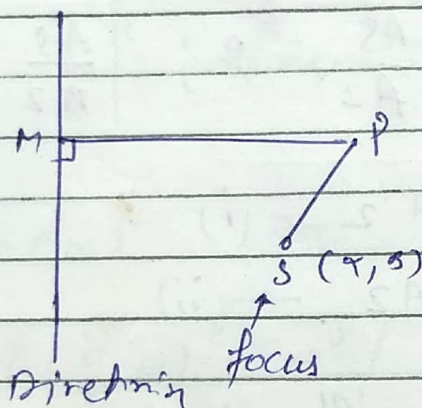


# SBG STUDY

## Ellipse And Hyperbola

A point  $P$  moves such that its dist. from fixed point (focus) to fixed line is constant then



$$\frac{PS}{PM} = e$$

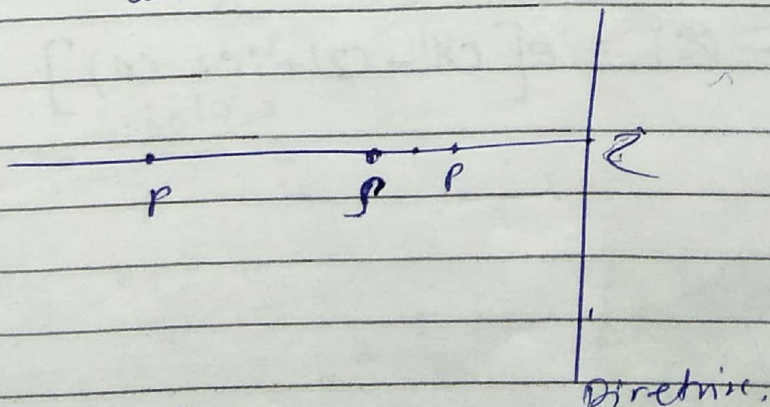
$0 < e < 1$

locus of  $P$  is an ellipse

$e > 1$

locus of

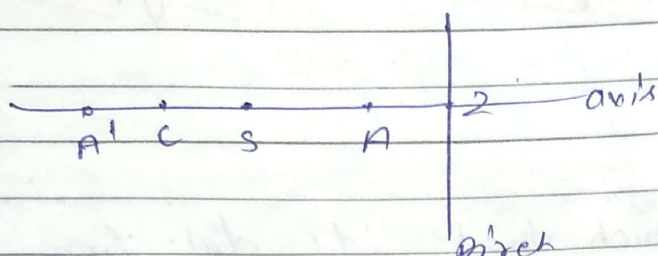
$P$  is Hyperbola.



$$\frac{PS}{PM}$$



\* In this case internal and ext. division of SM is possible



moving pt A is P.  
P is internal in A

$$\frac{PS}{PM} = e$$

$$\frac{AS}{A_2} = e; \quad \frac{A'S}{A'_2} = e$$

$$A'S = e A'_2 \quad \text{--- (i)}$$

$$AS = e A_2 \quad \text{--- (ii)}$$

Suppose, <sup>take</sup> C Middle Pt of A and A' ~~for~~

and  $\boxed{CA = CA' = a}$

(i) + (ii)

$$A'S + AS = e [A'_2 + A_2]$$

$$(CA' + CS) + (CA - CS) = e [CA' + C_2 + (C_2 - CA)]$$

$$2a = 2eC_2$$

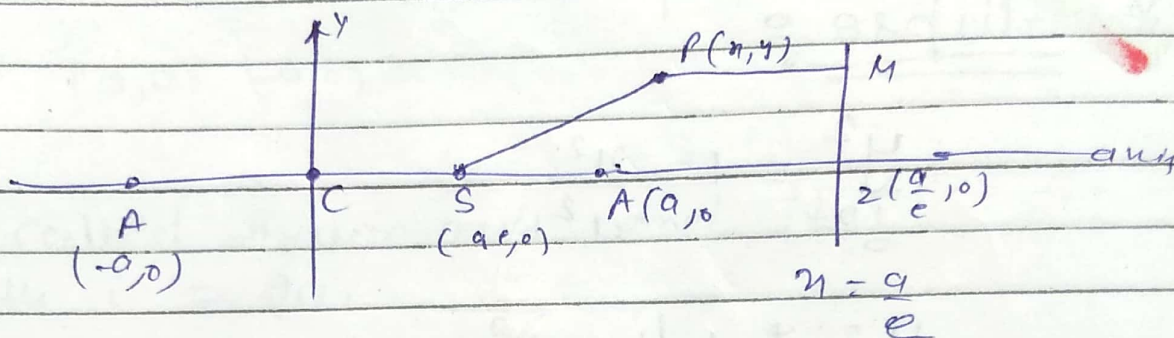
$$\boxed{C_2 = \frac{a}{e}}$$



Now Sub (i) and (ii)

$$cs = ae$$

take  $c$  as ~~center~~ origin  $C$  as  $x$ -axis



$$(PS)^2 = (ePM)^2$$

$$(x - ae)^2 + (y - 0)^2 = e^2 \left[ \frac{a}{e} - x \right]^2$$

$$\frac{x^2}{a^2} + \frac{y^2}{a^2(1-e^2)} = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Ellipse!

$$b^2 = a^2(1 - e^2)$$

Hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$b^2 = a^2(e^2 - 1)$$



## \* Ellipse!

Because eqn of curve contain given power of  $x$  and  $y$  therefore curve will be symmetrical ~~to be~~ about both axis and both have two focus and two directrix.

