2}{R}$$

$V \rightarrow$ पर डटक जाये ?

$$KE_A + PE_A = KE_B + PE_B$$

Equation को भरना सही-सही सीख लो

$$0 + Mg(4R) = \frac{1}{2}MV^2 + Mg(2R)$$

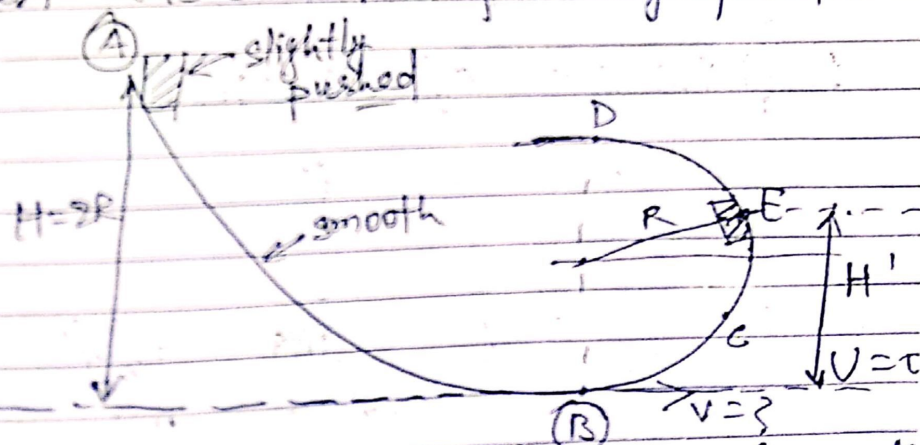
$$\boxed{V = 2\sqrt{gR}}$$

$$N + Mg = \frac{MV^2}{R}$$

$$N + Mg = \frac{M(4gR)}{R}$$

$$\boxed{N = 3Mg}$$

→ (c) If in the above question, if $R = 2R$ then answer the following parts—



- (i) will it complete its circular path
- (ii) where will it loose its contact.

$$KE_A + PE_A = KE_B + PE_B$$

अब तो सही सही करो

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Lecture No. 20+1

$$0 + mg(2R) = \frac{1}{2}mv^2 + 0$$

$$v^2 = 4gR$$

$$v = \sqrt{4gR}$$

यह तो $\sqrt{4gR}$ से बड़ा है
vertical circular नहीं

कहाँ E पर loose करेगा।

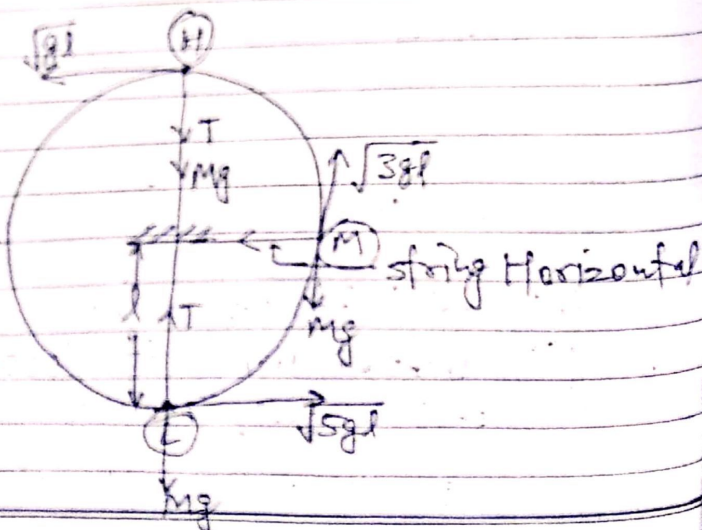
$$R < H' < 2R$$

Sheet

M.O.C. Q. 12, 13, 14 | H.C. Verma
S.C.C. Q. 037. | Q. no. 54.

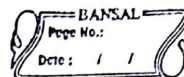
⇒ Some key points regarding vertical circular Motion

1. Discussion of tension :-



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(L)

$$T - Mg = \frac{MV^2}{l}$$

$$v = \sqrt{5gl}$$

$$T - Mg = \frac{M}{l}[5gl]$$

$$T - Mg = 5Mg$$

$$T = 6Mg$$

(M)

$$T = \frac{MV^2}{l}$$

$$v = \sqrt{3gl}$$

$$T = \frac{M}{l}[3gl]$$

$$T = 3Mg$$

(H) $T + Mg = \frac{MV^2}{l}$

$$v = \sqrt{gl}$$

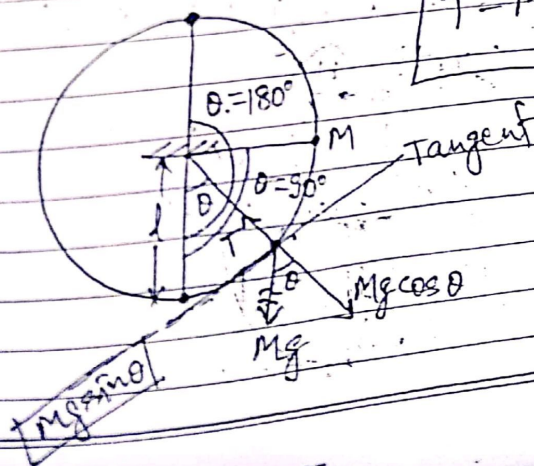
$$T + Mg = \frac{M}{l}[gl] \Rightarrow T = 0$$

