SOLVED PAPER 2017 **[OFFLINE]**

JEE MAIN

Joint Entrance Examination

INSTRUCTIONS

1. This test consists of 90 questions.
2. Each question is allotted 4 marks for correct response.
3. Candidates will be awarded marks as stated above in instruction no. 2 for correct response of each question. 1 mark will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
4. There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted according as per instructions.

Physics

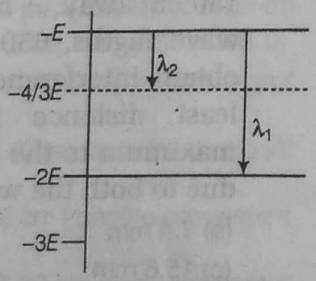
1. An observer is moving with half the speed of light towards a stationary microwave source emitting waves at frequency 10 GHz. What is the frequency of the microwave measured by the observer? (speed of light = 3×10^8 ms^{-1})

(a) 12.1 GHz (b) 17.3 GHz
(c) 15.3 GHz (d) 10.1 GHz

2. The following observations were taken for determining surface tension T of water by capillary method. Diameter of capillary, $d = 1.25 \times 10^{-2}$ m rise of water, $h = 1.45 \times 10^{-2}$m. Using $g = 9.80$ m/s^2 and the simplified relation $T = \frac{rhg}{2} \times 10^3$N/m, the possible error in surface tension is closest to

(a) 1.5% (b) 2.4%
(c) 10% (d) 0.15%

3. Some energy levels of a molecule are shown in the figure. The ratio of the wavelengths $r = \lambda_1 / \lambda_2$ is given by



(a) $r = \frac{2}{3}$ (b) $r = \frac{3}{4}$
(c) $r = \frac{1}{3}$ (d) $r = \frac{4}{3}$

4. A body of mass $m = 10^{-2}$kg is moving in a medium and experiences a frictional force $F = -kv^2$. Its initial speed is $v_0 = 10$ ms^{-1}. If, after 10 s, its energy is $\frac{1}{8}mv_0^2$, the value of k will be

(a) 10^{-3}kgs^{-1} (b) 10^{-4}kgm^{-1}
(c) 10^{-1}kgm^{-1}s^{-1} (d) 10^{-3}kgm^{-1}

5. C_p and C_V are specific heats at constant pressure and constant volume, respectively. It is observed that $C_p - C_V = a$ for hydrogen gas $C_p - C_V = b$ for nitrogen gas. The correct relation between a and b is

(a) $a = b$ (b) $a = 14b$
(c) $a = 28b$ (d) $a = \frac{1}{14}b$

6. The moment of inertia of a uniform cylinder of length l and radius R about its perpendicular bisector is I. What is the ratio l/R such that the moment of inertia is minimum?

(a) $\frac{\sqrt{3}}{2}$ (b) 1
(c) $\frac{3}{\sqrt{2}}$ (d) $\sqrt{\frac{3}{2}}$

7. A radioactive nucleus A with a half-life T, decays into a nucleus B. At $t = 0$, there is no nucleus B. After sometime t, the ratio of the number of B to that of A is 0.3. Then, t is given by

(a) $t = T\dfrac{\log 1.3}{\log_e 2}$ (b) $t = T \log 1.3$

(c) $t = \dfrac{T}{\log 1.3}$ (d) $t = \dfrac{T \log_e 2}{2 \log 1.3}$

8. Which of the following statements is false?

(a) In a balanced Wheatstone bridge, if the cell and the galvanometer are exchanged, the null point is disturbed

(b) A rheostat can be used as a potential divider

(c) Kirchhoff's second law represents energy conservation

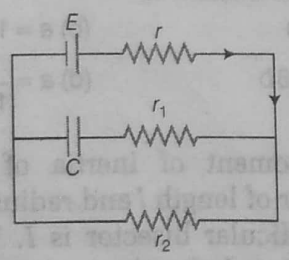
(d) Wheatstone bridge is the most sensitive when all the four resistances are of the same order of magnitude

9. A capacitance of 2 μF is required in an electrical circuit across a potential difference of 1kV. A large number of 1 μF capacitors are available which can withstand a potential difference of not more than 300 V. The minimum number of capacitors required to achieve this is

(a) 16 (b) 24

(c) 32 (d) 2

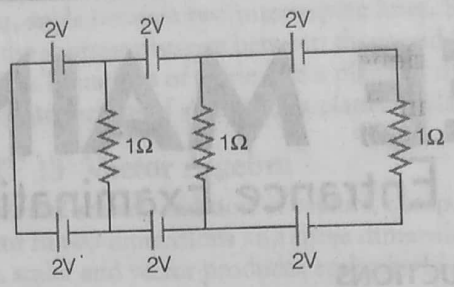
10. In the given circuit diagram, when the current reaches steady state in the circuit, the charge on the capacitor of capacitance C will be



(a) $CE\dfrac{r_1}{(r_2 + r)}$ (b) $CE\dfrac{r_2}{(r + r_2)}$

(c) $CE\dfrac{r_1}{(r_1 + r)}$ (d) CE

11. In the below circuit, the current in each resistance is



(a) 0.25 A (b) 0.5 A

(c) 0 A (d) 1 A

12. In amplitude modulation, sinusoidal carrier frequency used is denoted by ω_c and the signal frequency is denoted by ω_m. The bandwidth ($\Delta\omega_m$) of the signal is such that $\Delta\omega_m << \omega_c$. Which of the following frequencies is not contained in the modulated wave?

(a) ω_c (b) $\omega_m + \omega_c$

(c) $\omega_c - \omega_m$ (d) ω_m

13. In a common emitter amplifier circuit using an n-p-n transistor, the phase difference between the input and the output voltages will be

(a) 90° (b) 135°

(c) 180° (d) 45°

14. A copper ball of mass 100 g is at a temperature T. It is dropped in a copper calorimeter of mass 100 g, filled with 170 g of water at room temperature. Subsequently, the temperature of the system is found to be 75°C. T is (Given, room temperature = 30°C, specific heat of copper = 0.1 cal/g°C)

(a) 885°C (b) 1250°C

(c) 825°C (d) 800°C

15. In a Young's double slit experiment, slits are separated by 0.5 mm and the screen is placed 150 cm away. A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes on the screen. The least distance from the common central maximum to the point where the bright fringes due to both the wavelengths coincide, is

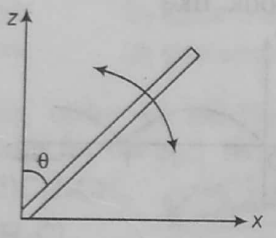
(a) 7.8 mm (b) 9.75 mm

(c) 15.6 mm (d) 1.56 mm

16. An electric dipole has a fixed dipole moment **p**, which makes angle θ with respect to *X*-axis. When subjected to an electric field $\mathbf{E}_1 = E\hat{\mathbf{i}}$, it experiences a torque $\mathbf{T}_1 = \tau\hat{\mathbf{k}}$. When subjected to another electric field $\mathbf{E}_2 = \sqrt{3}E_1\hat{\mathbf{j}}$, it experiences a torque $\mathbf{T}_2 = -\mathbf{T}_1$. The angle θ is

(a) 45° (b) 60° (c) 90° (d) 30°

17. A slender uniform rod of mass *M* and length *l* is pivoted at one end so that it can rotate in a vertical plane (see the figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical, is



(a) $\frac{2g}{3l}\sin\theta$ (b) $\frac{3g}{2l}\cos\theta$

(c) $\frac{2g}{3l}\cos\theta$ (d) $\frac{3g}{2l}\sin\theta$

18. An external pressure *P* is applied on a cube at 0°C so that it is equally compressed from all sides. *K* is the bulk modulus of the material of the cube and α is its coefficient of linear expansion. Suppose we want to bring the cube to its original size by heating. The temperature should be raised by

(a) $\frac{P}{\alpha K}$ (b) $\frac{3\alpha}{PK}$ (c) $3PK\alpha$ (d) $\frac{P}{3\alpha K}$

19. A diverging lens with magnitude of focal length 25 cm is placed at a distance of 15 cm from a converging lens of magnitude of focal length 20 cm. A beam of parallel light falls on the diverging lens. The final image formed is

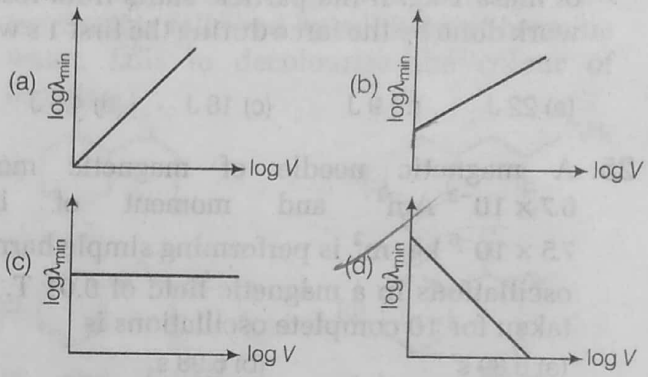
(a) virtual and at a distance of 40 cm from convergent lens

(b) real and at a distance of 40 cm from the divergent lens

(c) real and at a distance of 6 cm from the convergent lens

(d) real and at a distance of 40 cm from convergent lens

20. An electron beam is accelerated by a potential difference *V* to hit a metallic target to produce *X*-rays. It produces continuous as well as characteristic *X*-rays. If λ_{min} is the smallest possible wavelength of *X*-rays in the spectrum, the variation of log λ_{min} with log *V* is correctly represented in

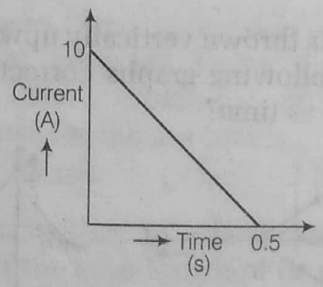


21. The temperature of an open room of volume 30 m^3 increases from 17°C to 27°C due to the sunshine. The atmospheric pressure in the room remains 1×10^5 Pa. If n_i and n_f are the number of molecules in the room before and after heating, then $n_f - n_i$ will be

(a) 1.38×10^{23} (b) 2.5×10^{25}

(c) -2.5×10^{25} (d) -1.61×10^{23}

22. In a coil of resistance 100 Ω, a current is induced by changing the magnetic flux through it as shown in the figure. The magnitude of change in flux through the coil is



(a) 225 Wb (b) 250 Wb

(c) 275 Wb (d) 200 Wb

23. When a current of 5 mA is passed through a galvanometer having a coil of resistance 15 Ω, it shows full scale deflection. The value of the resistance to be put in series with the

galvanometer to convert it into a voltmeter of range 0-10 V is

(a) $2.045 \times 10^3\ \Omega$ (b) $2.535 \times 10^3\ \Omega$

(c) $4.005 \times 10^3\ \Omega$ (d) $1.985 \times 10^3\ \Omega$

24. A time dependent force $F = 6t$ acts on a particle of mass 1 kg. If the particle starts from rest, the work done by the force during the first 1 s will be

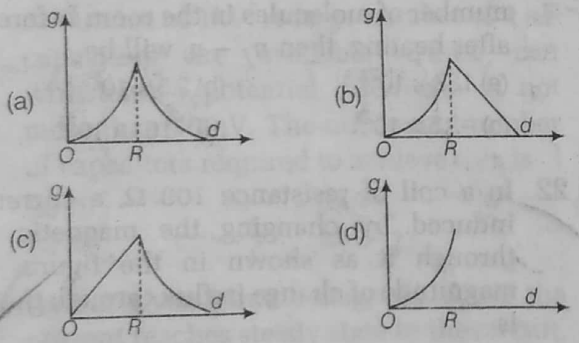
(a) 22 J (b) 9 J (c) 18 J (d) 4.5 J

25. A magnetic needle of magnetic moment $6.7 \times 10^{-2}\ Am^2$ and moment of inertia $7.5 \times 10^{-6}\ kg\ m^2$ is performing simple harmonic oscillations in a magnetic field of 0.01 T. Time taken for 10 complete oscillations is

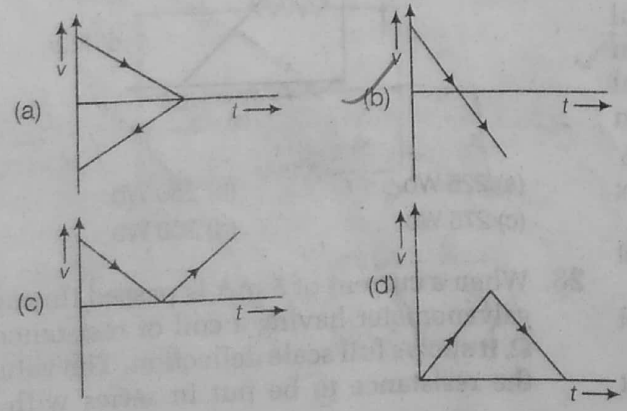
(a) 8.89 s (b) 6.98 s

(c) 8.76 s (d) 6.65 s

26. The variation of acceleration due to gravity g with distance d from centre of the Earth is best represented by (R = Earth's radius)



27. A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity *vs* time?

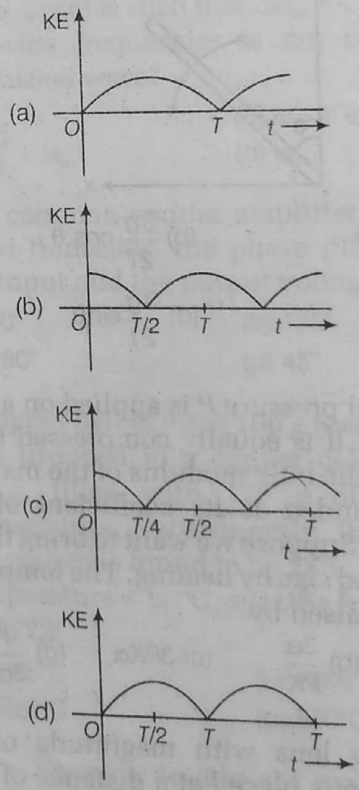


28. A particle A of mass m and initial velocity v collides with a particle B of mass $\frac{m}{2}$ which is at rest. The collision is head on, and elastic. The ratio of the de-Broglie wavelengths λ_A to λ_B after the collision is