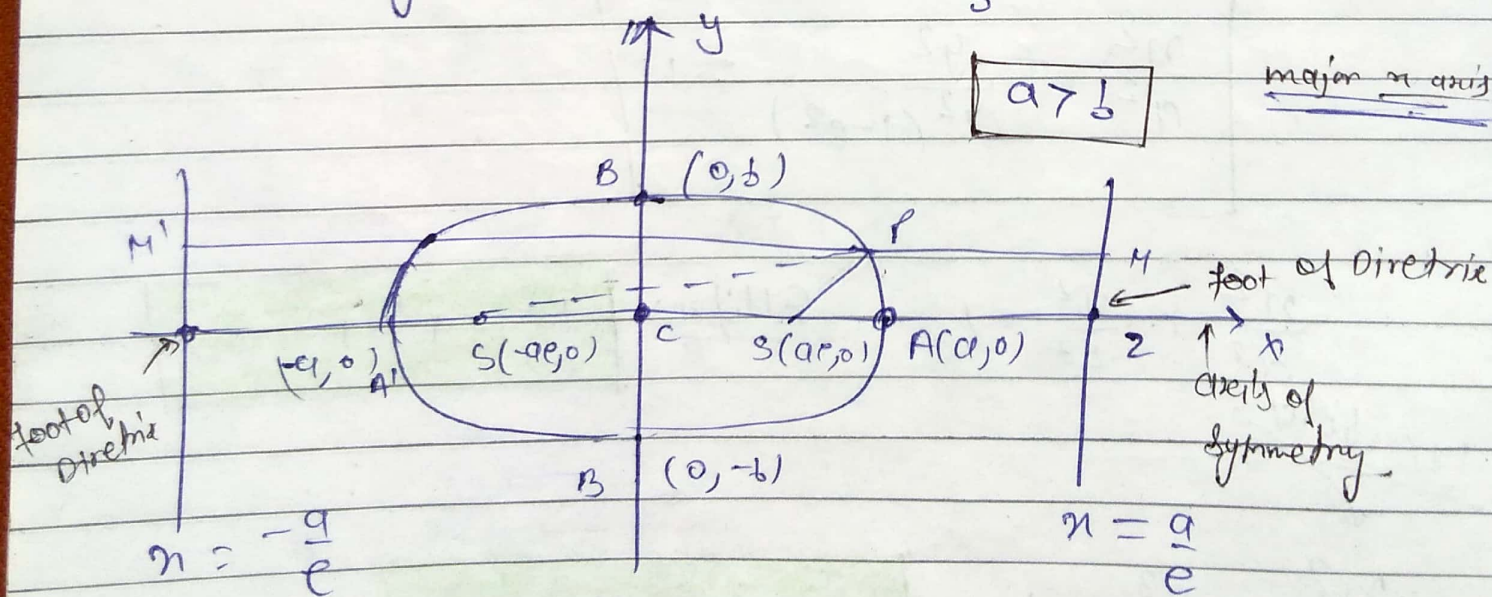
## \* Ellipse

$$\frac{y^2}{b^2} = 1 - \frac{x^2}{a^2}$$

$$y = \pm b \sqrt{1 - \frac{x^2}{a^2}}$$

for real  $y$ ,  $1 - \frac{x^2}{a^2} \geq 0 \Rightarrow -a \leq x \leq a$

Similarly for real  $x$ ;  $1 - \frac{y^2}{b^2} \geq 0 \Rightarrow -b \leq y \leq b$



$$\frac{PS}{PM} = e$$

$$b^2 = a^2(1 - e^2)$$



$$\frac{a}{e} + \frac{a}{e} = \frac{2a}{e}$$

$$\text{directi} = \frac{a}{e}$$

\* Dist. b/w two fixed line (directrix) =  $\frac{2a}{e}$

\* Dist. b/w two foci =  $2ae$

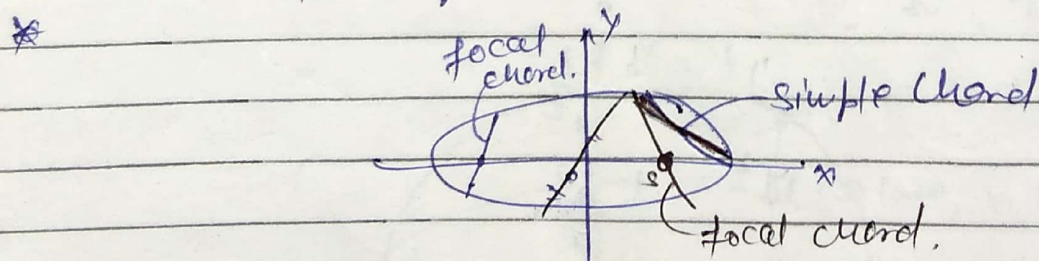
\* Line joining foci is called focal axis and point of intersection of curve with focal axis is called vertices of the ellipse.

$$(a, 0) \quad (-a, 0)$$

\*  $AA'$  is called major axis and its length is  $= 2a$ .

\*  $BB'$  is called minor axis and its length is  $2b$ .

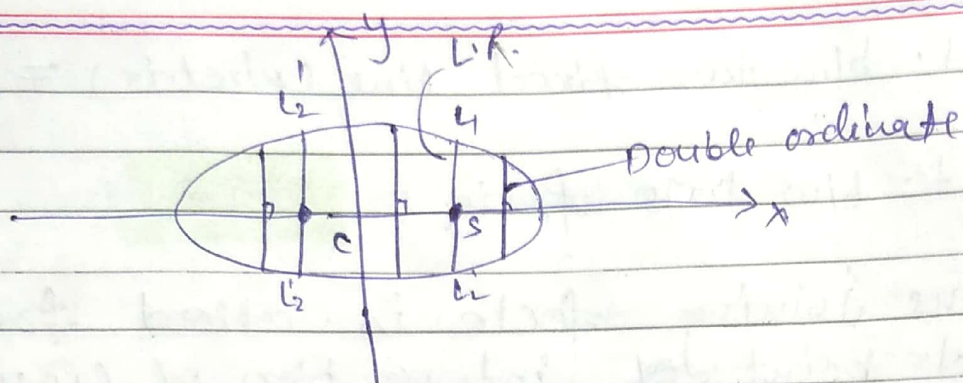
\* Major and minor axes together are called **Principal axes** of ellipse and intersection of these two axes is called centre of ellipse. All chord passes through centre of E/H get bisected at its centre.



\* Chord passes through its focus called focal chord.