Always True :-

min Tension $\rightarrow$ Highest point

max Tension $\rightarrow$ lowest point

2. Discussion about Tangential accn:



$$T - Mg \cos \theta = \frac{MV^2}{l}$$

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$$\text{Tangential force} = Mg \sin \theta$$

$$\text{Tangential acc'n} = \frac{\text{Force}}{\text{Mass}} = \frac{Mg \sin \theta}{M}$$

$$a_t = g \sin \theta$$

$$\rightarrow \text{At lowest, } \theta = 0^\circ \quad a_t = g \sin 0^\circ = 0$$

$$\rightarrow \text{At middle, } \theta = 90^\circ \quad a_t = g \sin 90^\circ = g$$

$$\rightarrow \text{At highest, } \theta = 180^\circ \quad a_t = g \sin 180^\circ = 0$$

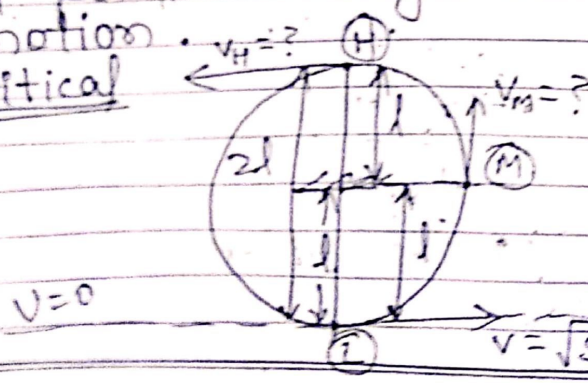
याद करो

Highest] $\Rightarrow$ Tangential acc'n
lowest] $\downarrow$
[minimum]

Middle] $\Rightarrow$ Tangential acc'n
 $\downarrow$
[maximum]

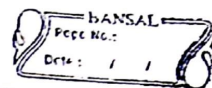
3. Calculation of velocity at different points during vertical circular motion.

Critical



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Land M

energy conserve

$$KE_L + PE_L + \cancel{PE_L} = KE_M + PE_M + \cancel{PE_M}$$

$$\frac{1}{2} M [\sqrt{5gl}]^2 + 0 = \frac{1}{2} M v_m^2 + Mgl$$

$$\frac{5gl}{2} - gl = \frac{v_m^2}{2} \Rightarrow \frac{3gl}{2} = \frac{v_m^2}{2}$$