(a) $\frac{\lambda_A}{\lambda_B} = 2$ (b) $\frac{\lambda_A}{\lambda_B} = \frac{2}{3}$

(c) $\frac{\lambda_A}{\lambda_B} = \frac{1}{2}$ (d) $\frac{\lambda_A}{\lambda_B} = \frac{1}{3}$

29. A particle is executing simple harmonic motion with a time period T. At time $t = 0$, it is at its position of equilibrium. The kinetic energy-time graph of the particle will look, like



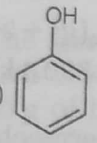
30. A man grows into a giant such that his linear dimensions increase by a factor of 9. Assuming that his density remains same, the stress in the leg will change by a factor of

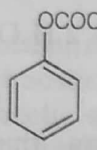
(a) $\frac{1}{9}$ (b) 81

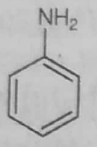
(c) $\frac{1}{81}$ (d) 9

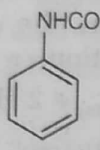
Chemistry

31. Which of the following compounds will give significant amount of *meta*-product during mononitration reaction?

(a) 

(b) 

(c) 

(d) 

32. ΔU is equal to
(a) isochoric work (b) isobaric work
(c) adiabatic work (d) isothermal work

33. The increasing order of reactivity of the following halides for the S_N1 reaction is
I. $CH_3CH(Cl)CH_2CH_3$
II. $CH_3CH_2CH_2Cl$
III. p-H_3CO— C_6H_4— CH_2Cl
(a) (III) < (II) < (I) (b) (II) < (I) < (III)
(c) (I) < (III) < (II) (d) (II) < (III) < (I)

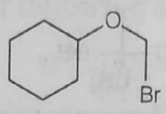
34. The radius of the second Bohr orbit for hydrogen atom is (Planck's constant (h) = 6.6262×10^{-34} Js; mass of electron = 9.1091×10^{-31} kg ; charge of electron (e) = 1.60210×10^{-19} C; permitivity of vacuum (ϵ_0) = $8.854185 \times 10^{-12} kg^{-1} m^{-3} A^2$)
(a) 1.65 Å (b) 4.76 Å (c) 0.529 Å (d) 2.12 Å

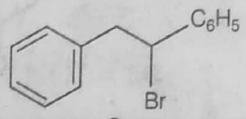
35. pK_a of a weak acid (HA) and pK_b of a weak base (BOH) are 3.2 and 3.4, respectively. The pH of their salt (AB) solution is
(a) 7.2 (b) 6.9 (c) 7.0 (d) 1.0

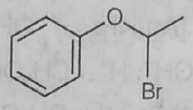
36. The formation of which of the following polymers involves hydrolysis reaction?
(a) Nylon-6 (b) Bakelite
(c) Nylon-6, 6 (d) Terylene

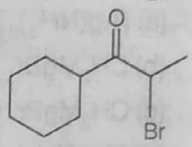
37. The most abundant elements by mass in the body of a healthy human adult are Oxygen (61.4%); Carbon (22.9%), Hydrogen (10.0 %); and Nitrogen (2.6%). The weight which a 75 kg person would gain if all ^1H atoms are replaced by ^2H atoms is
(a) 15 kg (b) 37.5 kg (c) 7.5 kg (d) 10 kg

38. Which of the following, upon treatment with *tert*-BuONa followed by addition of bromine water, fails to decolourise the colour of bromine ?

(a) 

(b) 

(c) 

(d) 

39. In the following reactions, ZnO is respectively acting as a/an
(i) $ZnO + Na_2O \longrightarrow Na_2ZnO_2$
(ii) $ZnO + CO_2 \longrightarrow ZnCO_3$
(a) base and acid (b) base and base
(c) acid and acid (d) acid and base

40. Both lithium and magnesium display several similar properties due to the diagonal relationship; however, the one which is incorrect is
(a) Both form basic carbonates
(b) Both form soluble bicarbonates
(c) Both form nitrides
(d) nitrates of both Li and Mg yield NO_2 and O_2 on heating

41. 3-methyl-pent-2-ene on reaction with HBr in presence of peroxide forms an addition product. The number of possible stereoisomers for the product is
(a) six (b) zero (c) two (d) four

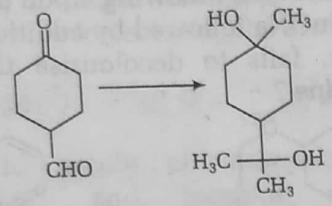
42. A metal crystallises in a face centred cubic structure. If the edge length of its unit cell is 'a', the closest approach between two atoms in metallic crystal will be
(a) $2a$ (b) $2\sqrt{2}\,a$ (c) $\sqrt{2}\,a$ (d) $\frac{a}{\sqrt{2}}$

43. Two reactions R_1 and R_2 have identical pre-exponential factors. Activation energy of R_1 exceeds that of R_2 by 10 kJ mol^{-1}. If k_1 and k_2 are rate constants for reactions R_1 and R_2,

respectively at 300 K, then $\ln\left(\frac{k_2}{k_1}\right)$ is equal to ($R = 8.314\ J\ mol^{-1}K^{-1}$)

(a) 8 (b) 12 (c) 6 (d) 4

44. The correct sequence of reagents for the following conversion will be



(a) $[Ag(NH_3)_2]^+\ OH^-, H^+ / CH_3OH, CH_3MgBr$

(b) $CH_3MgBr, H^+ / CH_3OH, [Ag(NH_3)_2]^+\ OH^-$

(c) $CH_3MgBr, [Ag(NH_3)_2]^+\ OH^-, H^+ / CH_3OH$

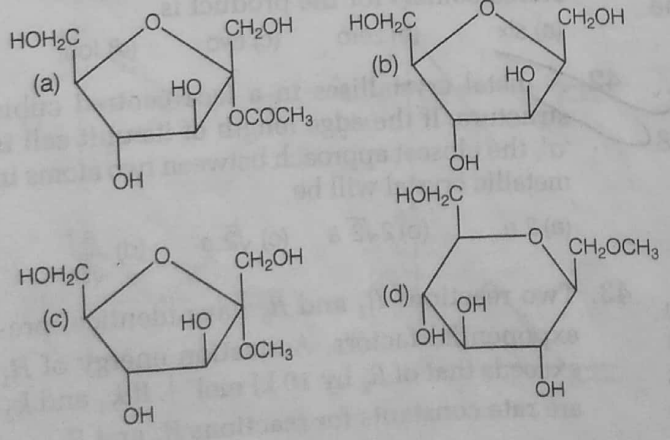
(d) $[Ag(NH_3)_2]^+\ OH^-, CH_3MgBr, H^+ / CH_3OH$

45. The Tyndall effect is observed only when following conditions are satisfied

1. The diameter of the dispersed particles is much smaller than the wavelength of the light used.
2. The diameter of the dispersed particle is not much smaller than the wavelength of the light used.
3. The refractive indices of the dispersed phase and dispersion medium are almost similar in magnitude.
4. The refractive indices of the dispersed phase and dispersion medium differ greatly in magnitude.

(a) 1 and 4 (b) 2 and 4 (c) 1 and 3 (d) 2 and 3

46. Which of the following compounds will behave as a reducing sugar in an aqueous KOH solution?



47. Given, $C_{(graphite)} + O_2(g) \longrightarrow CO_2(g);\ \Delta_r H^\circ = -393.5\ kJ\ mol^{-1}$

$H_2(g) + \frac{1}{2}O_2(g) \longrightarrow H_2O(l);\ \Delta_r H^\circ = -285.8\ kJ\ mol^{-1}$

$CO_2(g) + 2\ H_2O(l) \longrightarrow CH_4(g) + 2O_2(g);\ \Delta_r H^\circ = +890.3\ kJ\ mol^{-1}$

Based on the above thermochemical equations, the value of $\Delta_r H^\circ$ at 298 K for the reaction,

$C_{(graphite)} + 2\ H_2(g) \longrightarrow CH_4(g)$ will be

(a) $+78.8\ kJ\ mol^{-1}$ (b) $+144.0\ kJ\ mol^{-1}$

(c) $-74.8\ kJ\ mol^{-1}$ (d) $-144.0\ kJ\ mol^{-1}$

48. Which of the following reactions is an example of a redox reaction?

(a) $XeF_4 + O_2F_2 \longrightarrow XeF_6 + O_2$

(b) $XeF_2 + PF_5 \longrightarrow [XeF]^+PF_6^-$

(c) $XeF_6 + H_2O \longrightarrow XeOF_4 + 2HF$

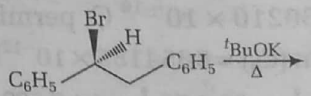
(d) $XeF_6 + 2H_2O \longrightarrow XeO_2F_2 + 4HF$

49. The products obtained when chlorine gas reacts with cold and dilute aqueous NaOH are

(a) ClO^- and ClO_3^- (b) ClO_2^- and ClO_3^-

(c) Cl^- and ClO^- (d) Cl^- and ClO_2^-

50. The major product obtained in the following reaction is



(a) (±) $C_6H_5CH(O^tBu)CH_2C_6H_5$

(b) $C_6H_5CH = CHC_6H_5$

(c) (+) $C_6H_5CH(O^tBu)CH_2C_6H_5$

(d) (−) $C_6H_5CH(O^tBu)CH_2C_6H_5$

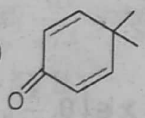
51. Sodium salt of an organic acid 'X' produces effervescence with conc. H_2SO_4. 'X' reacts with the acidified aqueous $CaCl_2$ solution to give a white precipitate which decolourises acidic solution of $KMnO_4$. 'X' is

(a) C_6H_5COONa (b) HCOONa

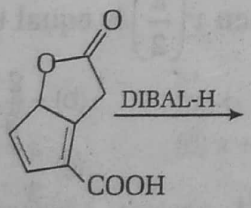
(c) CH_3COONa (d) $Na_2C_2O_4$

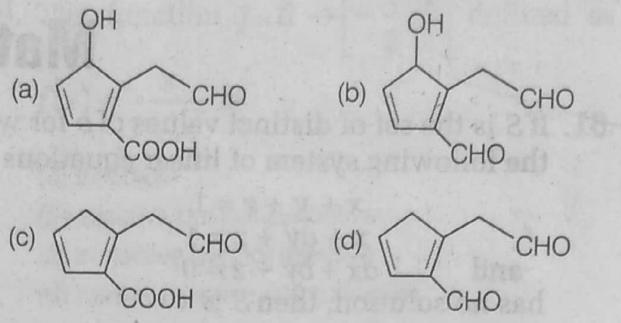
52. Which of the following species is not paramagnetic?
(a) NO
(b) CO
(c) O_2
(d) B_2

53. The freezing point of benzene decreases by 0.45°C when 0.2 g of acetic acid is added to 20 g of benzene. If acetic acid associates to form a dimer in benzene, percentage association of acetic acid in benzene will be (K_f for benzene = 5.12 K kg mol^{-1})
(a) 64.6 %
(b) 80.4 %
(c) 74.6 %
(d) 94.6 %

54. Which of the following molecules is least resonance stabilised?
(a) 
(b) 
(c) 
(d) 

55. On treatment of 100 mL of 0.1 M solution of $CoCl_3.6H_2O$ with excess of $AgNO_3$; 1.2×10^{22} ions are precipitated. The complex is
(a) $[Co(H_2O)_4Cl_2]\ Cl.2H_2O$
(b) $[Co(H_2O)_3Cl_3].3H_2O$
(c) $[Co(H_2O)_6]Cl_3$
(d) $[Co(H_2O)_5Cl]\ Cl_2.H_2O$

56. The major product obtained in the following reaction is




57. A water sample has ppm level concentration of following anions

$F^- = 10;\ SO_4^{2-} = 100;\ NO_3^- = 50$

the anion/anions that make/makes the water sample unsuitable for drinking is/are
(a) Only NO_3^-
(b) Both SO_4^{2-} and NO_3^-
(c) Only F^-
(d) Only SO_4^{2-}

58. 1 g of a carbonate (M_2CO_3) on treatment with excess HCl produces 0.01186 mole of CO_2. The molar mass of M_2CO_3 in g mol^{-1} is
(a) 1186
(b) 84.3
(c) 118.6
(d) 11.86

59. Given, $E^\circ_{Cl_2/Cl^-} = 1.36$ V, $E^\circ_{Cr^{3+}/Cr} = -0.74$ V
$E^\circ_{Cr_2O_7^{2-}/Cr^{3+}} = 1.33$ V, $E^\circ_{MnO_4^-/Mn^{2+}} = 1.51$ V
Among the following, the strongest reducing agent is
(a) Cr
(b) Mn^{2+}
(c) Cr^{3+}
(d) Cl^-

60. The group having isoelectronic species is
(a) O^{2-}, F^-, Na^+, Mg^{2+}
(b) O^-, F^-, Na, Mg^+
(c) O^{2-}, F^-, Na, Mg^{2+}
(d) O^-, F^-, Na^+, Mg^{2+}

OF N

Mathematics

61. If S is the set of distinct values of b for which the following system of linear equations

$$x + y + z = 1,$$
$$x + ay + z = 1$$
$$ax + by + z = 0$$

and has no solution, then S is

(a) an infinite set

(b) a finite set containing two or more elements

(c) singleton set

(d) a empty set

62. The statement $(p \to q) \to [(\sim p \to q) \to q]$ is

(a) a tautology

(b) equivalent to $\sim p \to q$

(c) equivalent to $p \to \sim q$

(d) a fallacy

63. If $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$, then the value of $\cos 4x$ is

(a) $-\frac{3}{5}$ (b) $\frac{1}{3}$ (c) $\frac{2}{9}$ (d) $-\frac{7}{9}$

64. For three events A, B and C, if P(exactly one of A or B occurs) = P(exactly one of B or C occurs) = P (exactly one of C or A occurs) = $\frac{1}{4}$ and P (all the three events occur simultaneously) = $\frac{1}{16}$, then the probability that atleast one of the events occurs, is