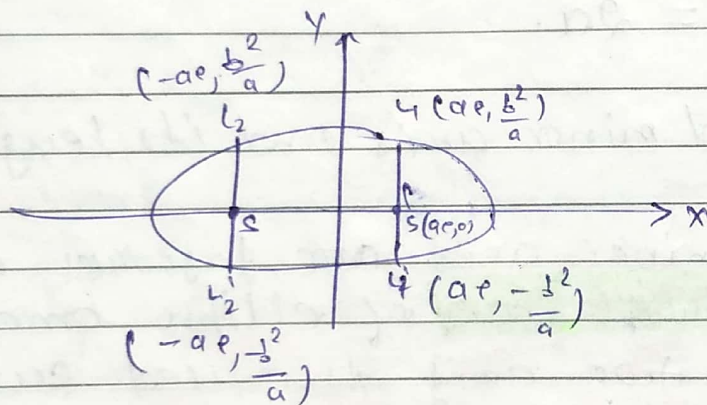
\*





\* chord  $\perp$  to its major axis is called its double ordinate.

\* when double ordinate passes through its focus then it is called Latus Rectum.



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{y^2}{b^2} = 1 - e^2$$

$$y^2 = b^2 \cdot \frac{b^2}{a^2}$$

$$y^2 = \pm \frac{b^2}{a}$$

\*  $L \cdot (L.R) = L_1 L_2$

$$= 2 \frac{b^2}{a}$$

$$= 2a \frac{b^2}{a^2}$$

$$= 2a(1 - e^2) = 2e \left( \frac{a}{e} - ae \right)$$



$$b^2 = a^2(1 - e^2)$$

$$L.R \Rightarrow \frac{2b^2}{a}$$

V.Dup

$$L(L.R) =$$

$$= 2e(cz - cs) = zs \cdot 2e$$

V.Dup.

$2e \times$  dist b/w correspond. focus and directrix

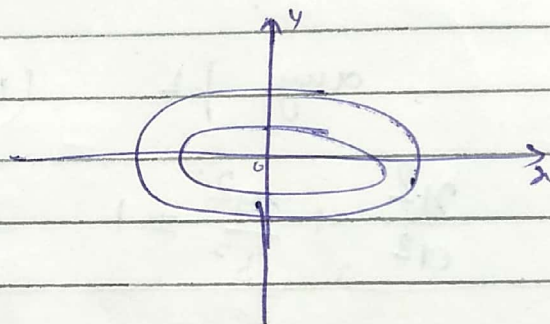
$$(2ex \frac{a}{e} - ae)$$

(Q21)

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

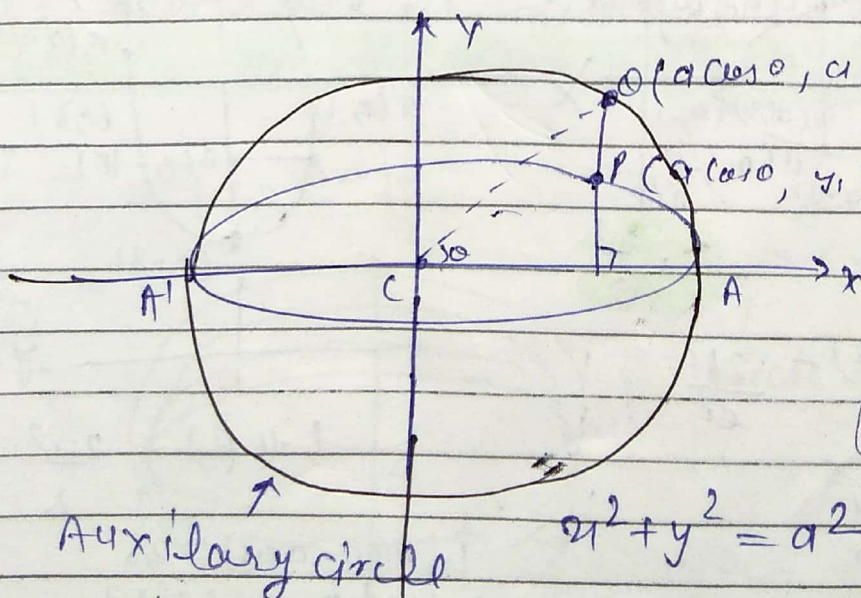
$$e = \frac{a}{b}$$

\* Two E/H are said to be similar if their  $e$  are same.



## \* Parametric Coordinate

Construct a circle concentric with given circle and whose radius = semi major axis of the ellipse



$$\frac{a^2 \cos^2 \theta}{a^2} + \frac{y^2}{b^2} = 1$$

$$y_1 = b \sin \theta$$

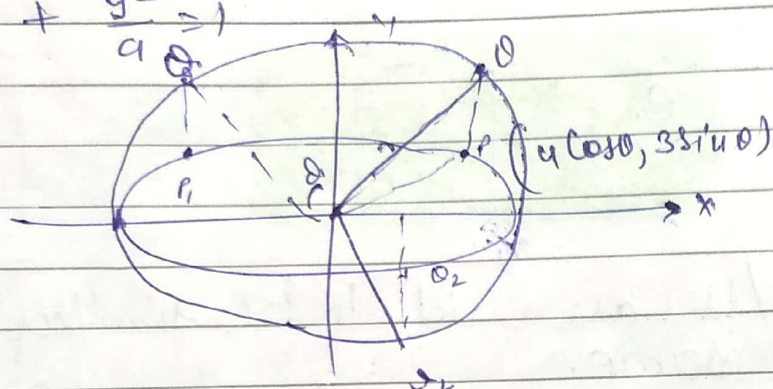
$$P(a \cos \theta, b \sin \theta)$$

where  $\theta$  is called eccentric angle of P



\* Let  $O$  be the Pt of the Auxilar Circle  
 Such that  $O$  be produce meet  $x$ -axis  
 then  $P$  and  $O$  are corresponding pt