$$\boxed{v_m = \sqrt{3gl}}$$

Land H

$$KE_L + PE_L = KE_H + PE_H$$

$$\frac{1}{2} M [\sqrt{5gl}]^2 + 0 = \frac{1}{2} M v_H^2 + Mg(2l)$$

$$\boxed{v_H = \sqrt{gl}}$$

Note :-

vertical circular path

को पूरा करने के लिए

Lowest point पर

minimum speed $\Rightarrow \sqrt{gl}$

and चक्कर पूरा करते समय

जब middle से pass $\Rightarrow \sqrt{3gl}$

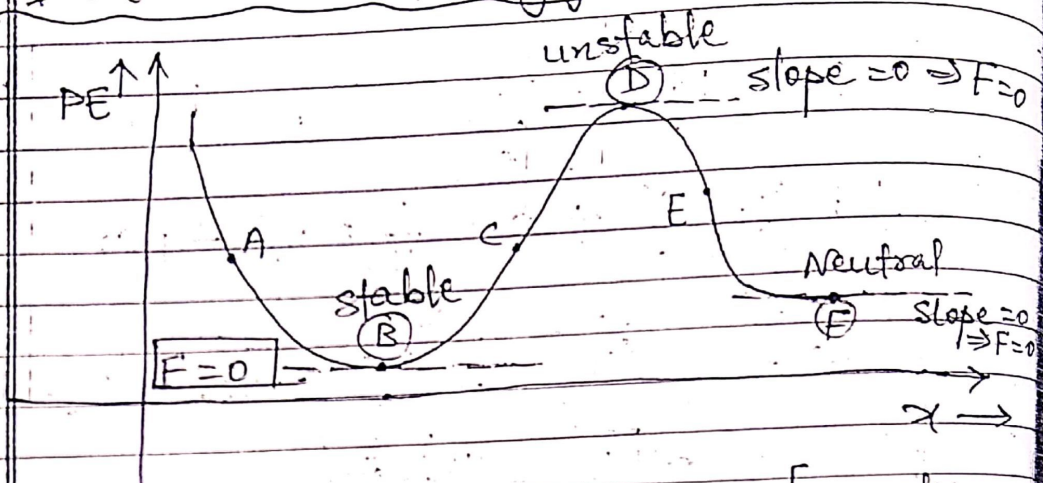
and at Highest point

speed $\Rightarrow \sqrt{gl}$

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⇒ Potential Energy curve :-



Slope of $v-t$ curve $\Rightarrow$ accelⁿ $\left[a = \frac{dv}{dt} \right]$

Slope of disp. time curve $\Rightarrow$ velocity $\left[v = \frac{dx}{dt} \right]$

Slope of $q-t$ curve $\Rightarrow$ current $\left[i = \frac{dq}{dt} \right]$

$\frac{dP}{dv}$ क्या है

↓
slope of $P-v$ curve

Slope of $P-t$ curve $\leftarrow$ क्या होगा $\leftarrow \frac{dP}{dt}$

Slope of curve $x-y$ की $\leftarrow \frac{dx}{dy}$

$$F = - \left(\frac{dU}{dx} \right)$$

$$F = - [\text{slope of } U-x \text{ curve}]$$

$$F = - [\text{slope of PE v/s distance curve}]$$

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$\left. \begin{matrix} B \\ D \\ F \end{matrix} \right\} \Rightarrow \text{slope} = \text{zero} \Rightarrow F = 0$

Equilibrium

$\Downarrow$
 $F = 0$

$B, D, F \Rightarrow$ 3 points $\Rightarrow$ Equilibrium

Stable	Unstable	Neutral
--------	----------	---------

If body is slightly displaced from mean position

$\downarrow$

If it has a tendency to move towards the mean.

$\downarrow$

If it tends to move away from the mean.

$\downarrow$

No effect

$\Rightarrow$ Golden Key points for Problem solving $\Rightarrow$

Stable	Unstable	Neutral
--------	----------	---------

① $F = 0$

② $\frac{dU}{dx} = 0$

③ $\frac{d^2U}{dx^2} > 0$

① $F = 0$

② $\frac{dU}{dx} = 0$

③ $\frac{d^2U}{dx^2} < 0$

① $F = 0$

② $\frac{dU}{dx} = 0$

③ $\frac{d^2U}{dx^2} = 0$

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Q.1. The potential Energy v/s distance function is given as $U = ax^2 + bx + c$. Find the equilibrium position and give the nature of equilibrium.
 $U = ax^2 + bx + c$ [a, b, c are positive]

Step ① $\frac{dU}{dx} = 0$

$$\frac{d[ax^2 + bx + c]}{dx} = 0$$

$$\frac{d}{dx} ax^2 + \frac{d}{dx} bx + \frac{dc}{dx} = 0$$

$$a[2x] + b[1] + 0 = 0$$

$$2ax + b = 0$$

$$x = \frac{-b}{2a}$$

Step ② $\frac{d^2U}{dx^2}$

या $\frac{d}{dx} [2ax + b]$

$$\frac{d}{dx} 2ax + \frac{d}{dx} b$$

$$2a \frac{d}{dx} x + \frac{d}{dx} b$$

$$2a(1) + 0$$

$$2a > 0$$

एसा point
जी equilibrium
पर है।

[Stable]

Q.2. If the PE function is given as $U = (x+1)(x+2)$, then find the position of equilibrium and tell about the nature of equilibrium.

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$$U = (x+1)(x+2)$$

↓

$$U = x^2 + 2x + x + 2$$

$$\Rightarrow U = x^2 + 3x + 2$$

Step ① $\frac{dU}{dx} = 0$

$$\Rightarrow \frac{d}{dx} [x^2 + 3x + 2] = 0$$

$$\frac{d}{dx} x^2 + \frac{d}{dx} 3x + \frac{d}{dx} 2 = 0$$

$$2x + 3(1) = 0$$

$$[2x + 3] = 0$$

इस point पर $\left[x = \frac{-3}{2} \right]$
[equilibrium]

Step ② इस Term को एक बार और differentiation करें

$$\frac{d}{dx} [2x + 3]$$

$$\frac{d}{dx} 2x + \frac{d}{dx} 3$$

$$2(1) + 0$$

$$2 > 0$$

↑

⇒ [stable]

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Q.8

If the PE function is given as $U = 20 + (x-2)^2$, then find the nature of equilibrium.