(a) $\frac{7}{32}$ (b) $\frac{7}{16}$ (c) $\frac{7}{64}$ (d) $\frac{3}{16}$

65. Let ω be a complex number such that $2\omega + 1 = z$, where $z = \sqrt{-3}$. If

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k, \text{ then } k \text{ is equal to}$$

(a) $-z$ (b) z

(c) -1 (d) 1

66. Let k be an integer such that the triangle with vertices $(k, -3k)$, $(5, k)$ and $(-k, 2)$ has area 28 sq units. Then, the orthocentre of this triangle is at the point

(a) $\left(2, -\frac{1}{2}\right)$ (b) $\left(1, \frac{3}{4}\right)$ (c) $\left(1, -\frac{3}{4}\right)$ (d) $\left(2, \frac{1}{2}\right)$

67. If 20 m of wire is available for fencing off a flower-bed in the form of a circular sector, then the maximum area (in sq m) of the flower-bed is

(a) 12.5 (b) 10

(c) 25 (d) 30

68. The area (in sq units) of the region $\{(x, y) : x \geq 0, x + y \leq 3, x^2 \leq 4y$ and $y \leq 1 + \sqrt{x}\}$ is

(a) $\frac{59}{12}$ (b) $\frac{3}{2}$

(c) $\frac{7}{3}$ (d) $\frac{5}{2}$

69. If the image of the point $P(1, -2, 3)$ in the plane $2x + 3y - 4z + 22 = 0$ measured parallel to the line $\frac{x}{1} = \frac{y}{4} = \frac{z}{5}$ is Q, then PQ is equal to

(a) $3\sqrt{5}$ (b) $2\sqrt{42}$

(c) $\sqrt{42}$ (d) $6\sqrt{5}$

70. For $x \in \left(0, \frac{1}{4}\right)$, if the derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1 - 9x^3}\right)$ is $\sqrt{x} \cdot g(x)$, then $g(x)$ equals

(a) $\frac{9}{1 + 9x^3}$ (b) $\frac{3x\sqrt{x}}{1 - 9x^3}$

(c) $\frac{3x}{1 - 9x^3}$ (d) $\frac{3}{1 + 9x^3}$

71. If $(2 + \sin x)\frac{dy}{dx} + (y + 1)\cos x = 0$ and $y(0) = 1$, then $y\left(\frac{\pi}{2}\right)$ is equal to

(a) $\frac{1}{3}$ (b) $-\frac{2}{3}$

(c) $-\frac{1}{3}$ (d) $\frac{4}{3}$

72. Let a vertical tower AB have its end A on the level ground. Let C be the mid-point of AB and P be a point on the ground such that $AP = 2AB$. If $\angle BPC = \beta$, then $\tan\beta$ is equal to

(a) $\frac{6}{7}$ (b) $\frac{1}{4}$ (c) $\frac{2}{9}$ (d) $\frac{4}{9}$

73. If $A=\begin{bmatrix}2 & -3\\ -4 & 1\end{bmatrix}$, then $\text{adj}(3A^2+12A)$ is equal to

(a) $\begin{bmatrix}72 & -84\\ -63 & 51\end{bmatrix}$ (b) $\begin{bmatrix}51 & 63\\ 84 & 72\end{bmatrix}$

(c) $\begin{bmatrix}51 & 84\\ 63 & 72\end{bmatrix}$ (d) $\begin{bmatrix}72 & -63\\ -84 & 51\end{bmatrix}$

74. For any three positive real numbers a, b and c, if $9(25a^2+b^2)+25(c^2-3ac)=15b(3a+c)$, then

(a) b, c and a are in GP (b) b, c and a are in AP

(c) a, b and c are in AP (d) a, b and c are in GP

75. The distance of the point $(1, 3, -7)$ from the plane passing through the point $(1, -1, -1)$ having normal perpendicular to both the lines $\frac{x-1}{1}=\frac{y+2}{-2}=\frac{z-4}{3}$ and $\frac{x-2}{2}=\frac{y+1}{-1}=\frac{z+7}{-1}$, is

(a) $\frac{20}{\sqrt{74}}$ units (b) $\frac{10}{\sqrt{83}}$ units

(c) $\frac{5}{\sqrt{83}}$ units (d) $\frac{10}{\sqrt{74}}$ units

76. Let $I_n=\int \tan^n x\,dx\,(n>1)$. If $I_4+I_6=a\tan^5 x+bx^5+C$, where C is a constant of integration, then the ordered pair (a, b) is equal to

(a) $\left(-\frac{1}{5}, 1\right)$ (b) $\left(\frac{1}{5}, 0\right)$

(c) $\left(\frac{1}{5}, -1\right)$ (d) $\left(-\frac{1}{5}, 0\right)$

77. The eccentricity of an ellipse whose centre is at the origin is 1/2. If one of its directrices is $x=-4$, then the equation of the normal to it at $\left(1, \frac{3}{2}\right)$ is

(a) $2y-x=2$ (b) $4x-2y=1$

(c) $4x+2y=7$ (d) $x+2y=4$

78. If a hyperbola passes through the point $P(\sqrt{2}, \sqrt{3})$ and has foci at $(\pm 2, 0)$, then the tangent to this hyperbola at P also passes through the point

(a) $(3\sqrt{2}, 2\sqrt{3})$ (b) $(2\sqrt{2}, 3\sqrt{3})$

(c) $(\sqrt{3}, \sqrt{2})$ (d) $(-\sqrt{2}, -\sqrt{3})$

79. The function $f: R \to \left[-\frac{1}{2}, \frac{1}{2}\right]$ defined as $f(x)=\frac{x}{1+x^2}$ is

(a) invertible

(b) injective but not surjective

(c) surjective but not injective

(d) neither injective nor surjective

80. $\lim\limits_{x\to\pi/2}\frac{\cot x-\cos x}{(\pi-2x)^3}$ equals

(a) $\frac{1}{24}$ (b) $\frac{1}{16}$ (c) $\frac{1}{8}$ (d) $\frac{1}{4}$

81. Let $\mathbf{a}=2\hat{\mathbf{i}}+\hat{\mathbf{j}}-2\hat{\mathbf{k}}$, $\mathbf{b}=\hat{\mathbf{i}}+\hat{\mathbf{j}}$ and $\mathbf{c}$ be a vector such that $|\mathbf{c}-\mathbf{a}|=3$, $|(\mathbf{a}\times\mathbf{b})\times\mathbf{c}|=3$ and the angle between $\mathbf{c}$ and $\mathbf{a}\times\mathbf{b}$ is 30°. Then, $\mathbf{a}\cdot\mathbf{c}$ is equal to

(a) $\frac{25}{8}$ (b) 2 (c) 5 (d) $\frac{1}{8}$

82. The normal to the curve $y(x-2)(x-3)=x+6$ at the point, where the curve intersects the Y-axis passes through the point

(a) $\left(-\frac{1}{2}, -\frac{1}{2}\right)$ (b) $\left(\frac{1}{2}, \frac{1}{2}\right)$

(c) $\left(\frac{1}{2}, -\frac{1}{3}\right)$ (d) $\left(\frac{1}{2}, \frac{1}{3}\right)$

83. If two different numbers are taken from the set $\{0, 1, 2, 3, \ldots, 10\}$, then the probability that their sum as well as absolute difference are both multiple of 4, is

(a) $\frac{6}{55}$ (b) $\frac{12}{55}$

(c) $\frac{14}{45}$ (d) $\frac{7}{55}$

84. A man X has 7 friends, 4 of them are ladies and 3 are men. His wife Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y have no common friends. Then, the total number of ways in which X and Y together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of X and Y are in this party, is

(a) 485 (b) 468 (c) 469 (d) 484

85. The value of $({}^{21}C_1 - {}^{10}C_1) + ({}^{21}C_2 - {}^{10}C_2) + ({}^{21}C_3 - {}^{10}C_3) + ({}^{21}C_4 - {}^{10}C_4) + \ldots + ({}^{21}C_{10} - {}^{10}C_{10})$ is

(a) $2^{21} - 2^{11}$ (b) $2^{21} - 2^{10}$ (c) $2^{20} - 2^9$ (d) $2^{20} - 2^{10}$

86. A box contains 15 green and 10 yellow balls. If 10 balls are randomly drawn one-by-one with replacement, then the variance of the number of green balls drawn is

(a) $\frac{12}{5}$ (b) 6 (c) 4 (d) $\frac{6}{25}$

87. Let $a, b, c \in R$. If $f(x) = ax^2 + bx + c$ be such that $a + b + c = 3$ and $f(x + y) = f(x) + f(y) + xy, \forall\ x, y \in R$, then $\sum_{n=1}^{10} f(n)$ is equal to

(a) 330 (b) 165 (c) 190 (d) 255

88. The radius of a circle having minimum area, which touches the curve $y = 4 - x^2$ and the lines $y = |x|$, is

(a) $2(\sqrt{2} + 1)$ (b) $2(\sqrt{2} - 1)$
(c) $4(\sqrt{2} - 1)$ (d) $4(\sqrt{2} + 1)$

89. For a positive integer n, if the quadratic equation, $x(x+1) + (x+1)(x+2) + \ldots + (x + n - 1)(x + n) = 10n$ has two consecutive integral solutions, then n is equal to

(a) 12 (b) 9
(c) 10 (d) 11

90. $\int_{\pi/4}^{3\pi/4} \frac{dx}{1 + \cos x}$ is equal to

(a) -2 (b) 2
(c) 4 (d) -1

Answer with Explanation

Physics

1. *(b)* As the observer is moving towards the source, so frequency of waves emitted by the source will be given by the formula

$$f_{\text{observed}} = f_{\text{actual}} \cdot \left(\frac{1 + v/c}{1 - v/c}\right)^{1/2}$$

Here, frequency $\frac{v}{c} = \frac{1}{2}$

So, $f_{\text{observed}} = f_{\text{actual}} \left(\frac{3/2}{1/2}\right)^{1/2}$

$\therefore$ $f_{\text{observed}} = 10 \times \sqrt{3} = 17.3\ \text{GHz}$

2. *(a)* By ascent formula, we have surface tension,

$$T = \frac{rhg}{2} \times 10^3\ \frac{\text{N}}{\text{m}} = \frac{dhg}{4} \times 10^3\ \frac{\text{N}}{\text{m}} \quad \left(\because r = \frac{d}{2}\right)$$

$$\Rightarrow \quad \frac{\Delta T}{T} = \frac{\Delta d}{d} + \frac{\Delta h}{h} \qquad \text{[given, } g \text{ is constant]}$$

So, percentage $= \frac{\Delta T}{T} \times 100 = \left(\frac{\Delta d}{d} + \frac{\Delta h}{h}\right) \times 100$

$$= \left(\frac{0.01 \times 10^{-2}}{1.25 \times 10^{-2}} + \frac{0.01 \times 10^{-2}}{1.45 \times 10^{-2}}\right) \times 100$$

$= 1.5\%$

$\therefore$ $\frac{\Delta T}{T} \times 100 = 1.5\%$

3. *(c)* We have, $\lambda = \frac{hc}{\Delta E}$

So, ratio of wave lengths

$$\frac{\lambda_1}{\lambda_2} = \frac{hc/\Delta E_1}{hc/\Delta E_2} = \frac{\Delta E_2}{\Delta E_1} = \frac{\left(\frac{4}{3}E - E\right)}{2E - E} = \frac{1}{3}$$

4. *(b)* Given, force, $F = -kv^2$

$\therefore$ Acceleration, $a = \frac{-k}{m}v^2$

or $\frac{dv}{dt} = \frac{-k}{m}v^2 \Rightarrow \frac{dv}{v^2} = -\frac{k}{m}.dt$

Now, with limits, we have

$$\int_{10}^{v} \frac{dv}{v^2} = -\frac{k}{m}\int_0^t dt$$

$$\Rightarrow \quad \left(-\frac{1}{v}\right)_{10}^{v} = -\frac{k}{m}t \Rightarrow \frac{1}{v} = 0.1 + \frac{kt}{m}$$

$$\Rightarrow \quad v = \frac{1}{0.1 + \frac{kt}{m}} = \frac{1}{0.1 + 1000k}$$

$$\Rightarrow \quad \frac{1}{2} \times m \times v^2 = \frac{1}{8} m v_0^2 \Rightarrow v = \frac{v_0}{2} = 5$$

$$\Rightarrow \quad \frac{1}{0.1 + 1000\,k} = 5 \quad \Rightarrow \quad 1 = 0.5 + 5000\,k$$

$$\Rightarrow \quad k = \frac{0.5}{5000} \quad \Rightarrow \quad k = 10^{-4}\,\text{kg/m}$$

5. *(b)* By Mayor's relation, for 1 g mole of a gas,

$$C_p - C_V = R$$

So, when n gram moles are given,

$$C_p - C_V = \frac{R}{n}$$

As per given question,

$$a = C_p - C_V = \frac{R}{2}; \text{ for } H_2 \qquad \ldots (i)$$

$$b = C_p - C_V = \frac{R}{28}; \text{ for } N_2 \qquad \ldots (ii)$$

From Eqs. (i) and (ii), we get

$$a = 14b$$

6. *(d)* MI of a solid cylinder about its perpendicular bisector of length is

$$I = m\left(\frac{l^2}{12} + \frac{R^2}{4}\right)$$

$$\Rightarrow \quad I = \frac{mR^2}{4} + \frac{ml^2}{12} = \frac{m^2}{4\pi\rho l} + \frac{ml^2}{12} \quad [\because \rho\pi R^2 l = m]$$

For I to be maximum,

$$\frac{dI}{dl} = -\frac{m^2}{4\pi\rho}\left(\frac{1}{l^2}\right) + \frac{ml}{6} = 0$$

$$\Rightarrow \quad \frac{m^2}{4\pi\rho} = \frac{ml^3}{6}$$

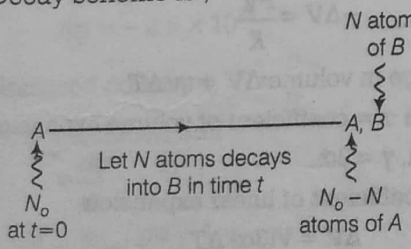
Now, putting $m = \rho\pi R^2 l$

$$\therefore \quad l^3 = \frac{3}{2\pi\rho} \cdot \rho\pi R^2 l$$

$$\frac{l^2}{R^2} = \frac{3}{2}$$

$$\therefore \quad \frac{l}{R} = \sqrt{\frac{3}{2}}$$

7. *(a)* Decay scheme is ,



Given, $\quad \frac{N_B}{N_A} = 0.3 = \frac{3}{10}$

$$\Rightarrow \quad \frac{N_B}{N_A} = \frac{30}{100}$$

So, $\quad N_0 = 100 + 30 = 130$ atoms

By using $\quad N = N_0 e^{-\lambda t}$

We have, $\quad 100 = 130\, e^{-\lambda t}$

$$\Rightarrow \quad \frac{1}{1.3} = e^{-\lambda t} \Rightarrow \log 1.3 = \lambda t$$

If T is half-life, then $\lambda = \frac{\log_e 2}{T}$

$$\Rightarrow \quad \log 1.3 = \frac{\log_e 2}{T} \cdot t$$

$$\therefore \quad t = \frac{T \cdot \log (1.3)}{\log_e 2}$$

8. *(a)* In a balanced Wheatstone bridge, there is no effect on position of null point, if we exchange the battery and galvanometer. So, option (a) is incorrect.