$$\Rightarrow \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

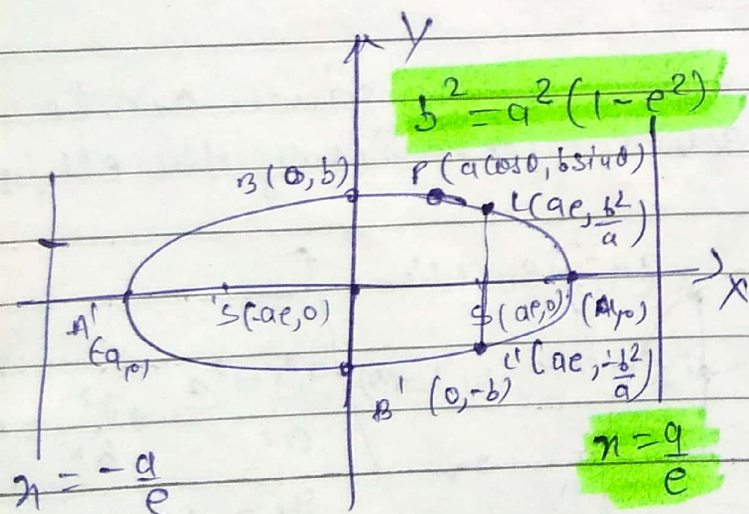


any pt.  $(a \cos \theta, b \sin \theta)$

$$* \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$a > b$$

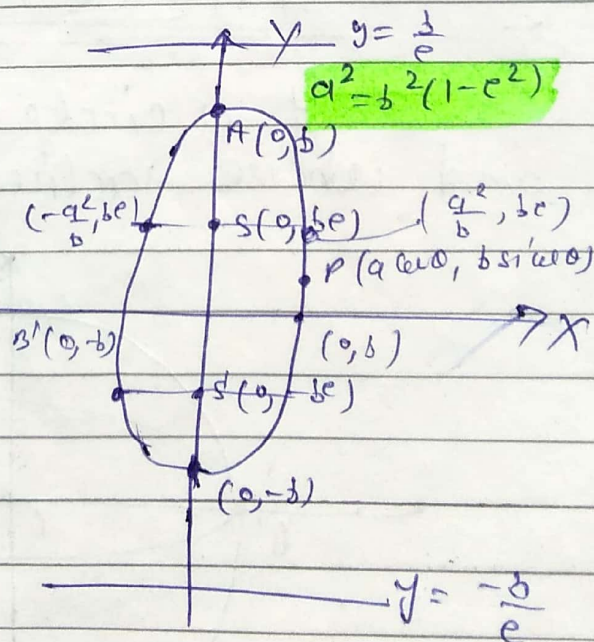
$$a < b$$



$$L(LR) = \frac{2b^2}{a}$$

$$L(\text{major axis}) = 2a$$

$$L(\text{minor axis}) = 2b$$



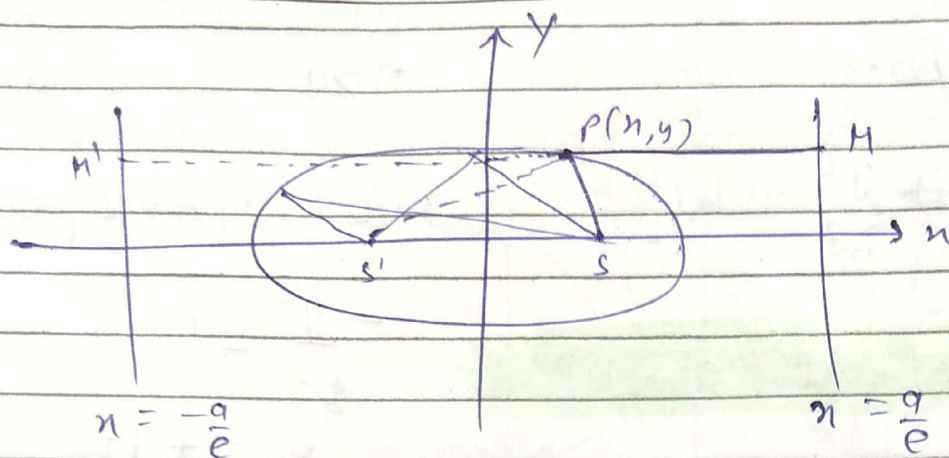
$$L(LR) = \frac{2a^2}{b}$$

$$L(\text{major axis}) = 2b$$

$$L(\text{minor axis}) = 2a$$



## Focal directrix property:



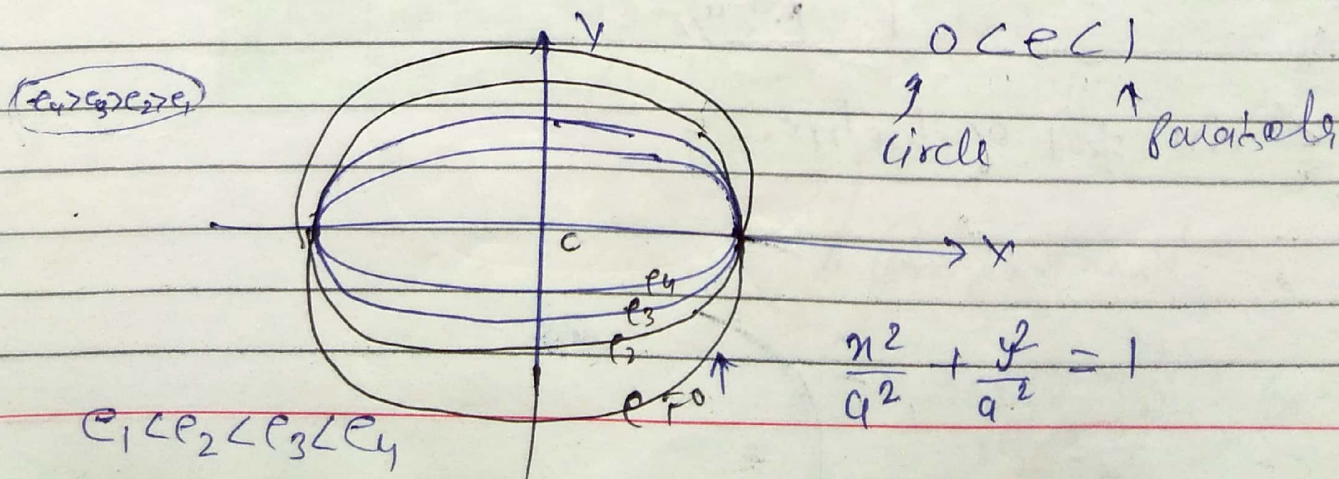
$$\begin{aligned} * \quad PS &= e PM \\ &= e \left( \frac{a}{e} - x \right) \end{aligned}$$

$$\begin{aligned} PS' &= e PM' \\ &= e \left( \frac{a}{e} + x \right) \end{aligned}$$

$$PS + PS' = 2a$$

Hence ellipse is locus of pt which moves such that sum of its distance from two fixed points (foci) is always const (i.e. major axis).