$$U = 20 + (x-2)^2 = 20 + x^2 - 4x + 4$$

$$\Rightarrow U = x^2 - 4x + 24$$

Step ① $\frac{dU}{dx} = 0$

$$\frac{d}{dx} [x^2 - 4x + 24] = 0$$

$$2x - 4 = 0$$

Equilibrium $\Rightarrow x = 2$

Step ② पता करने के लिए एक बार और

$$\frac{d}{dx} [2x - 4] \Rightarrow 2(1) = 2$$

$$\Rightarrow 2 > 0$$

↑ stable

$\Rightarrow$ Important points regarding equilibrium based questions:-

① Stable

$$\Rightarrow F = 0$$

$$\Rightarrow \frac{dU}{dx} = 0$$

$$\Rightarrow \frac{d^2U}{dx^2} > 0$$

$$\Rightarrow PE \rightarrow \min$$

② Unstable

$$\Rightarrow F = 0$$

$$\Rightarrow \frac{dU}{dx} = 0$$

$$\Rightarrow \frac{d^2U}{dx^2} < 0$$

$$\Rightarrow PE \rightarrow \max$$

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Lecture No. 22

Q.1. The potential energy function for a conservative force is given as $U = (x+2)(x+3)$ then find out the following parts -

- equilibrium position
- nature of equilibrium
- minimum value of PE

$$U = (x+2)(x+3)$$

$$\Rightarrow U = x^2 + 3x + 2x + 6$$

$$\Rightarrow U = x^2 + 5x + 6$$

Step ① $\frac{dU}{dx} = 0 \Rightarrow \frac{d}{dx}[x^2 + 5x + 6] = 0$

$$2x' + 5(1) + 0 = 0$$

इस point $\rightarrow x = -\frac{5}{2}$

है जहाँ

Equilibrium

Step-② एक बार और

$$\frac{d}{dx}(2x' + 5) \Rightarrow 2 > 0$$

↑
Stable

$$U = (x+2)(x+3)$$

$$x = -\frac{5}{2}$$

$$U = \left[-\frac{5}{2} + 2\right]\left[-\frac{5}{2} + 3\right]$$

$$U = \left[-\frac{1}{2}\right]\left[\frac{1}{2}\right]$$

$$U_{\min} = -\frac{1}{4} \text{ J}$$

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Q.2. The function of PE for a conservative force is given as $U = \frac{9}{x^2} - \frac{2}{x}$ then, find out the

point when speed can be max.

(a) $x = 6m$ (b) $x = 9m$ (c) $x = -3m$ (d) $x = +5m$

Speed $\rightarrow$ max

$$KE = \frac{1}{2}mv^2$$

KE max

PE min

उदात्त point
उदात्त stable

$$U = \frac{9}{x^2} - \frac{2}{x}$$

$$\frac{dU}{dx} = 0 \Rightarrow \frac{d}{dx} \left[9x^{-2} - 2x^{-1} \right] = 0$$

$$\Rightarrow 9[-2]x^{-2-1} - 2(-1)x^{-1-1} = 0$$

$$\Rightarrow -18x^{-3} + 2x^{-2} = 0$$

$$2x^{-2} = 18x^{-3}$$

$$\frac{2}{x^2} = \frac{18}{x^3} \Rightarrow 2 = \frac{18}{x}$$

$$x = 9m$$

Q.3. The function of PE is given as $U = \frac{x^4}{4}$ and if mass of the body is 1 kg and total mechanical energy

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is 2 Joule. Calculate max speed.

$$U = \frac{x^4}{4} - \frac{x^2}{2}$$

Step-①

$$\frac{dU}{dx} = 0 \Rightarrow \frac{d}{dx} \left[\frac{x^4}{4} - \frac{x^2}{2} \right] = 0$$

$$\Rightarrow \frac{1}{4}[4x^3] - \frac{1}{2}[2x] = 0$$

$$\Rightarrow x^3 - x = 0 \Rightarrow x(x^2 - 1) = 0$$

$$\Rightarrow \boxed{x=0} \quad \boxed{x=1} \quad \boxed{x=-1}$$

Step-②

$$\frac{d}{dx} [x^3 - x] \Rightarrow 3x^2 - 1$$

$$\boxed{x=0} \Rightarrow -1 < 0 \quad (\text{unstable})$$

$$\boxed{x=1} \Rightarrow 3(1)^2 - 1$$

$$2 > 0 \quad (\text{stable})$$

$$\boxed{x=-1} \Rightarrow 3(-1)^2 - 1$$

$$2 > 0 \quad (\text{stable})$$

$$x=0 \Rightarrow \text{Unstable}$$

$$x=1 \Rightarrow \text{stable}$$

$$x=-1 \Rightarrow \text{stable}$$

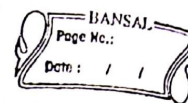
हमें पता है $\Rightarrow$ उत्तर equilibrium