9. *(c)* As each capacitors cannot withstand more than 300 V, so there should be four capacitors in each row became in this condition 1 kV i.e. 1000 V will be divided by 4 (i.e. 250 not more than 300 V).

Now, equivalent capacitance of one row

$$= \frac{1}{4} \times 1\mu F = 0.25\mu\text{F}$$

$$[\because \text{in series combination, } C_{eq} = \frac{C}{n}]$$

Now, we need equivalent of 2μF, so let we need n such rows

$$\therefore \quad n \times 0.25 = 2\mu\text{F}$$

$$[\because \text{in parallel combination } C_{eq} = nc]$$

$$n = \frac{2}{0.25}$$

$$= 8$$

$\therefore$ Total number of capacitors = number of rows × number of capacitors in each row

$$= 8 \times 4 = 32$$

10. *(b)* In steady state, no current flows through the capacitor. So, resistance r_p becomes ineffective.

So, the current in circuit, $I = \frac{E}{r + r_2}$ (Total Resistance)

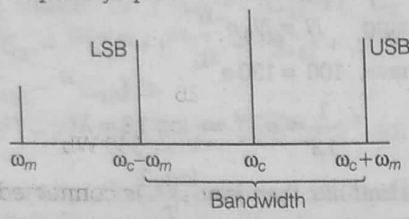
$\because$ Potential drop across capacitor = Potential drop across r_2

$$= Ir_2 = \frac{Er_2}{r + r_2}$$

$\therefore$ Stored charge of capacitor, $Q = CV = CE\frac{r_2}{r + r_2}$

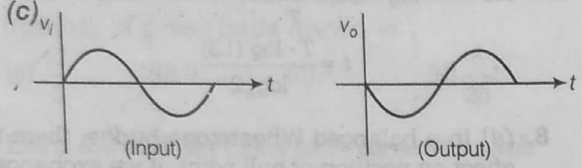
11. *(c)* Each resistance is converted with two cells combined in opposite direction, so potential drop across each resistor is zero. Hence the current through each of resistor is zero.

12. *(d)* Frequency spectrum of modulated wave is



Clearly, ω_m is not included in the spectrum.

13. *(c)*



In a CE *npn* transistor amplifier output is 180° out of phase with input.

14. *(a)* Heat gained (water + calorimeter) = Heat lost by copper ball

$$\Rightarrow \quad m_w s_w \Delta T + m_c s_c \Delta T = m_B s_B \Delta T$$

$$\Rightarrow \quad 170 \times 1 \times (75-30) + 100 \times 0.1 \times (75-30)$$
$$= 100 \times 0.1 \times (T-75)$$

$$\therefore \quad T = 885°C$$

15. *(a)* Let n_1th fringe formed due to first wavelength and n_2th fringe formed due to second wavelength coincide i.e. their distance from common central maxima will be same

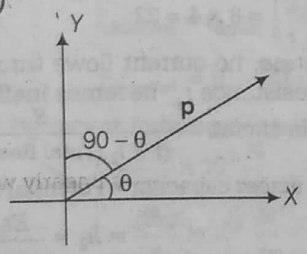
i.e. $\quad y_{n_1} = y_{n_2}$

$$\Rightarrow \quad \frac{n_1\lambda_1 D}{d} = \frac{n_2\lambda_2 D}{d} \quad \Rightarrow \quad \frac{n_1}{n_2} = \frac{\lambda_1}{\lambda_2} = \frac{520}{650} = \frac{4}{5}$$

Hence, distance of the point of coincidence from the central maxima is

$$y = \frac{n_1\lambda_1 D}{d} = \frac{n_2\lambda_2 D}{d} = \frac{4 \times 650 \times 10^{-9} \times 1.5}{0.5 \times 10^{-3}} = 7.8 \text{ mm}$$

16. *(b)*



Torque applied on a dipole $\tau = pE \sin\theta$

where θ = angle between axis of dipole and electric field.

For electric field $E_1 = E\hat{i}$

it means field is directed along positive X direction, so angle between dipole and field will remain θ, therefore torque in this direction

$$E_1 = pE_1 \sin\theta$$

In electric field $E_2 = \sqrt{3}\, E\hat{j}$, it means field is directed along positive Y-axis, so angle between dipole and field will be $90 - \theta$.

Torque in this direction $T_2 = pE \sin(90-\theta)$
$= p\sqrt{3}\, E_1 \cos\theta$

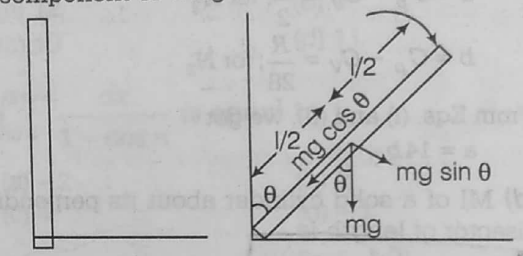
According to question $\tau_2 = -\tau_1 \Rightarrow |\tau_2| = |\tau_1|$

$$\therefore \quad pE_1 \sin\theta = p\sqrt{3}\, E_1 \cos\theta$$

$$\tan\theta = \sqrt{3} \Rightarrow \tan\theta = \tan 60°$$

$$\therefore \quad \theta = 60°$$

17. *(d)* As the rod rotates in vertical plane so a torque is acting on it, which is due to the vertical component of weight of rod.



Now, Torque τ = force × perpendicular distance of line of action of force from axis of rotation

$$= mg \sin\theta \times \frac{l}{2}$$

Again, Torque $\tau = I\alpha$

Where, I = moment of inertia $= \frac{ml^2}{3}$ [Force and Torque frequency along axis of rotation passing through in end]

α = angular acceleration

$$\therefore \quad mg \sin\theta \times \frac{l}{2} = \frac{ml^2}{3}\alpha$$

$$\therefore \quad \alpha = \frac{3g \sin\theta}{2l}$$

18. *(d)* $K = \frac{P}{(-\Delta V/V)} \Rightarrow -\frac{\Delta V}{V} = \frac{P}{K}$

$$\Rightarrow \quad -\Delta V = \frac{PV}{K}$$

Change in volume $\Delta V = \gamma v \Delta T$

Where r = coefficient of volume expansion

Again, $\gamma = 3\alpha$

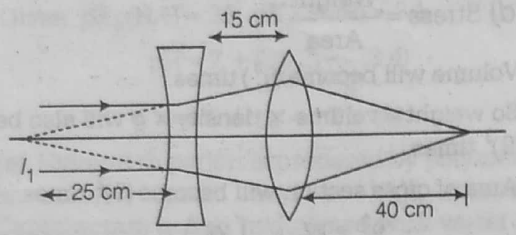
α is coefficient of linear expansion

$$\therefore \quad \Delta V = V(3\alpha)\,\Delta T$$

$$\therefore \quad \frac{PV}{K} = V(3\alpha)\,\Delta T$$

$$\therefore \quad \Delta T = \frac{P}{3\alpha K}$$

19. *(d)* Focal length of diverging lens is 25 cm.
As the rays are coming parallel, so the image (I_1) will be formed at the focus of diverging lens i.e. at 25 cm towards left of diverging lens.



Now, the image (I_1) will work as object for converging lens.
For convergying lens, distance of object u
(i.e. distance of I_1) $= -(25 + 15)$
$= -40$ cm
$f = 20$ cm
$\therefore$ From len's formula $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$$\frac{1}{20} = \frac{1}{v} - \frac{1}{-40} \Rightarrow \frac{1}{v} = \frac{1}{20} - \frac{1}{40}$$

$$\frac{1}{v} = \frac{1}{90} \Rightarrow v = 40 \text{ cm}$$

v is positive so image will be real and will form at right side of converging lens at 40 cm.

20. *(d)* $\lambda_{min} = \frac{hc}{eV}$

$$\log(\lambda_{min}) = \log\left(\frac{hc}{e}\right) - \log V$$

$$y = c - mx$$

So, the required graph is given in option (d).

21. *(c)* From $pV = nRT = \frac{N}{N_A}RT$

We have, $n_f - n_i = \frac{pVN_A}{RT_f} - \frac{pVN_A}{RT_i}$

$$\Rightarrow n_f - n_i = \frac{10^5 \times 30}{8.3} \times 6.02 \times 10^{23} \cdot \left(\frac{1}{300} - \frac{1}{290}\right)$$

$$= -2.5 \times 10^{25}$$

$\therefore \quad \Delta n = -2.5 \times 10^{25}$

22. *(b)* Induced constant, $I = \frac{e}{R}$
Here, e = induced emf

$$= \frac{d\phi}{dt}$$

$$I = \frac{e}{R} = \left(\frac{d\phi}{dt}\right) \cdot \frac{1}{R}$$

$$d\phi = IRdt$$

$$\phi = \int IRdt$$

$\therefore$ Here, R is constant

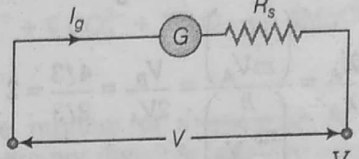
$$\therefore \quad \phi = R\int Idt$$

$\int I \cdot dt$ = Area under $I - t$ graph

$$= \frac{1}{2} \times 10 \times 0.5 = 2.5$$

$$\therefore \quad \phi = R \times 2.5 = 100 \times 2.5 = 250 \text{ Wb}.$$

23. *(d)* Suppose a resistance R_s is connected in series with galvanometer to convert it into voltmeter.



$$I_g(G + R_s) = V \Rightarrow R = \frac{V}{I_g} - G$$

$$\Rightarrow R = 1985 = 1.985 \text{ k}\Omega \text{ or } R = 1.985 \times 10^3 \ \Omega$$

24. *(d)* From Newton's second law, $\frac{\Delta p}{\Delta t} = F$

$$\Rightarrow \quad \Delta p = F\Delta t$$

$$\therefore \quad p = \int dp = \int_0^1 F\,dt \Rightarrow p = \int_0^1 6t\,dt = 3 \text{ kg}\left(\frac{\text{m}}{\text{s}}\right)$$

Also, change in kinetic energy

$$\Delta k = \frac{\Delta p^2}{2m} = \frac{3^2}{2 \times 1} = 4.5$$

From work-energy theorem, work done = change in kinetic energy.
So, work done = $\Delta k = 4.5$ J

25. *(d)* Time period of oscillation is

$$T = 2\pi\sqrt{\frac{I}{MB}}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{7.5 \times 10^{-6}}{6.7 \times 10^{-2} \times 0.01}} = 0.665 \text{ s}$$

Hence, time for 10 oscillations is $t = 6.65$ s.

26. *(c)* Inside the earth surface $g = \frac{GM}{R^3}r$

i.e. $\quad g \propto r$

Out the earth surface $g = \frac{Gm}{r^2}$

i.e. $\quad g \propto \frac{1}{r^2}$

So, till earth surface 'g' increases linearly with distance r, shown only in graph (c).

27. *(b)* Initially velocity keeps on decreasing at a constant rate, then it increases in negative direction with same rate.

28. *(a)* For elastic collision,

$$p_{\text{before collision}} = p_{\text{after collision}}$$

$$mv = mv_A + \frac{m}{2}v_B$$

$$2v = 2v_A + v_B \qquad \ldots(i)$$

Now, coefficient of restitution, $e = \frac{v_B - v_A}{u_A - v_B}$

Here, $u_B = 0$ (Particle at rest) and for elastic collisione = 1

$\therefore \quad 1 = \frac{v_B - v_A}{v} \Rightarrow v = v_B - v_A \quad \ldots$ (ii)

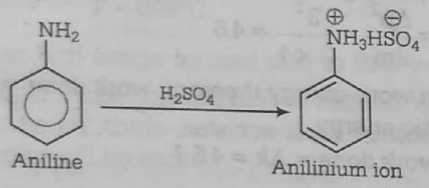
From Eq. (i) and Eq. (ii)

$$v_A = \frac{v}{3} \text{ and } v_B = \frac{4v}{3}$$

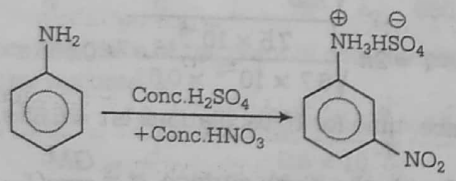
Hence, $\frac{\lambda_A}{\lambda_B} = \frac{\left(\frac{h}{mV_A}\right)}{\left(\frac{h}{\frac{m}{2}.V_B}\right)} = \frac{V_B}{2V_A} = \frac{4/3}{2/3} = 2$

Chemistry

31. *(c)* Aniline in presence of nitrating mixture (conc. HNO_3 + conc. H_2SO_4) gives significance amount ($\approx 47\%$) of meta-product because in presence of H_2SO_4 its protonation takes place and anilinium ion is formed.