$$* \quad b^2 = a^2(1 - e^2)$$





$$\text{ellips} \Rightarrow b^2 = a^2(1 - e^2) \Rightarrow$$

$$\text{hyp} \Rightarrow b^2 = a^2(e^2 - 1)$$

## \* Hyperbola !

Same fact

$$e > 1$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

meet y-axis ( $x=0$ )

$$-\frac{y^2}{b^2} = 1$$

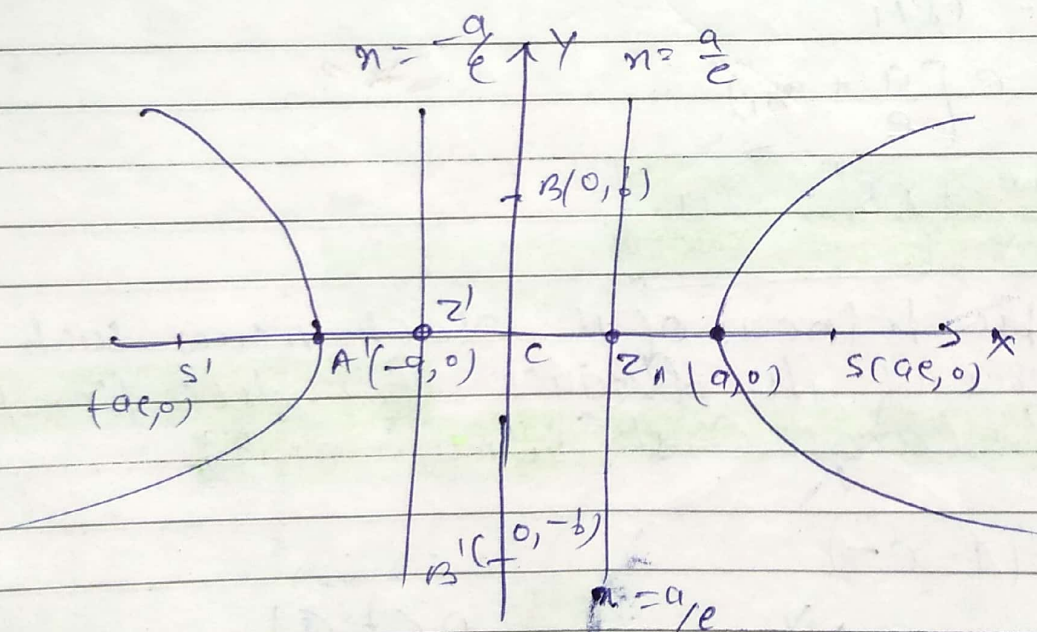
$$y = \pm ib$$

hyperbola

$$b^2 = a^2(e^2 - 1)$$

$$\frac{x^2}{a^2} = \left(1 + \frac{y^2}{b^2}\right)$$

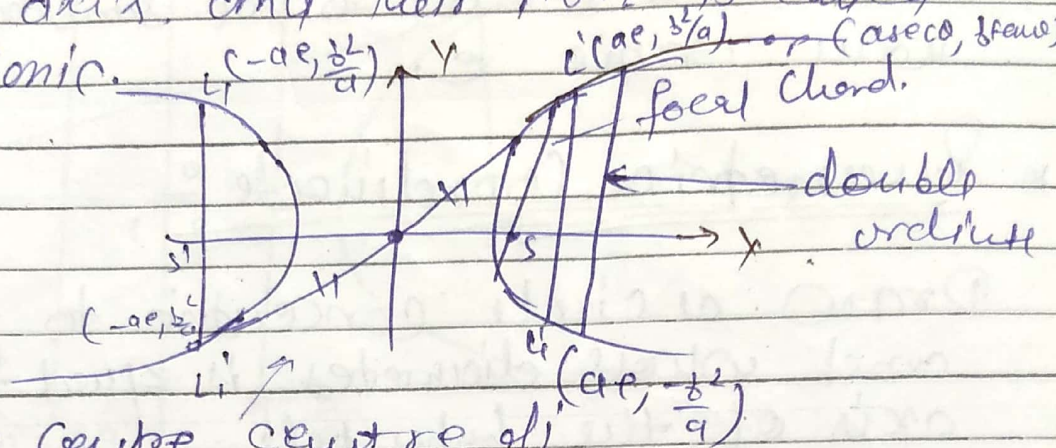
$$x = \pm a \sqrt{1 + \frac{y^2}{b^2}}$$



$Z$  &  $Z'$  are foot of directrix.



\*  $AA'$  is called Transverse axis.  $l(TA) = 2a$   
 and  $BB'$  is called Conjugate axis.  $l(CA) = 2b$ .  
 Together transverse or conjugate axis are called  
 Principle axis, and their P.O.C is called  
 centre of conic.



Chord passes centre centre of

\* dist. b/w both foci is called its  
 focal axis and dist. b/w them is  
 called its focal length. ( $L = 2ae$ )

\* Chord passes through its focus is called  
 its focal chord.