stable है तो $PE \Rightarrow \min$

and min. value निकालने के लिए x की value को U के function में रखेंगे

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$$U = \frac{x^4}{4} - \frac{x^2}{2}$$

$$\left. \begin{array}{l} x=1 \\ x=-1 \end{array} \right\} U_{\min} = \frac{1^4}{4} - \frac{1^2}{2}$$

$$U_{\min} = -\frac{1}{4} \text{ J}$$

$$T.E. = KE_{\max} + PE_{\min}$$

$$2 = KE_{\max} - \frac{1}{4}$$

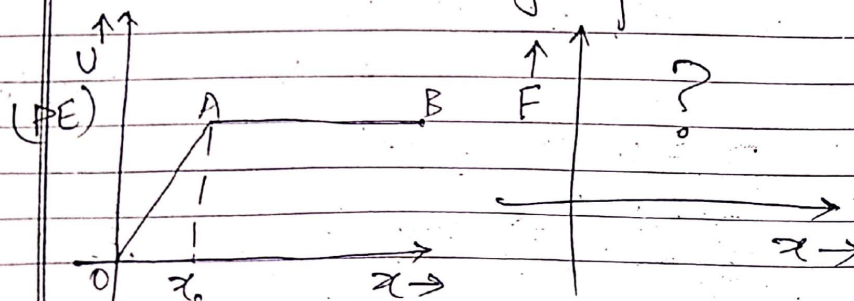
$$2 + \frac{1}{4} = KE_{\max}$$

$$\frac{9}{4} = \frac{1}{2} m (v_{\max})^2$$

$$\frac{9}{2} = 1 \cdot v_{\max}^2$$

$$v = \frac{3}{\sqrt{2}} \text{ m/sec.}$$

Q.4. Convert the following graph into Force-distance graph.



$$F = -\frac{dU}{dx}$$

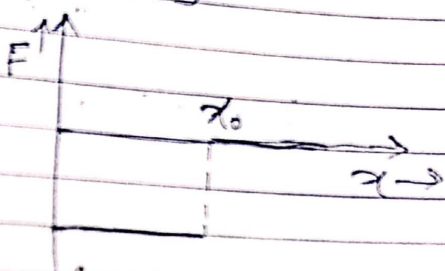
$$F = -(\text{slope of } U-x \text{ curve})$$

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0 से A
Slope = const
 $F = -[\text{slope of } U-x]$
 $F = -[\text{constant}]$
 $[F = -\text{const}]$

A से B
Slope = 0
 $F = -[\text{slope of } U-x]$
 $F = -[0]$
 $[F = 0]$



Q.5. The PE function for a conservative force is given as $U = 20 + (x + (-2))^2$ then calculate the following parts -

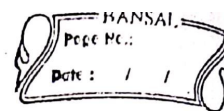
- if at $x = 5\text{m}$, the KE = 20 Joule
- equilibrium position
- nature of equilibrium
- Total mechanical energy
- Max KE.

$U = 20 + (x + (-2))^2$
 $= 20 + x^2 - 4x + 4$
 $\Rightarrow U = x^2 - 4x + 24$

Step - (i) $\frac{dU}{dx} = 0$
 $\Rightarrow \frac{d}{dx} [x^2 - 4x + 24] = 0$

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$$2x - 4 = 0$$

$$\text{Equilibrium} \leftarrow [x = 2]$$

Step-02 एक बार और differentiate करें

$$\frac{d}{dx} [2x - 4] \rightarrow 2(1) - 0$$

$$\Rightarrow 2 > 0$$

↑
[Stable]

↓
मतलब [PE = min]

Step-03 PE की min value निकालने के लिए x की value ques में रख दें

$$U = 20 + (x - 2)^2$$

$$\text{For } x = 2 \text{ m}$$

$$U_{\min} = 20 + (2 - 2)^2$$

$$U_{\min} = 20 \text{ Joule}$$

Given $x = 5 \text{ m}$; $KE = 20 \text{ Joule}$

$$U = 20 + (x - 2)^2$$

$$x = 5 \text{ m} \quad U = 20 + (5 - 2)^2 = 29 \text{ J}$$

$$\text{Total ME} = KE + PE$$

$$[\text{Total Energy} = 49 \text{ J}]$$

$$TE = PE_{\min} + KE_{\max}$$

$$49 = 20 + KE_{\max}$$

$$KE_{\max} = 29 \text{ Joule}$$

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Q.6. The PE of a particle is given as
 $U = 20 + (x-2)^2$ and total ME of the particle is 36 Joule

(a) Find equilibrium position
 (b) Find the max KE of the particle

$U = 20 + (x-2)^2$
 $= 20 + x^2 - 4x + 4$
 $\Rightarrow U = x^2 - 4x + 24$

Step-① $\frac{dU}{dx} = 0$
 $\Rightarrow \frac{d}{dx} [x^2 - 4x + 24] = 0$
 $\Rightarrow \frac{d}{dx} x^2 - \frac{d}{dx} 4x + \frac{d}{dx} 24 = 0$
 $\Rightarrow 2x' - 4(1) + 0 = 0$
 $\Rightarrow 2x - 4 = 0$
 $\boxed{x = 2m}$