Here, anilinium ion is strongly deactivating group and *meta*-directing in nature. So, it gives *meta*-nitration product.



32. *(c)* According to first law of thermodynamics,

$$\Delta U = q + W = q - p\Delta V$$

In isochoric process ($\Delta V = 0$),

$$\Delta U = q$$

In isobaric process ($\Delta p = 0$),

$$\Delta U = q$$

In adiabatic process ($q = 0$)

$$\Delta U = W$$

In isothermal process ($\Delta T = 0$) and $\Delta U = 0$

$\therefore$ ΔU is equal to adiabatic work.

33. *(b) (i) The rate of S_N1 reaction depends only upon the concentration of the alkyl halide.*

(ii) S_N1 reaction proceeds through the formation of carbocation.

The reactivity is decided by ease of dissociation of alkyl halide.

29. *(c)* KE is maximum at mean position and minimum at extreme position $\left(\text{at } t = \frac{T}{4}\right)$.

30. *(d)* Stress $= \frac{\text{Weight}}{\text{Area}}$

Volume will become (9^3) times.

So weight = volume × density × g will also become $(9)^3$ times.

Area of cross section will become $(9)^2$ times.

$$= \frac{9^3 \times W_0}{9^2 \times A_0} = 9\left(\frac{W_0}{A_0}\right)$$

Hence, the stress increases by a factor of 9.

$$R{-}X \rightleftharpoons R^{\oplus} + X^{\ominus}$$

Higher the stability of R^+ (carbocation), higher would be the reactivity towards S_N1 reaction.

p-$H_3CO{-}C_6H_4{-}CH_2^{\oplus}$ is the most stable carbocation due to resonance and then $CH_3\overset{\oplus}{C}HCH_2CH_3$ (2° carbocation) while $CH_3CH_2\overset{\oplus}{C}H_2$ (1°) is least stable.

Thus, the correct increasing order of the reactivity of the given halides towards the S_N1 reaction is

$$\underset{\text{(II)}}{CH_3CH_2CH_2Cl} < \underset{\text{(I)}}{CH_3\underset{\displaystyle Cl}{\underset{|}{C}}HCH_2CH_3} < \underset{\text{(III)}}{p\text{-}H_3CO{-}C_6H_4{-}CH_2Cl}$$

34. *(d)* Bohr radius $(r_n) = \epsilon_0\, n^2h^2$

$$r_n = \frac{n^2h^2}{4\pi^2me^2kZ}$$

$$k = \frac{1}{4\pi\,\epsilon_0}$$

$\therefore$ $$r_n = \frac{n^2h^2\,\epsilon_0}{\pi me^2Z} = n^2\frac{a_0}{Z}$$

where, m = mass of electron

e = charge of electron

h = Planck's constant

k = Coulomb constant

$$r_n = \frac{n^2 \times 0.53}{Z}\,\text{Å}$$

Radius of n^{th} Bohr orbit for H-atom $= 0.53\, n^2$ Å [$Z = 1$ for H-atom]

$\therefore$ Radius of 2^{nd} Bohr orbit for H-atom $= 0.53 \times (2)^2 = 2.12$ Å

35. *(b)* For a salt of weak acid and weak base,

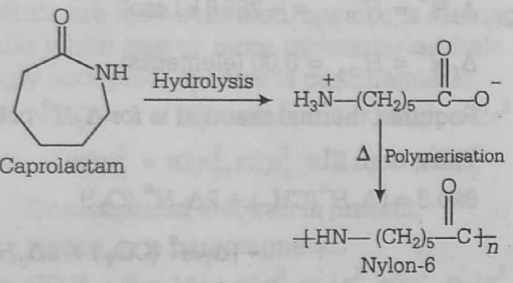
$$pH = 7 + \frac{1}{2}pK_a - \frac{1}{2}pK_b$$

Given, $pK_a(HA) = 3.2$, $pK_a(BOH) = 3.4$

$$\therefore \quad pH = 7 + \frac{1}{2}(3.2) - \frac{1}{2}(3.4)$$

$$= 7 + 1.6 - 1.7 = 6.9$$

36. *(a)* Nylon-6 or perlon is prepared by polymerisation of amino caproic acid at high temperature. Caprolactam is first hydrolysed with water to form amino acid which on heating undergoes polymerisation to give nylon-6.



Caprolactam

Nylon-6

37. *(c)* Given, abundance of elements by mass oxygen = 61.4%, carbon = 22.9%, hydrogen = 10% and nitrogen = 2.6%

Total weight of person = 75 kg

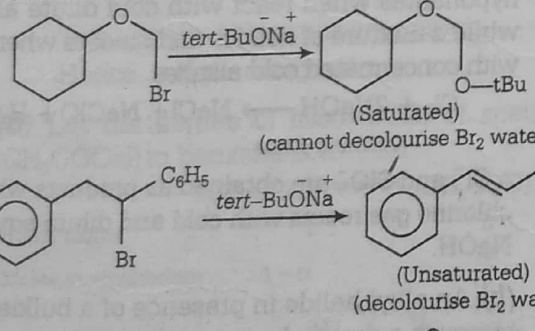
$$\text{Mass due to } {}^1H = \frac{75 \times 10}{100} = 7.5 \text{ kg}$$

${}^1H$ atoms are replaced by ${}^2H$ atoms,

Mass due to ${}^2H = (7.5 \times 2)$ kg

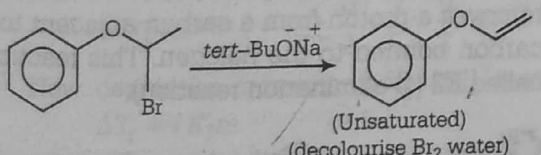
$\therefore$ Mass gain by person = 7.5 kg

38. *(a)* To show decolourisation, compound must be unsaturated.

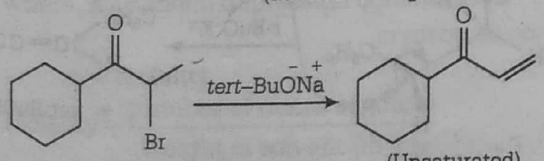


(Saturated)
(cannot decolourise Br_2 water)

(Unsaturated)
(decolourise Br_2 water)



(Unsaturated)
(decolourise Br_2 water)



(Unsaturated)
(decolourise Br_2 water)

39. *(d)* Zinc oxide (ZnO) when react with Na_2O it act as acid while with CO_2 it act as base. Therefore, it is an amphoteric oxide.

$$\underset{\text{Acid}}{ZnO} + \underset{\text{Base}}{Na_2O} \longrightarrow \underset{\text{Salt}}{Na_2ZnO_2}$$

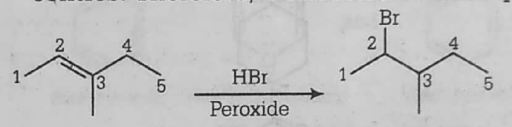
$$\underset{\text{Base}}{ZnO} + \underset{\text{Acid}}{CO_2} \longrightarrow \underset{\text{Salt}}{ZnCO_3}$$

40. *(a)* **Mg** can form basic carbonate while **Li** cannot.

$$5\,Mg^{2+} + 6\,CO_3^{2-} + 7H_2O \longrightarrow 4MgCO_3 \cdot Mg(OH)_2 \cdot 5H_2O + 2\,HCO_3^-$$

41. *(d)* The number of stereoisomers in molecules which are not divisible into two equal halves and have n number of asymmetric C-atoms = 2^n.

3-methyl-pent-2-ene on reaction with HBr in presence of peroxide forms an addition product i.e. 2-bromo-3-methyl pentane. It has two chiral centres. Therefore, 4 stereoisomers are possible



3-methylpent-2-ene

Anti-Markownikoff's addition
Four stereoisomers are possible
(As molecule has two chiral centres and asymmetric).

42. *(d)* For fcc arrangement, $4r = \sqrt{2}a$

where, r = radius and a = edge length

$$\therefore \text{ Closest distance} = 2r = \frac{\sqrt{2}\,a}{2} = \frac{a}{\sqrt{2}}$$

43. *(d)* According to Arrhenius equation

$$k = Ae^{-E_a/RT}$$

where, A = collision number or pre-exponential factor.

R = gas constant

T = absolute temperature

E_a = energy of activation

For reaction R_1, $k_1 = Ae^{-E_{a_1}/RT}$ ...(i)

For reaction R_2, $k_2 = Ae^{-E_{a_2}/RT}$ ...(ii)

On dividing Eq. (ii) by Eq. (i), we get

$$\frac{k_2}{k_1} = e^{-\frac{(E_{a_2} - E_{a_1})}{RT}} \quad \ldots(iii)$$

[$\because$ Pre-exponential factor 'A' is same for both reactions]

Taking ln on both the sides of Eq. (iii), we get

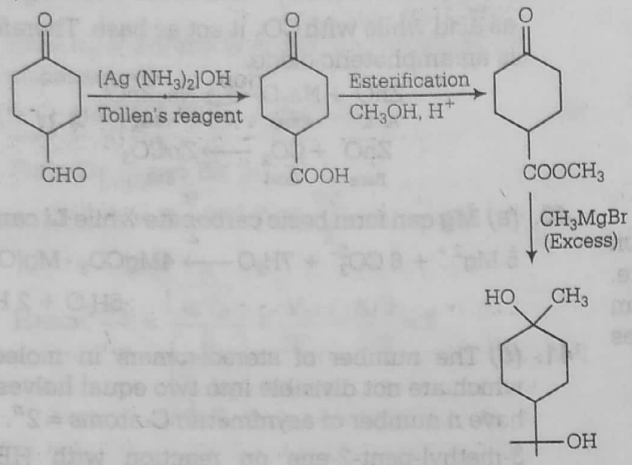
$$\ln\left(\frac{k_2}{k_1}\right) = \frac{E_{a_1} - E_{a_2}}{RT}$$

Given, $E_{a_1} = E_{a_2} + 10 \text{ kJ mol}^{-1}$

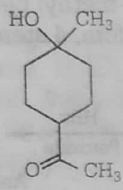
$$= E_{a_2} + 10{,}000 \text{ J mol}^{-1}$$

$$\therefore \quad \ln\frac{k_2}{k_1} = \frac{10{,}000 \text{ J mol}^{-1}}{8.314 \text{ J mol}^{-1}\text{K}^{-1} \times 300 \text{ K}} = 4$$

44. *(a)*



Before final product is formed, intermediate is

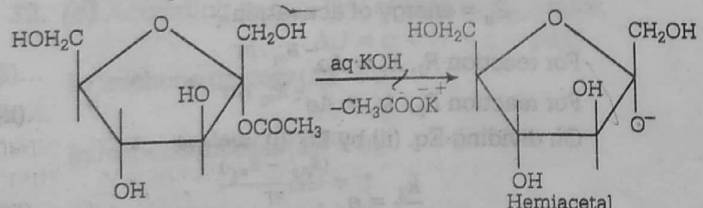


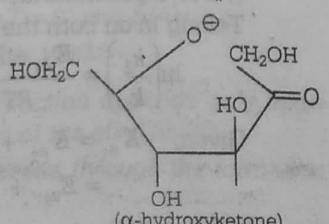
45. *(b)* Colloidal solutions show Tyndall effect due to scattering of light by colloidal particles in all directions in space. It is observed only under the following conditions.

(i) The diameter of the colloids should not be much smaller than the wavelength of light used.

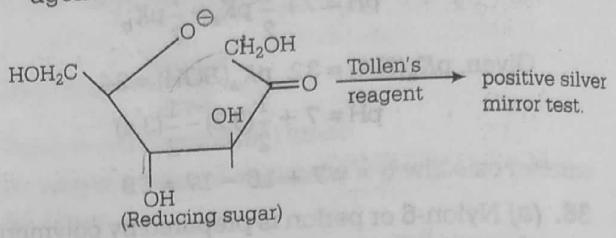
(ii) The refractive indices of the dispersed phase and dispersion medium should differ greatly in magnitude.

46. *(a)* Sugars that have an aldehyde, a ketone, a hemiacetal or a hemiketal group is able to reduce an oxidising agent. These sugars are classified as reducing sugars.





Hemiacetal can be easily reduced by oxidising agent such as Tollen's reagent.



47. *(c)* Based on given $\Delta_f H^\circ$

$\Delta_f H^\circ = H^\circ_{CO_2} = -393.5\ \text{kJ mol}^{-1}$ ...(i)

$\Delta_f H^\circ = H^\circ_{H_2O} = -285.8\ \text{kJ mol}^{-1}$ ...(ii)

$\Delta_f H^\circ = H^\circ_{O_2} = 0.00$ (elements) ...(iii)

Required thermal reaction is for $\Delta_f H^\circ$ of CH_4

Thus, from III

$$890.3 = [\Delta_f H^\circ(CH_4) + 2\Delta_f H^\circ(O_2)] - [\Delta_f H^\circ(CO_2) + 2\Delta_f H^\circ(H_2O)]$$

$$= \Delta_f H^\circ(CH_4) + 0] - [-393.5 - 2 \times 285.5]$$

$$\therefore \quad \Delta_f H^\circ(CH_4) = -74.8\ \text{kJ/mol}$$

48. *(a)* The reaction in which oxidation and reduction occur simultaneously are termed as redox reaction.

$$\overset{+4}{(X)}eF_4 + \overset{+1}{O_2}(F_2) \longrightarrow \overset{+6}{X}eF_6 + \overset{0}{O_2}$$

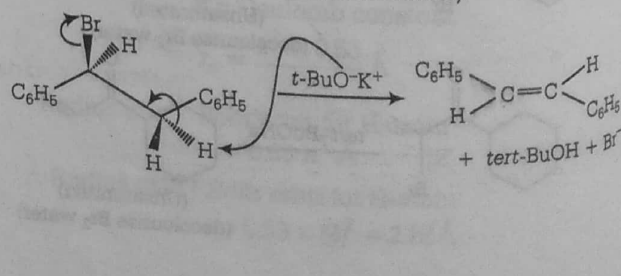
Since, Xe undergoes oxidation while O undergoes reduction. So, it is an example of redox reaction.