\* Chord  $\perp$  to its transverse axis is called  
 its double ordinate. and when double  
 ordinate passes through its focal then  
 it is called its latus rectum.

$$l(L \cdot R) = \frac{2b^2}{a}$$

$$= 2e \left[ \text{dist. b/w correspond. focus \& direction} \right]$$

V. Important

$$2ex \left( \frac{a}{e} - ae \right) = 2a - 2ae^2$$

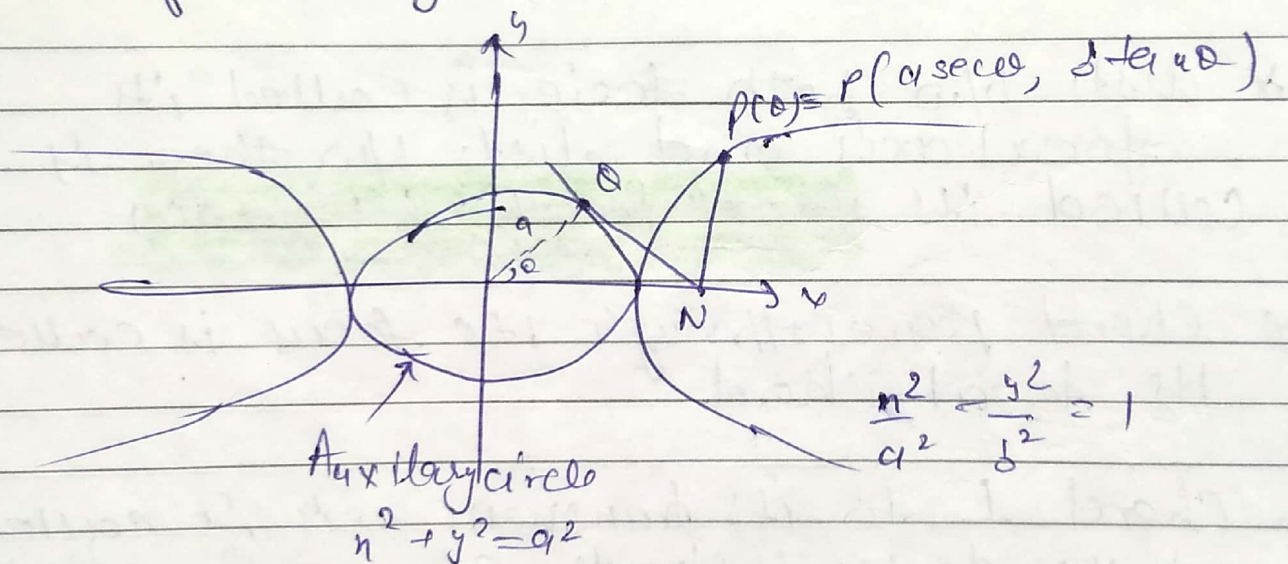


\* In case hyperbola, unlike ellipse.  
 $a > b$  or  $a < b$  is meaningless.

\* Two H are said to be similar if they have same  $e$ .

\* Parametric Coordinate :-

Draw a circle concentric to given hyperbola and whose diameter is equal to major axis of the hyperbola.



Now P & Q are the corresponding points.

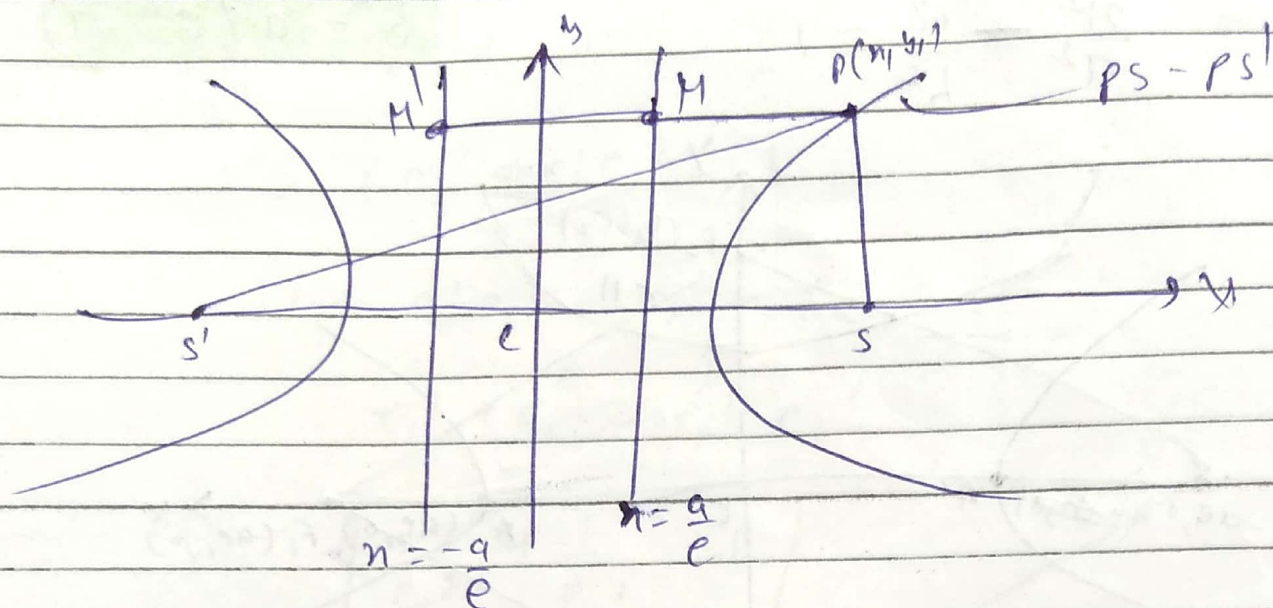
$$\cos \theta = \frac{a}{CN}$$

$$CN = a \sec \theta$$

$$\frac{a^2 \sec^2 \theta}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow y = b \tan \theta$$



# \* focal directrix Property of Hyperbola \*



$$PS = e PM = e \left( n_1 - \frac{a}{e} \right)$$

$$PS' = e PM' = e \left[ \frac{a}{e} + n_1 \right]$$

$$\boxed{PS' - PS = 2a}$$

Here Hyp. is locus of Pt. which moves such that diff. of its dist. from two fixed pt. (foci) is always const. (transverse axis).