Step-② $\frac{d}{dx} (2x - 4)$
 $2 > 0$
 $\uparrow$ stable

Step-③
 $x = 2m$ में & में रखें
 $U = 20 + (x-2)^2$
 $U_{\min} = 20 \text{ Joule}$
 $TE = PE_{\min} + KE_{\max} \rightarrow$

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$$36 = 20 + KE_{\max}$$

$$KE_{\max} = 16 \text{ Joule}$$

Q. 7. The force acting on the particle is given as $F = 2x^2 - 3x - 2$, then find equilibrium.

$$F = 2x^2 - 3x - 2$$

$$2(2x) - 3(1) = 0 \quad \text{differentiate}$$

$$4x - 3 = 0$$

$$x = \frac{3}{4}$$

नहीं होगी क्योंकि PE की function नहीं है

function F की है।

$$F = -\frac{dU}{dx} = 0$$

$$F = 0$$

$$2x^2 - 3x - 2 = 0$$

$$2x^2 - 4x + x - 2 = 0$$

$$2x(x-2) + 1(x-2) = 0$$

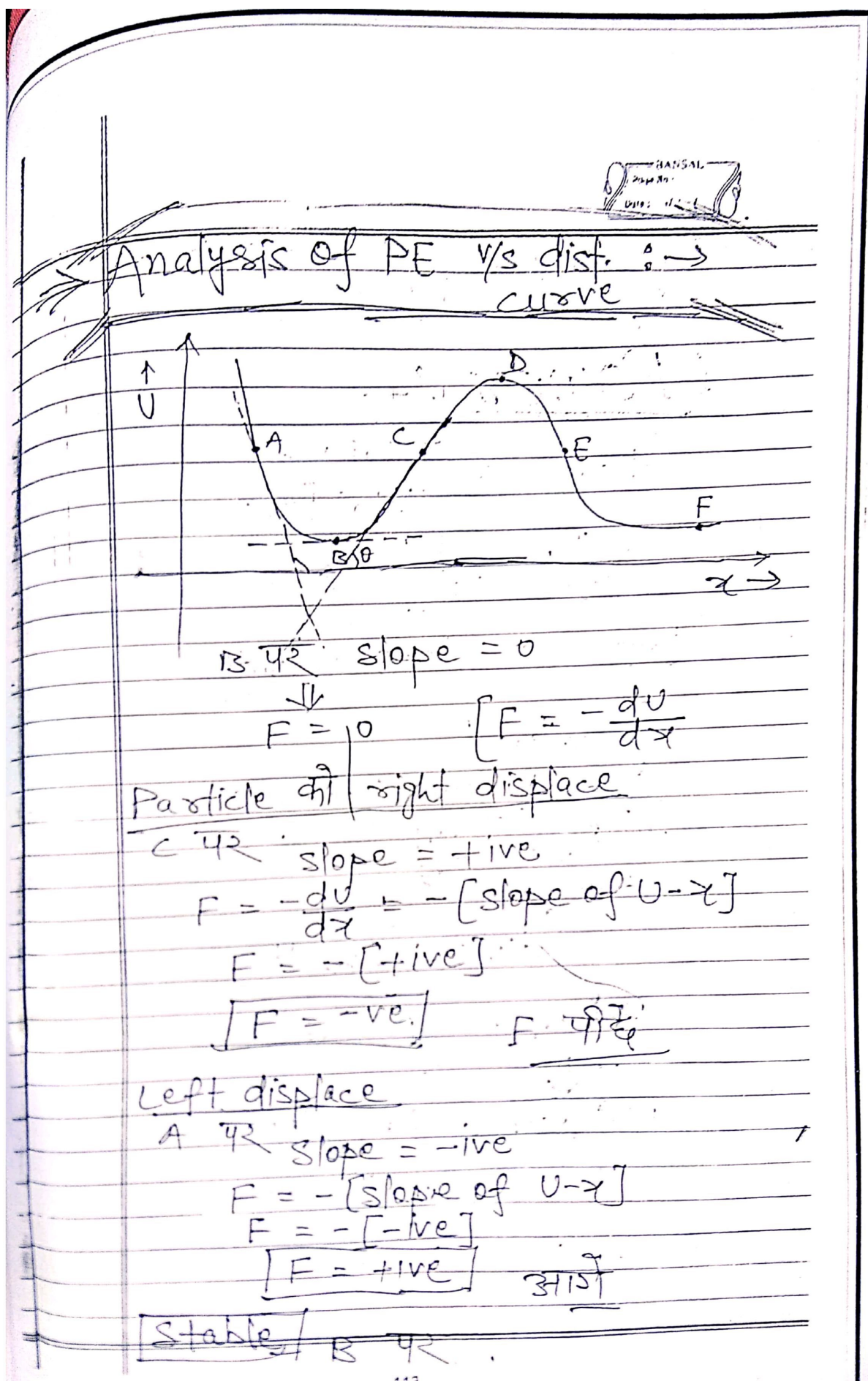
$$(x-2)(2x+1) = 0$$

$$x = -\frac{1}{2}, x = 2$$

Equilibrium

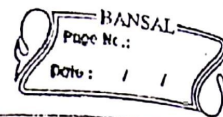
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→ The rate of doing work is defined as power.

Power :-

$$\text{Power} = \frac{\text{Total Work}}{\text{Total time}}$$

$$P = \frac{W}{t}$$

Unit $P = \frac{\text{Joule}}{\text{sec}} = \text{Watt}$

Scalar

Instantaneous Power :-

$$P_{\text{inst.}} = \frac{dW}{dt}$$

$$\int dW = \int P_{\text{inst.}} dt$$

$$W = \int P_{\text{inst.}} dt$$

$$\Delta KE = \int P_{\text{inst.}} dt$$

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Q.1. The inst. Power varies with time is given as $P = 3t^2 - 2t + 1$, then find the ΔKE b/w $t = 2$ sec to $t = 4$ sec.

$\Delta KE = \int_{t=2}^{t=4} P_{ins} dt$

P की value - रखो

$\Delta KE = \int_2^4 (3t^2 - 2t + 1) dt$