49. *(c)* Cl_2, Br_2 and I_2 form a mixture of halide and hypohalites when react with cold dilute alkalies while a mixture of halides and haloate when react with concentrated cold alkalies.

$$Cl_2 + 2NaOH \xrightarrow[\text{Cold and dilute}]{} NaCl + NaClO + H_2O$$

$\therefore Cl^-$ and ClO^- are obtained as products when chlorine gas reacts with cold and dilute aqueous NaOH.

50. *(b)* An alkyl halide in presence of a bulkier base removes a proton from a carbon adjacent to the carbon bonded to the halogen. This reaction is called E2 (β-elimination reaction).



51. *(d)* The reaction takes place as follows

$$\underset{(X)}{Na_2C_2O_4} + \underset{(conc.)}{H_2SO_4} \longrightarrow Na_2SO_4 + H_2O + \underset{\text{Effervescence}}{CO\uparrow + CO_2\uparrow}$$

$$\underset{(X)}{Na_2C_2O_4} + CaCl_2 \longrightarrow \underset{\text{White ppt.}}{CaC_2O_4} + 2NaCl$$

$$5CaC_2O_4 + 2\underset{\text{Purple}}{KMnO_4} + 8H_2SO_4 \longrightarrow K_2SO_4 + 5CaSO_4 + \underset{\text{Colourless}}{2MnSO_4} + 10CO_2 + 8H_2O$$

Hence, X is $Na_2C_2O_4$.

52. *(b)* To identify the magnetic nature we need to check the molecular orbital configuration. If all orbitals are fully occupied, species is diamagnetic while when one or more molecular orbitals is/are singly occupied, species is paramagnetic.

(a) NO $(7 + 8 = 15) - \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2,$

$$\pi 2p_x^2 = \pi 2p_y^2, \pi 2p_z^2, \pi^* 2p_x^1 = \pi^* 2p_y^0$$

One unpaired electron is present.

Hence, it is paramagnetic.

(b) CO $(6 + 8 = 14) - \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2,$

$$\pi 2p_x^2 = \pi 2p_y^2, \sigma 2p_z^2$$

No unpaired electron is present.

Hence, it is diamagnetic.

(c) O_2

$(8 + 8 = 16) - \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2p_z^2,$

$$\pi 2p_x^2 = \pi 2p_x^2, \pi^* 2p_x^1 = \pi^* 2p_y^1$$

Two unpaired electrons are present.

Hence, it is paramagnetic.

(d) $B_2 (5 + 5) - \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \pi 2p_x^1$

$$= \pi 2p_y^1$$

Two unpaired electrons are present.

Hence, it is paramagnetic.

53. *(d)* Let the degree of association of acetic acid (CH_3COOH) in benzene is α, then

$$2CH_3COOH \rightleftharpoons (CH_3COOH)_2$$

Initial moles	1	0
Moles at equilibrium	$1-\alpha$	$\frac{\alpha}{2}$

$$\therefore \text{ Total moles } = 1 - \alpha + \frac{\alpha}{2} = 1 - \frac{\alpha}{2} \text{ or } i = 1 - \frac{\alpha}{2}$$

Now, depression in freezing point (ΔT_f) is given as

$$\Delta T_f = i K_f m \quad \ldots (i)$$

where, K_f = molal depression constant or cryoscopic constant.

m = molality

$$\text{Molality} = \frac{\text{number of moles of solute}}{\text{weight of solvent (in kg)}} = \frac{0.2}{60} \times \frac{1000}{20}$$

Putting the values in Eq. (i)

$$\therefore \quad 0.45 = \left[1 - \frac{\alpha}{2}\right](5.12)\left[\frac{0.2}{60} \times \frac{1000}{20}\right]$$

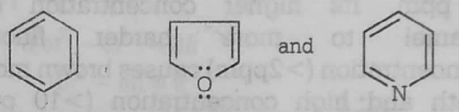
$$1 - \frac{\alpha}{2} = \frac{0.45 \times 60 \times 20}{5.12 \times 0.2 \times 1000}$$

$$\Rightarrow \quad 1 - \frac{\alpha}{2} = 0.527 \quad \Rightarrow \quad \frac{\alpha}{2} = 1 - 0.527$$

$$\therefore \quad \alpha = 0.946$$

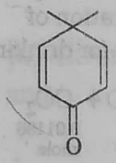
Thus, percentage of association = 94.6%

54. *(d)* Aromatic compounds are stable due to resonance while non-aromatics are not. According to Huckel's rule (or $4n + 2$ rule), "For a planar, cyclic compound to be aromatic, its π cloud must contain $(4n + 2)\pi$ electrons, where, n is any whole number." Thus,



($6\pi e^-$ system) ($4\pi e^- + 2e^-$ system) ($6\pi e^-$ system)

are aromatic and stabilised by resonance. They follow Huckel's rule.



is non-aromatic, hence, least stabilised by resonance.

55. *(d)* Molarity $(M) = \frac{\text{Number of moles of solute}}{\text{Volume of solution (in L)}}$

$\therefore$ Number of moles of complex

$$= \frac{\text{Molarity} \times \text{volume (in mL)}}{1000} = \frac{0.1 \times 100}{1000} = 0.01 \text{ mole}$$

Number of moles of ions precipitate

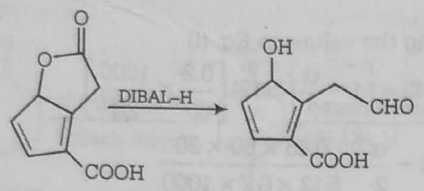
$$= \frac{1.2 \times 10^{22}}{6.02 \times 10^{23}} = 0.02 \text{ moles}$$

$\therefore$ Number of Cl^- present in ionisation sphere

$$= \frac{\text{Number of moles of ions precipitated}}{\text{Number of moles of complex}} = \frac{0.02}{0.01} = 2$$

$\therefore 2\ Cl^-$ are present outside the square brackets, i.e. in ionisation sphere. Thus, the formula of complex is $[Co(H_2O)_5Cl]Cl_2 \cdot H_2O$.

56. *(a)* DIBAL-H (Di-isobutyl aluminium hydride) is a reducing agent with formula. This is generally used for the preparation of aldehydes. Using DIBAL —H, Lactones are reduced directly to aldehydes.



57. *(c)* **NO_3^-** The maximum limit of nitrate (NO_3^-) in drinking water is 50 ppm and its source is fertilisers. If the maximum limit is increased in water it will cause methemoglobinemia (blue baby syndrome.)

SO_4^{2-} The maximum limit of sulphate (SO_4^{2-}) according to WHO is 500 pm and its sources are acid rain, industries. Excess SO_4^{2-} has laxative effect.

F^- The maximum limit of fluoride (F^-) is about 1.5 ppm. Its higher concentration converts enamel to more harder fluorapatite. Concentration (>2ppm) causes brown mottling of teeth and high concentration (>10 ppm) are harmful for bones and teeth.

$\therefore SO_4^{2-}$ (100 ppm) and NO_3^- (50 ppm) in water is suitable for drinking but the concentration of F^- (10 ppm) makes water unsuitable for drinking.

58. *(b)* $\underset{1\,g}{M_2CO_3} + 2HCl \longrightarrow 2MCl + H_2O + \underset{0.01186\ \text{mole}}{CO_2}$

Number of moles of M_2CO_3 reacted = Number of moles of CO_2 evolved

$$\frac{1}{M} = 0.01186 \qquad [M = \text{molar mass of } M_2CO_3]$$

$$M = \frac{1}{0.01186} = 84.3\ \text{g mol}^{-1}$$

59. *(a)* The substances which have lower reduction potentials are stronger reducing agents. Therefore, Cr ($E^\circ_{Cr^{3+}/Cr} = -0.74$ V) is the strongest reducing agent among all the other given options.

60. *(a)* Isoelectronic species are those which contains same number of electrons.

Species	Atomic number	Number of electrons
O^{2-}	8	10
F^-	9	10
Na^+	11	10
Mg^{2+}	12	10
O^-	8	9
Na	11	11
Mg^+	12	11

$\therefore$ Option (a) is correct which contains isoelectronic species O^{2-}, F^-, Na^+, Mg^{2+}.

Mathematics

61. *(d)* $\because \Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ a & b & 1 \end{vmatrix} = 1(a-b) - 1(1-a) + 1(b-a^2)$

$$= -(a-1)^2$$

$$\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ 0 & b & 1 \end{vmatrix} = 1(a-b) - 1(1) + 1(b)$$

$$= -(a-1)$$

$$\Delta_2 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ a & 0 & 1 \end{vmatrix} = 1(1) - 1(1-a) + 1(0-a) = 0$$

and $\Delta_3 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ a & b & 0 \end{vmatrix} = 1(-b) - 1(-a) + 1(b-a^2)$

$$= -a(a-1)$$

For $a = 1$

$$\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$$

Δ for $b = 1$ only

$$x + y + z = 1,$$
$$x + y + z = 1$$

and $x + y + z = 0$

i.e. no solution ($\because$ RHS is not equal)

Hence, for no solution $b = 1$ only

62. *(a)* The truth table of the given expression is given below :

p	q	$x \equiv p \to q$	$\sim p$	$\sim p \to q$	$y \equiv (\sim p \to q) \to q$	$x \to y$
T	T	T	F	T	T	T
T	F	F	F	T	F	T
F	T	T	T	T	T	T
F	F	T	T	F	T	T

Hence, it is a tautology.

63. *(d)* Given, $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$

$$\Rightarrow 5\left(\frac{1-\cos 2x}{1+\cos 2x} - \frac{1+\cos 2x}{2}\right) = 2\cos 2x + 9$$

Put $\cos 2x = y$, we have

$$5\left(\frac{1-y}{1+y} - \frac{1+y}{2}\right) = 2y + 9$$

$\Rightarrow 5(2-2y-1-y^2-2y)=2(1+y)(2y+9)$

$\Rightarrow 5(1-4y-y^2)=2(2y+9+2y^2+9y)$

$\Rightarrow 5-20y-5y^2=22y+18+4y^2$

$\Rightarrow 9y^2+42y+13=0$

$\Rightarrow 9y^2+3y+39y+13=0$

$\Rightarrow 3y(3y+1)+13(3y+1)=0$

$\Rightarrow (3y+1)(3y+13)=0$

$\Rightarrow y=-\frac{1}{3},-\frac{13}{3}$

$\therefore \cos 2x=-\frac{1}{3},-\frac{13}{3}$

$\Rightarrow \cos 2x=-\frac{1}{3} \quad \left[\because \cos 2x \neq -\frac{13}{3}\right]$

Now, $\cos 4x=2\cos^2 2x-1=2\left(-\frac{1}{3}\right)^2-1$

$=\frac{2}{9}-1=-\frac{7}{9}$

64. ***(b)*** We have, P (exactly one of A or B occurs)

$=P(A\cup B)-P(A\cap B)$

$=P(A)+P(B)-2P(A\cap B)$

According to the question,

$P(A)+P(B)-2P(A\cap B)=\frac{1}{4}$ ...(i)

$P(B)+P(C)-2P(B\cap C)=\frac{1}{4}$ ...(ii)

and $P(C)+P(A)-2P(C\cap A)=\frac{1}{4}$ ...(iii)

On adding Eqs. (i), (ii) and (iii), we get

$2[P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(C\cap A)]=\frac{3}{4}$

$\Rightarrow P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(C\cap A)=\frac{3}{8}$

$\therefore P$ (atleast one event occurs)

$=P(A\cup B\cup C)$

$=P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(C\cap A)+P(A\cap B\cap C)$

$=\frac{3}{8}+\frac{1}{16}=\frac{7}{16} \quad \left[\because P(A\cap B\cap C)=\frac{1}{16}\right]$

65. ***(a)*** Given, $2\omega+1=z$

$\Rightarrow 2\omega+1=\sqrt{-3} \quad [\because z=\sqrt{-3}]$

$\Rightarrow \omega=\frac{-1+\sqrt{3}i}{2}$

Since, ω is cube root of unity.

$\therefore \omega^2=\frac{-1-\sqrt{3}i}{2}$ and $\omega^{3n}=1$

Now, $\begin{vmatrix}1 & 1 & 1\\ 1 & -\omega^2-1 & \omega^2\\ 1 & \omega^2 & \omega^7\end{vmatrix}=3k$

$\Rightarrow \begin{vmatrix}1 & 1 & 1\\ 1 & \omega & \omega^2\\ 1 & \omega^2 & \omega\end{vmatrix}=3k$

$[\because 1+\omega+\omega^2=0$ and $\omega^7=(\omega^3)^2\cdot\omega=\omega]$

On applying $R_1\to R_1+R_2+R_3$, we get

$\begin{vmatrix}3 & 1+\omega+\omega^2 & 1+\omega+\omega^2\\ 1 & \omega & \omega^2\\ 1 & \omega^2 & \omega\end{vmatrix}=3k$

$\Rightarrow \begin{vmatrix}3 & 0 & 0\\ 1 & \omega & \omega^2\\ 1 & \omega^2 & \omega\end{vmatrix}=3k$

$\Rightarrow 3(\omega^2-\omega^4)=3k$

$\Rightarrow (\omega^2-\omega)=k$

$\therefore k=\left(\frac{-1-\sqrt{3}i}{2}\right)-\left(\frac{-1+\sqrt{3}i}{2}\right)$

$=-\sqrt{3}i=-z$

66. ***(d)*** Given, vertices of triangle are $(k,-3k),(5,k)$ and $(-k,2)$.