$PS - PS'$  will give locus of this branch of hyperbola.

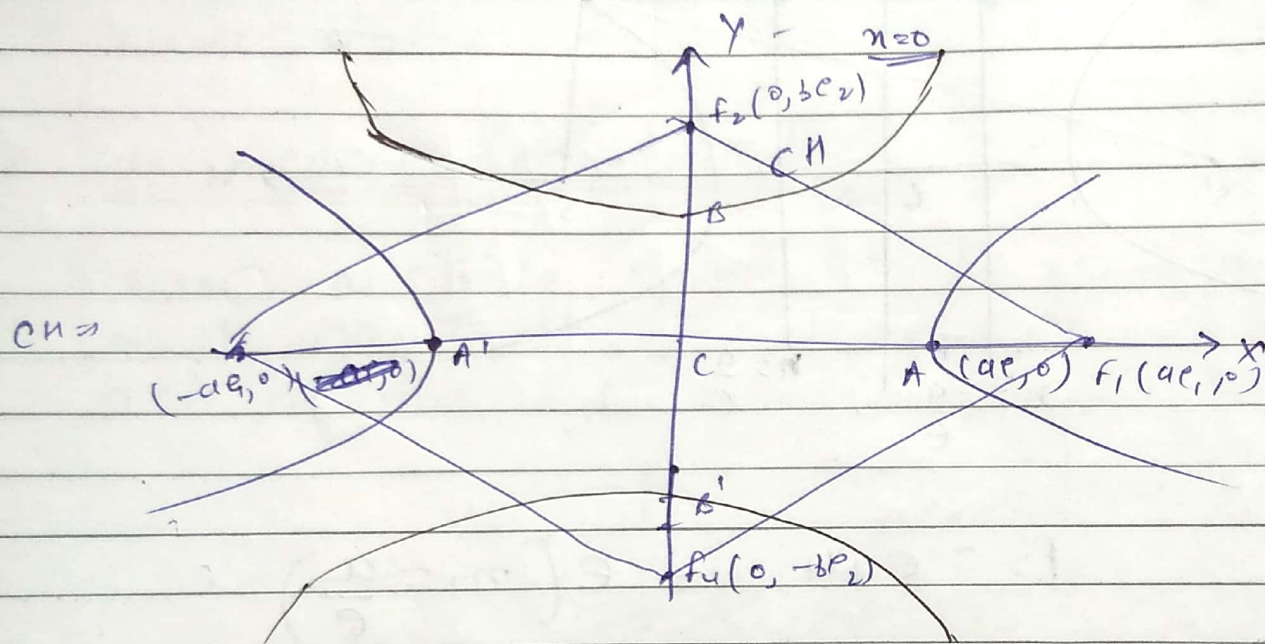
$|PS - PS'|$  will give both branch of hyperbola.



## \* Conjugate of Hyperbola!

$$\text{Hyp.} \rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$b^2 = a^2(e^2 - 1)$$



corresponding to every Hyp. there exist a hyperbola such that the conjugate A and TA of one is equal to TA and CA of other. Such Hyp. are known as Conj. to each other

$$\text{CH} \rightarrow \frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$$

or

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

$$\frac{x^2}{(-a)^2} - \frac{y^2}{(-b)^2} = 1$$

$$a^2 = b^2(e^2 - 1)$$



or

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

$$b^2 = a^2(e^2 - 1)$$

$$\frac{b^2}{a^2} = e_1^2 - 1 \quad (i)$$

$$\frac{a^2}{b^2} = e_2^2 - 1 \quad (ii)$$

$$1 = (e_1^2 - 1)(e_2^2 - 1)$$

$$1 = e_1^2 e_2^2 - e_1^2 - e_2^2 + 1$$

$$e_1^2 + e_2^2 = e_1^2 e_2^2$$

$$\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1$$

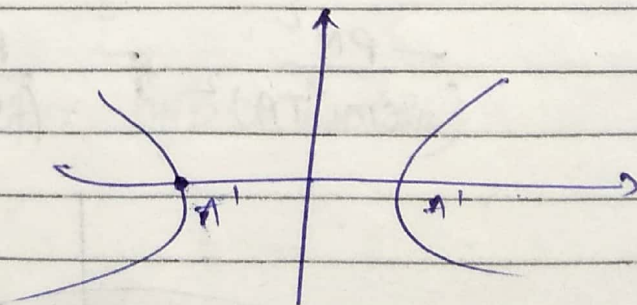
Note 1 foci of Hyp. and its conjugate are coincident and form vertices of a square.

i.e.  $f_1, f_2, f_3, f_4$  is a square.

\* Rectangular Hyp.

On Hyp. if  $a = b$  then it becomes rectangular Hyp.