$\Delta KE = \int_2^4 3t^2 dt - \int_2^4 2t dt + \int_2^4 1 dt$

$\Delta KE = 3 \left[\frac{t^3}{3} \right]_2^4 - 2 \left[\frac{t^2}{2} \right]_2^4 + 1 \left[t \right]_2^4$

$= 3 \left[\frac{(4)^3}{3} - \frac{(2)^3}{3} \right] - 2 \left[\frac{(4)^2}{2} - \frac{(2)^2}{2} \right] + [4 - 2]$

$= 3 \left[\frac{64}{3} - \frac{8}{3} \right] - 2 \left[\frac{16}{2} - \frac{4}{2} \right] + 2$

$= 3 \left[\frac{56}{3} \right] - 2 \left[\frac{12}{2} \right] + 2$

$= 56 - 12 + 2 = 46 \text{ Joule}$

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Lecture No. 23

Q.1.

Chain Problem :-

A uniform chain of mass 'm' and length 'L' overhang a table with its 2/3rd part on the table. find the work to be done by a person to put the hanging part back on the table.

$U=0$

$$[KE_i = 0]$$

$$[KE_f = 0]$$

L length of chain ki dikhti mass = M

Unit length ki dikhti mass = $\frac{M}{L}$

dx or dx ki mass = $\left(\frac{M}{L} \cdot dx\right)$

→ dx dikhti ki mass

$$W_{\text{ext}} = \Delta KE + \Delta PE$$

$$W_{\text{ext}} = \Delta PE = PE_f - PE_i$$

$$\text{3rd dikhti ki PE} = - \left[\frac{M}{L} \cdot dx \right] [g] [x]$$

$$U_i = \int_{L/3}^0 - \frac{M}{L} dx \cdot g \cdot x$$

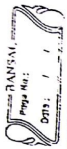
$$U_i = - \frac{M}{L} g \left[\frac{x^2}{2} \right]_{L/3}^0$$

$$U_i = - \frac{M}{L} g \left[0 - \frac{L^2}{18} \right]$$

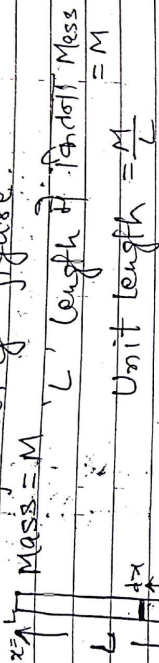
$$U_i = - \frac{Mg}{18} L$$

$$[W = \frac{MgL}{18}]$$

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Q.2 Calculate the gravitational PE of the rod in the following figure.