$\therefore \frac{1}{2}\begin{vmatrix}k & -3k & 1\\ 5 & k & 1\\ -k & 2 & 1\end{vmatrix}=\pm 28$

$\Rightarrow \begin{vmatrix}k & -3k & 1\\ 5 & k & 1\\ -k & 2 & 1\end{vmatrix}=\pm 56$

$\Rightarrow k(k-2)+3k(5+k)+1(10+k^2)=\pm 56$

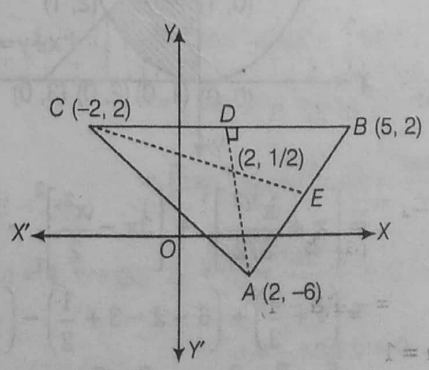
$\Rightarrow k^2-2k+15k+3k^2+10+k^2=\pm 56$

$\Rightarrow 5k^2+13k+10=\pm 56$

$\Rightarrow 5k^2+13k-66=0$ or $5k^2+13k-46=0$

$\Rightarrow k=2 \quad [\because k\in I]$

Thus, the coordinates of vertices of triangle are $A(2,-6)$, $B(5,2)$ and $C(-2,2)$.



Now, equation of altitude from vertex A is

$$y-(-6)=\frac{-1}{\left(\frac{2-2}{-2-5}\right)}(x-2)\Rightarrow x=2 \quad \ldots(i)$$

Equation of altitude from vertex C is

$$y-2=\frac{-1}{\left[\frac{2-(-6)}{5-2}\right]}[x-(-2)]$$

$$\Rightarrow \quad 3x+8y-10=0 \quad \ldots(ii)$$

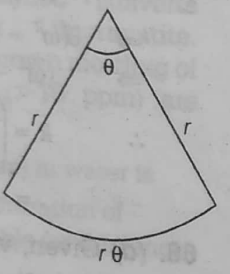
On solving Eqs. (i) and (ii), we get $x=2$ and $y=\frac{1}{2}$.

$\therefore$ Orthocentre $=\left(2,\frac{1}{2}\right)$

67. *(c)* Total length $=2r+r\theta=20$

$$\Rightarrow \quad \theta=\frac{20-2r}{r}$$

Now, area of flower-bed,



$$A=\frac{1}{2}r^2\theta$$

$$\Rightarrow \quad A=\frac{1}{2}r^2\left(\frac{20-2r}{r}\right)$$

$$\Rightarrow \quad A=10r-r^2$$

$$\therefore \quad \frac{dA}{dr}=10-2r$$

For maxima or minima, put $\frac{dA}{dr}=0$.

$$\Rightarrow \quad 10-2r=0$$

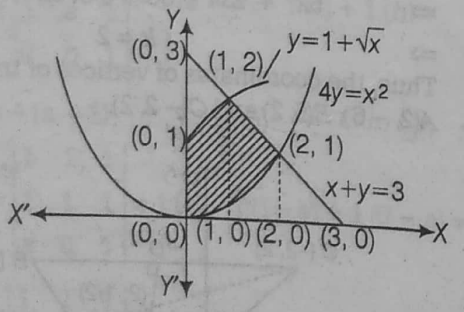
$$\Rightarrow \quad r=5$$

$$\therefore \quad A_{max}=\frac{1}{2}(5)^2\left[\frac{20-2(5)}{5}\right]$$

$$=\frac{1}{2}\times 25\times 2=25 \text{ sq m}$$

68. *(d)* Required area

$$=\int_0^1(1+\sqrt{x})dx+\int_1^2(3-x)dx-\int_0^2\frac{x^2}{4}dx$$

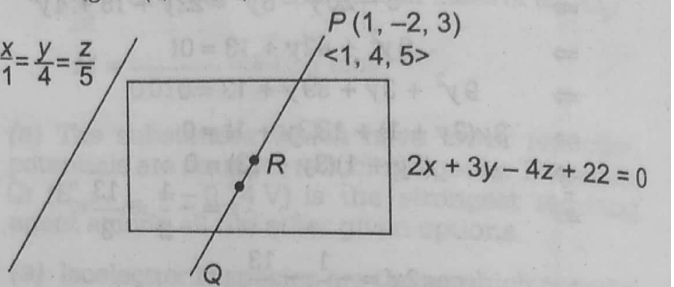


$$=\left[x+\frac{x^{3/2}}{3/2}\right]_0^1+\left[3x-\frac{x^2}{2}\right]_1^2-\left[\frac{x^3}{12}\right]_0^2$$

$$=\left(1+\frac{2}{3}\right)+\left(6-2-3+\frac{1}{2}\right)-\left(\frac{8}{12}\right)$$

$$=\frac{5}{3}+\frac{3}{2}-\frac{2}{3}=1+\frac{3}{2}=\frac{5}{2} \text{ sq units}$$

69. *(b)* Any line parallel to $\frac{x}{1}=\frac{y}{4}=\frac{z}{5}$ and passing through $P(1,-2,3)$ is



$$\frac{x-1}{1}=\frac{y+2}{4}=\frac{z-3}{5}=\lambda \quad \text{(say)}$$

Any point on above line can be written as $(\lambda+1,4\lambda-2,5\lambda+3)$.

$\therefore$ Coordinates of R are $(\lambda+1,4\lambda-2,5\lambda+3)$.

Since, point R lies on the above plane.

$$\therefore 2(\lambda+1)+3(4\lambda-2)-4(5\lambda+3)+22=0$$

$$\Rightarrow \quad \lambda=1$$

So, point R is $(2,2,8)$.

Now, $PR=\sqrt{(2-1)^2+(2+2)^2+(8-3)^2}=\sqrt{42}$

$$\therefore \quad PQ=2PR=2\sqrt{42}$$

70. *(a)* Let $y=\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)=\tan^{-1}\left[\frac{2\cdot(3x^{3/2})}{1-(3x^{3/2})^2}\right]$

$$=2\tan^{-1}(3x^{3/2})\left[\because 2\tan^{-1}x=\tan^{-1}\frac{2x}{1-x^2}\right]$$

$$\therefore \quad \frac{dy}{dx}=2\cdot\frac{1}{1+(3x^{3/2})^2}\cdot 3\times\frac{3}{2}(x)^{1/2}$$

$$=\frac{9}{1+9x^3}\cdot\sqrt{x}$$

$$\therefore \quad g(x)=\frac{9}{1+9x^3}$$

71. *(a)* We have, $(2+\sin x)\frac{dy}{dx}+(y+1)\cos x=0$

$$\Rightarrow \quad \frac{dy}{dx}+\frac{\cos x}{2+\sin x}y=\frac{-\cos x}{2+\sin x}$$

which is a linear differential equation.

$$\therefore \quad \text{IF}=e^{\int\frac{\cos x}{2+\sin x}dx}=e^{\log(2+\sin x)}=2+\sin x$$

$\therefore$ Required solution is given by

$$y\cdot(2+\sin x)=\int\frac{-\cos x}{2+\sin x}\cdot(2+\sin x)dx+C$$

$$\Rightarrow \quad y(2+\sin x)=-\sin x+C$$

Also, $y(0) = 1$

$\therefore \quad 1(2 + \sin 0) = -\sin 0 + C$

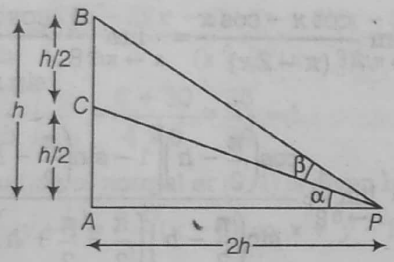
$\Rightarrow \quad C = 2$

$\therefore \quad y = \dfrac{2 - \sin x}{2 + \sin x}$

$\Rightarrow \quad y\left(\dfrac{\pi}{2}\right) = \dfrac{2 - \sin\dfrac{\pi}{2}}{2 + \sin\dfrac{\pi}{2}} = \dfrac{1}{3}$

72. *(c)* Let $AB = h$, then $AD = 2h$ and $AC = BC = \dfrac{h}{2}$

Again, let $\angle CPA = \alpha$



Now, in ΔABP, $\tan(\alpha + \beta) = \dfrac{AB}{AP} = \dfrac{h}{2h} = \dfrac{1}{2}$

Also, in ΔACP, $\tan\alpha = \dfrac{AC}{AP} = \dfrac{\frac{h}{2}}{2h} = \dfrac{1}{4}$

Now, $\tan\beta = \tan[(\alpha + \beta) - \alpha]$

$$= \frac{\tan(\alpha + \beta) - \tan\alpha}{1 + \tan(\alpha + \beta)\tan\alpha} = \frac{\frac{1}{2} - \frac{1}{4}}{1 + \frac{1}{2} \times \frac{1}{4}} = \frac{\frac{1}{4}}{\frac{9}{8}}$$

$$= \frac{2}{9}$$

73. *(b)* We have, $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$

$\therefore \quad A^2 = A \cdot A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 4 + 12 & -6 - 3 \\ -8 - 4 & 12 + 1 \end{bmatrix} = \begin{bmatrix} 16 & -9 \\ -12 & 13 \end{bmatrix}$$

Now, $3A^2 + 12A = 3\begin{bmatrix} 16 & -9 \\ -12 & 13 \end{bmatrix} + 12\begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 48 & -27 \\ -36 & 39 \end{bmatrix} + \begin{bmatrix} 24 & -36 \\ -48 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$$

$\therefore \text{adj}(3A^2 + 12A) = \begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$

74. *(c)* We have,

$225a^2 + 9b^2 + 25c^2 - 75ac - 45ab - 15bc = 0$

$\Rightarrow (15a)^2 + (3b)^2 + (5c)^2 - (15a)(5c) - (15a)(3b) - (3b)(5c) = 0$

$\Rightarrow \dfrac{1}{2}[(15a - 3b)^2 + (3b - 5c)^2 + (5c - 15a)^2] = 0$

$\Rightarrow \quad 15a = 3b,\ 3b = 5c \text{ and } 5c = 15a$

$\therefore \quad 15a = 3b = 5c$

$\Rightarrow \quad \dfrac{a}{1} = \dfrac{b}{5} = \dfrac{c}{3} = \lambda \text{ (say)}$

$\Rightarrow \quad a = \lambda,\ b = 5\lambda,\ c = 3\lambda$

Hence, a, b and c are in AP.

75. *(b)* Given, equations of lines are

$$\frac{x - 1}{1} = \frac{y + 2}{-2} = \frac{z - 4}{3} \text{ and}$$

$$\frac{x - 2}{2} = \frac{y + 1}{-1} = \frac{z + 7}{-1}$$

Let $\mathbf{n}_1 = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ and $\mathbf{n}_2 = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}$

$\therefore$ Any vector $\mathbf{n}$ perpendicular to both $\mathbf{n}_1$, $\mathbf{n}_2$ is given by

$$\mathbf{n} = \mathbf{n}_1 \times \mathbf{n}_2$$

$$\Rightarrow \quad \mathbf{n} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & -2 & 3 \\ 2 & -1 & -1 \end{vmatrix} = 5\hat{\mathbf{i}} + 7\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$$

$\therefore$ Equation of a plane passing through $(1, -1, -1)$ and perpendicular to $\mathbf{n}$ is given by

$$5(x - 1) + 7(y + 1) + 3(z + 1) = 0$$

$$\Rightarrow \quad 5x + 7y + 3z + 5 = 0$$

$\therefore$ Required distance $= \left|\dfrac{5 + 21 - 21 + 5}{\sqrt{5^2 + 7^2 + 3^2}}\right| = \dfrac{10}{\sqrt{83}}$ units

76. *(b)* We have, $I_n = \int \tan^n x\, dx$

$\therefore \quad I_n + I_{n+2} = \int \tan^n x\, dx + \int \tan^{n+2} x\, dx$

$$= \int \tan^n x(1 + \tan^2 x)\, dx$$

$$= \int \tan^n x \sec^2 x\, dx = \frac{\tan^{n+1} x}{n + 1} + C$$

Put $n = 4$, we get $I_4 + I_6 = \dfrac{\tan^5 x}{5} + C$

$\therefore \quad a = \dfrac{1}{5}$ and $b = 0$

77. *(b)* We have, $e=\frac{1}{2}$ and $\frac{a}{e}=4$

$\therefore \quad a=2$

Now, $b^2=a^2(1-e^2)=(2)^2\left[1-\left(\frac{1}{2}\right)^2\right]=4\left(1-\frac{1}{4}\right)=3$

$\Rightarrow \quad b=\sqrt{3}$

$\therefore$ Equation of the ellipse is $\frac{x^2}{(2)^2}+\frac{y^2}{(\sqrt{3})^2}=1$

$\Rightarrow \quad \frac{x^2}{4}+\frac{y^2}{3}=1$

Now, the equation of normal at $\left(1,\frac{3}{2}\right)$ is

$$\frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2-b^2$$

$$\Rightarrow \quad \frac{4x}{1}-\frac{3y}{(3/2)}=4-3$$

$$\Rightarrow \quad 4x-2y=1$$

78. *(b)* Let the equation of hyperbola be $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.

$\therefore \quad ae=2\Rightarrow a^2e^2=4$

$\Rightarrow \quad a^2+b^2=4\Rightarrow b^2=4-a^2$

$\therefore \quad \frac{x^2}{a^2}-\frac{y^2}{4-a^2}=1$

Since, $(\sqrt{2},\sqrt{3})$ lie on hyperbola.

$\therefore \quad \frac{2}{a^2}-\frac{3}{4-a^2}=1$

$\Rightarrow \quad 8-2a^2-3a^2=a^2(4-a^2)$

$\Rightarrow \quad 8-5a^2=4a^2-a^4$

$\Rightarrow \quad a^4-9a^2+8=0$

$\Rightarrow \quad (a^4-8)(a^4-1)=0$

$\Rightarrow \quad a^4=8, a^4=1$

$\therefore \quad a=1$

Now, equation of hyperbola is $\frac{x^2}{1}-\frac{y^2}{3}=1$.

$\therefore$ Equation of tangent at $(\sqrt{2},\sqrt{3})$ is given by

$$\sqrt{2}x-\frac{\sqrt{3}y}{3}=1\Rightarrow\sqrt{2}x-\frac{y}{\sqrt{3}}=1$$

which passes through the point $(2\sqrt{2},3\sqrt{3})$.

79. *(c)* We have, $f(x)=\frac{x}{1+x^2}$

$\therefore \quad f\left(\frac{1}{x}\right)=\frac{\frac{1}{x}}{1+\frac{1}{x^2}}=\frac{x}{1+x^2}=f(x)$

$\therefore \quad f\left(\frac{1}{2}\right)=f(2)$ or $f\left(\frac{1}{3}\right)=f(3)$ and so on.

So, $f(x)$ is many-one function.

Again, let $\quad y=f(x)\Rightarrow y=\frac{x}{1+x^2}$

$\Rightarrow \quad y+x^2y=x\Rightarrow yx^2-x+y=0$

As, $\quad x\in R$

$\therefore \quad (-1)^2-4(y)(y)\geq 0 \quad\Rightarrow 1-4y^2\geq 0$

$\Rightarrow \quad y\in\left[\frac{-1}{2},\frac{1}{2}\right]$

$\therefore$ Range = Codomain $=\left[\frac{-1}{2},\frac{1}{2}\right]$

So, $f(x)$ is surjective.

Hence, $f(x)$ is surjective but not injective.

80. *(b)* $\lim\limits_{x\to\pi/2}\frac{\cot x-\cos x}{(\pi-2x)^3}=\lim\limits_{x\to\pi/2}\frac{1}{8}\cdot\frac{\cos x(1-\sin x)}{\sin x\left(\frac{\pi}{2}-x\right)^3}$

$$=\lim_{h\to 0}\frac{1}{8}\cdot\frac{\cos\left(\frac{\pi}{2}-h\right)\left[1-\sin\left(\frac{\pi}{2}-h\right)\right]}{\sin\left(\frac{\pi}{2}-h\right)\left(\frac{\pi}{2}-\frac{\pi}{2}+h\right)^3}$$

$$=\frac{1}{8}\lim_{h\to 0}\frac{\sin h(1-\cos h)}{\cos h\cdot h^3}$$

$$=\frac{1}{8}\lim_{h\to 0}\frac{\sin h\left(2\sin^2\frac{h}{2}\right)}{\cos h\cdot h^3}$$

$$=\frac{1}{4}\lim_{h\to 0}\frac{\sin h\cdot\sin^2\left(\frac{h}{2}\right)}{h^3\cos h}$$

$$=\frac{1}{4}\lim_{h\to 0}\left(\frac{\sin h}{h}\right)\left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right)^2\cdot\frac{1}{\cos h}\cdot\frac{1}{4}=\frac{1}{4}\times\frac{1}{4}$$

$$=\frac{1}{16}$$

81. *(b)* We have, $\mathbf{a}=2\hat{\mathbf{i}}+\hat{\mathbf{j}}-2\hat{\mathbf{k}}$

$\Rightarrow \quad |\mathbf{a}|=\sqrt{4+1+4}=3$

and $\quad \mathbf{b}=\hat{\mathbf{i}}+\hat{\mathbf{j}}\Rightarrow|\mathbf{b}|=\sqrt{1+1}=\sqrt{2}$

Now, $\quad |\mathbf{c}-\mathbf{a}|=3\Rightarrow|\mathbf{c}-\mathbf{a}|^2=9$

$\Rightarrow \quad (\mathbf{c}-\mathbf{a})\cdot(\mathbf{c}-\mathbf{a})=9$

$\Rightarrow |\mathbf{c}|^2+|\mathbf{a}|^2-2\mathbf{c}\cdot\mathbf{a}=9$

Again, $\quad |(\mathbf{a}\times\mathbf{b})\times\mathbf{c}|=3$

$\Rightarrow \quad |\mathbf{a}\times\mathbf{b}||\mathbf{c}|\sin 30^\circ=3\Rightarrow|\mathbf{c}|=\frac{6}{|\mathbf{a}\times\mathbf{b}|}$

But $\mathbf{a}\times\mathbf{b}=\begin{vmatrix}\hat{\mathbf{i}}&\hat{\mathbf{j}}&\hat{\mathbf{k}}\\2&1&-2\\1&1&0\end{vmatrix}=2\hat{\mathbf{i}}-2\hat{\mathbf{j}}+\hat{\mathbf{k}}$

$$\therefore \quad |\mathbf{c}| = \frac{6}{\sqrt{4+4+1}} = 2 \qquad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$(2)^2 + (3)^2 - 2\mathbf{c}\cdot\mathbf{a} = 9 \Rightarrow 4 + 9 - 2\mathbf{c}\cdot\mathbf{a} = 9$$

$$\Rightarrow \quad \mathbf{c}\cdot\mathbf{a} = 2$$

82. *(b)* Given curve is

$$y(x-2)(x-3) = x + 6 \qquad \ldots(i)$$

Put $x = 0$ in Eq. (i), we get

$$y(-2)(-3) = 6 \Rightarrow y = 1$$

So, point of intersection is (0, 1).

Now, $$y = \frac{x+6}{(x-2)(x-3)}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1(x-2)(x-3) - (x+6)(x-3+x-2)}{(x-2)^2(x-3)^2}$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{(0,\,1)} = \frac{6+30}{4\times 9} = \frac{36}{36} = 1$$

$\therefore$ Equation of normal at (0, 1) is given by

$$y - 1 = \frac{-1}{1}(x - 0) \Rightarrow x + y - 1 = 0$$

which passes through the point $\left(\frac{1}{2}, \frac{1}{2}\right)$.

83. *(a)* Total number of ways of selecting 2 different numbers from $\{0, 1, 2, \ldots, 10\} = {}^{11}C_2 = 55$

Let two numbers selected be x and y.

Then, $$x + y = 4m \qquad \ldots(i)$$

and $$x - y = 4n \qquad \ldots(ii)$$

$$\Rightarrow \quad 2x = 4(m+n) \text{ and } 2y = 4(m-n)$$

$$\Rightarrow \quad x = 2(m+n) \text{ and } y = 2(m-n)$$

$\therefore$ x and y both are even numbers.

x	y
0	4, 8
2	6, 10
4	0, 8
6	2, 10
8	0, 4
10	2, 6

$\therefore$ Required probability $= \frac{6}{55}$

84. *(a)* Given, X has 7 friends, 4 of them are ladies and 3 are men while Y has 7 friends, 3 of them are ladies and 4 are men.

$\therefore$ Total number of required ways

$$= {}^3C_3 \times {}^4C_0 \times {}^4C_0 \times {}^3C_3 + {}^3C_2 \times {}^4C_1 \times {}^4C_1 \times {}^3C_2$$
$$+ {}^3C_1 \times {}^4C_2 \times {}^4C_2 \times {}^3C_1 + {}^3C_0 \times {}^4C_3 \times {}^4C_3 \times {}^3C_0$$
$$= 1 + 144 + 324 + 16 = 485$$

85. *(d)* $({}^{21}C_1 - {}^{10}C_1) + ({}^{21}C_2 - {}^{10}C_2) + ({}^{21}C_3 - {}^{10}C_3) + \ldots + ({}^{21}C_{10} - {}^{10}C_{10})$

$$= ({}^{21}C_1 + {}^{21}C_2 + \ldots + {}^{21}C_{10}) - ({}^{10}C_1 + {}^{10}C_2 + \ldots + {}^{10}C_{10})$$

$$= \frac{1}{2}({}^{21}C_1 + {}^{21}C_2 + \ldots + {}^{21}C_{20}) - (2^{10} - 1)$$

$$= \frac{1}{2}({}^{21}C_1 + {}^{21}C_2 + \ldots + {}^{21}C_{21} - 1) - (2^{10} - 1)$$

$$= \frac{1}{2}(2^{21} - 2) - (2^{10} - 1) = 2^{20} - 1 - 2^{10} + 1$$

$$= 2^{20} - 2^{10}$$

86. *(a)* Given box contains 15 green and 10 yellow balls.

$\therefore$ Total number of balls = 15 + 10 = 25

P(green balls) $= \frac{15}{25} = \frac{3}{5} = p$ = Probability of success

P(yellow balls) $= \frac{10}{25} = \frac{2}{5} = q$ = Probability of unsuccess and $n = 10$ = Number of trials.

$\therefore$ Variance $= npq = 10 \times \frac{3}{5} \times \frac{2}{5} = \frac{12}{5}$

87. *(a)* We have, $f(x) = ax^2 + bx + c$

Now, $f(x + y) = f(x) + f(y) + xy$

Put $y = 0 \Rightarrow f(x) = f(x) + f(0) + 0$

$$\Rightarrow \quad f(0) = 0 \Rightarrow c = 0$$

Again, put $y = -x$

$$\therefore \quad f(0) = f(x) + f(-x) - x^2$$

$$\Rightarrow \quad 0 = ax^2 + bx + ax^2 - bx - x^2$$

$$\Rightarrow \quad 2ax^2 - x^2 = 0$$

$$\Rightarrow \quad a = \frac{1}{2}$$

Also, $a + b + c = 3$

$$\Rightarrow \quad \frac{1}{2} + b + 0 = 3 \Rightarrow b = \frac{5}{2}$$

$$\therefore \quad f(x) = \frac{x^2 + 5x}{2}$$

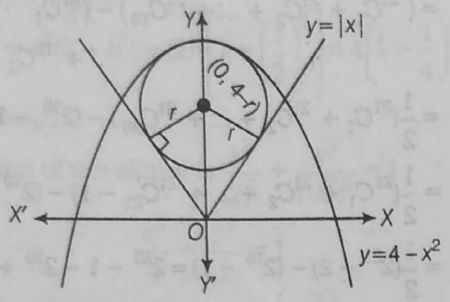
Now, $$f(n) = \frac{n^2 + 5n}{2} = \frac{1}{2}n^2 + \frac{5}{2}n$$

$$\therefore \quad \sum_{n=1}^{10} f(n) = \frac{1}{2}\sum_{n=1}^{10} n^2 + \frac{5}{2}\sum_{n=1}^{10} n$$

$$= \frac{1}{2} \cdot \frac{10 \times 11 \times 21}{6} + \frac{5}{2} \times \frac{10 \times 11}{2}$$

$$= \frac{385}{2} + \frac{275}{2} = \frac{660}{2} = 330$$

88. *(c)* Let the radius of circle with least area be r.
Then, then coordinates of centre = $(0, 4 - r)$.



Since, circle touches the line $y = x$ in first quadrant.

$$\therefore \quad \left|\frac{0-(4-r)}{\sqrt{2}}\right| = r$$

$$\Rightarrow \quad r - 4 = \pm r\sqrt{2}$$

$$\Rightarrow \quad r = \frac{4}{\sqrt{2}+1} \text{ or } \frac{4}{1-\sqrt{2}}$$

But $$r \neq \frac{4}{1-\sqrt{2}} \quad \left[\because \frac{4}{1-\sqrt{2}} < 0\right]$$

$$\therefore \quad r = \frac{4}{\sqrt{2}+1} = 4(\sqrt{2}-1)$$

89. *(d)* Given quadratic equation is

$x(x + 1) + (x + 1)(x + 2) + \ldots + (x + \overline{n-1})$

$(x + n) = 10n$

$$\Rightarrow (x^2 + x^2 + \ldots + x^2) + [(1 + 3 + 5 + \ldots + (2n - 1)]x + [(1 \cdot 2 + 2 \cdot 3 + \ldots + (n-1)n] = 10n$$

$$\Rightarrow nx^2 + n^2x + \frac{n(n^2-1)}{3} - 10n = 0$$

$$\Rightarrow \quad x^2 + nx + \frac{n^2-1}{3} - 10 = 0$$

$$\Rightarrow \quad 3x^2 + 3nx + n^2 - 31 = 0$$

Let α and β be the roots.

Since, α and β are consecutive.

$$\therefore \quad |\alpha - \beta| = 1$$

$$\Rightarrow \quad (\alpha - \beta)^2 = 1$$

Again, $$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$$

$$\Rightarrow \quad 1 = \left(\frac{-3n}{3}\right)^2 - 4\left(\frac{n^2-31}{3}\right)$$

$$\Rightarrow \quad 1 = n^2 - \frac{4}{3}(n^2 - 31)$$

$$\Rightarrow \quad 3 = 3n^2 - 4n^2 + 124$$

$$\Rightarrow \quad n^2 = 121$$

$$\Rightarrow \quad n = \pm 11$$

$$\therefore \quad n = 11 \qquad [\because n > 0]$$

90. *(b)* Let $I = \int_{\pi/4}^{3\pi/4} \frac{dx}{1+\cos x} = \int_{\pi/4}^{3\pi/4} \frac{1-\cos x}{1-\cos^2 x} dx$

$$= \int_{\pi/4}^{3\pi/4} \frac{1-\cos x}{\sin^2 x} dx$$

$$= \int_{\pi/4}^{3\pi/4} (\text{cosec}^2 x - \text{cosec } x \cot x) dx$$

$$= [-\cot x + \text{cosec } x]_{\pi/4}^{3\pi/4}$$

$$= [(1 + \sqrt{2}) - (-1 + \sqrt{2})] = 2$$