'dx' element ki mass

$$= \frac{M}{L} \cdot dx$$

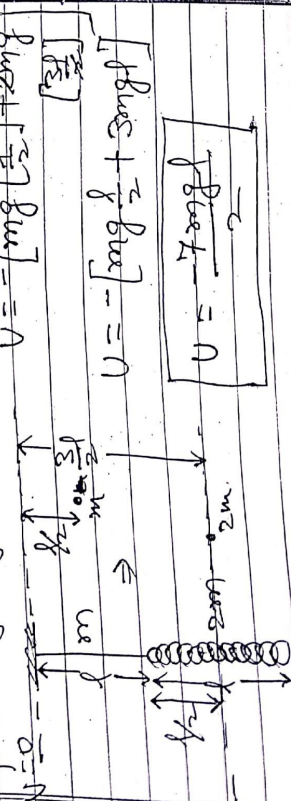
3rd part ki PE
 $= \int_0^L \left[\frac{M}{L} dx \right] g (x)$

$$U = \int_0^L \frac{M}{L} \cdot dx \cdot g \cdot x$$

$$U = \frac{M}{L} g \left[\frac{x^2}{2} \right]_0^L = \frac{M}{L} \cdot g \left[\frac{L^2}{2} - 0 \right]$$

$$U = Mg \left[\frac{L}{2} \right]$$

Q.3 Find out the gravitational PE of the following system.



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$U = 0$

Chain reverse
→

$$U = - \left[2m \cdot g \cdot \frac{L}{2} \right] + \left[Mg \frac{3L}{2} \right]$$

$$U = - \frac{5mgl}{2}$$

$$\Delta U = - \frac{5mgl}{2} - \left[- \frac{7mgl}{2} \right]$$

$$\Delta U = - \frac{5mgl}{2} + \frac{7mgl}{2}$$

$$\boxed{\Delta U = + mgl}$$

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Spring - Block System

Natural length

Equilibrium

$$Mg = kx'$$

$$x' = \frac{Mg}{k}$$

$v=0$

maxim extension

Block rest

Net Force = 0

$a=0$

$V = \max$

बीच वाली

$v = \max$

$\downarrow$

$acc = 0$

$\downarrow$

Force = 0

$\downarrow$

$Mg = kx'$

JEE-2002 last

maximum elongation

$$0 + 0 + 0 = KE_B + PE_{Block} + PE_{Spring}$$

$$0 = 0 + (-mgx) + \frac{1}{2}kx^2$$

$$mgx = \frac{1}{2}kx^2$$

JEE 2002

$x = \frac{2mg}{k}$

$\downarrow$

maximum elongation

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Speed $\rightarrow \max / v = \max$

$\Downarrow$

$acc = 0$

$\Downarrow$

Net Force $= 0$

Q. 22

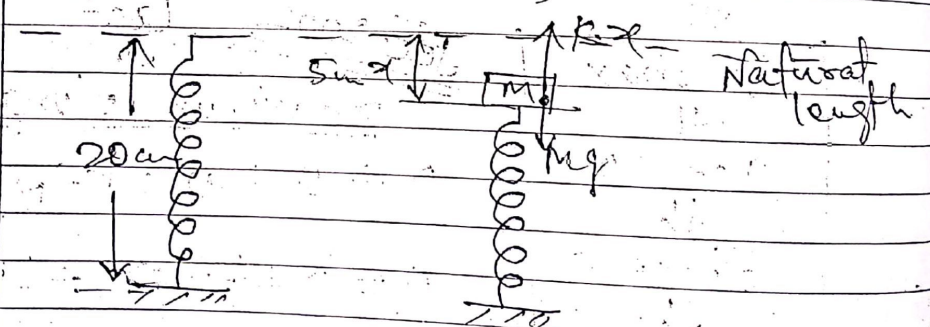
A light spring of length 20 cm and force const. 200 N/m is placed vertically on a table. A small block of mass 1 kg falls on it. The length H from the surface of the table at which the ball will have the ~~maximum~~ maximum velocity is

(a) 20 cm

(c) 10 cm

(b) 15 cm

(d) 5 cm



$$mg = Kx$$

नीचे से

$$1[10] = [200]x$$

$$x = \frac{10}{200} = \frac{1}{20} \text{ m}$$

$$x = 5 \text{ cm}$$

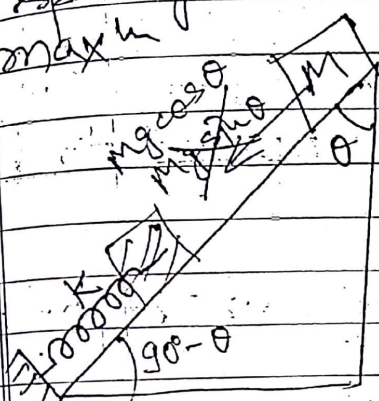
$$20 - 5$$

$$= [15 \text{ cm}]$$

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Q.5 Calculate the compression in the spring when speed of the block is max



a) $\frac{mg \sin \theta}{K}$

b) $\frac{mg \cos \theta}{K}$

c) $\frac{mg}{K}$

d) None of these

Speed $\rightarrow$ max



Net Force = 0



मतलब

Forces को Balance कर दो

$$Mg \cos \theta = Kx$$

$$x = \frac{Mg \cos \theta}{K}$$

Forces को Balance out कि दो ?