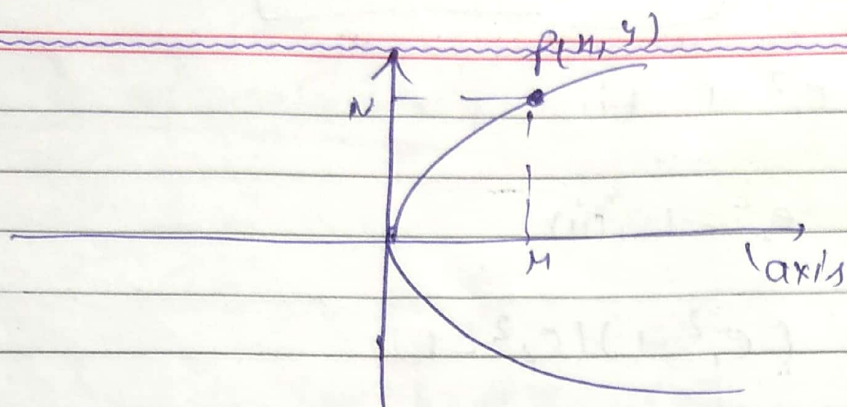
$$x^2 + y^2 = a^2$$



$$e = \sqrt{2}$$



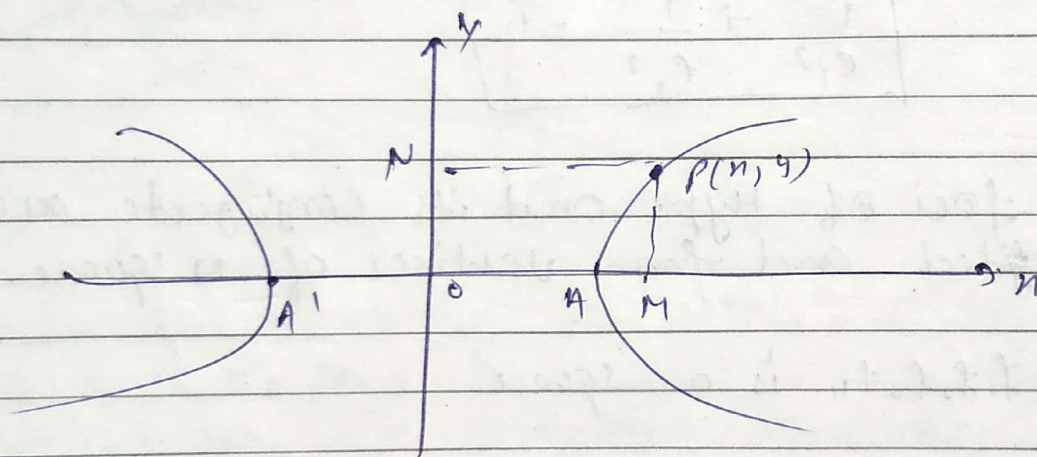
\*



$$y^2 = 4ax$$

$$PM^2 = L(L+R)PN$$

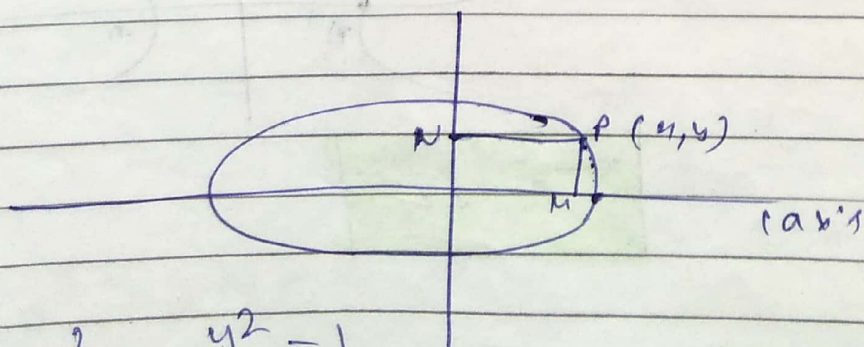
\*



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{PN^2}{(\text{semi} TA)^2} - \frac{PM^2}{(\text{semi} CA)^2} = 1$$

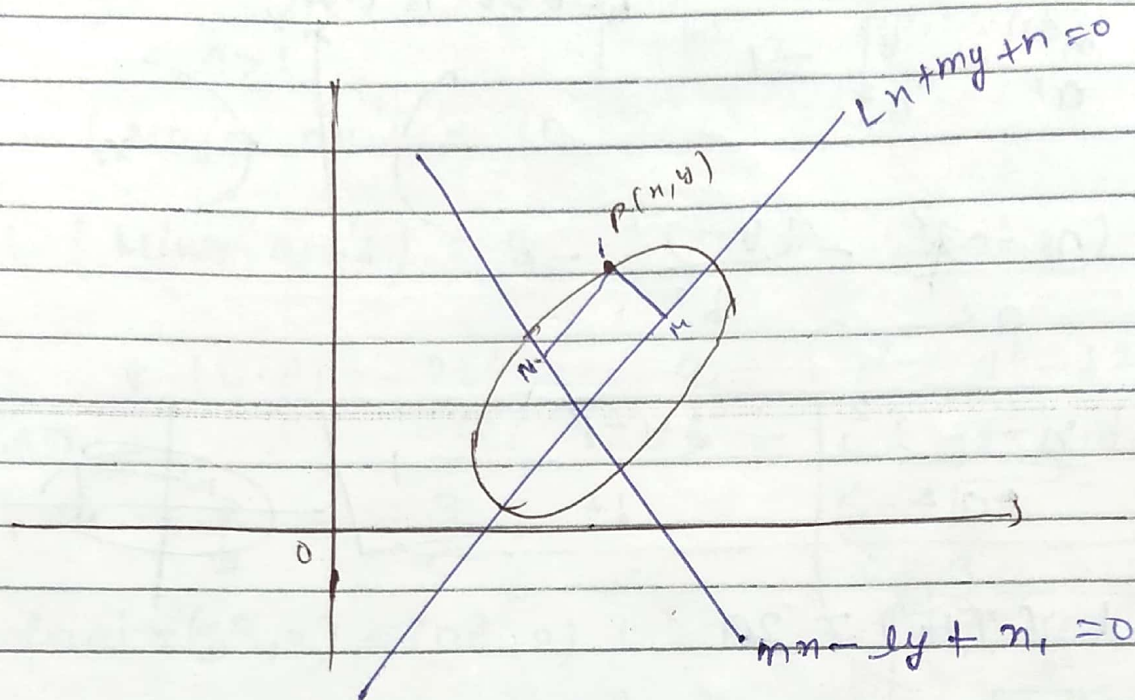
\*



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



$$\frac{(\text{Perp dist from } P \text{ to minor axis})^2}{(\text{semi MA})^2} + \frac{(\text{Perp dist from } P \text{ to major axis})^2}{(\text{semi major axis})^2} = 1$$



$$\frac{PN^2}{a^2} + \frac{PM^2}{b^2} = 1$$

$$\Rightarrow \frac{\left( \frac{mx - ly + n_1}{\sqrt{m^2 + l^2}} \right)^2}{a^2} + \frac{\left( \frac{Ln + my + n}{\sqrt{l^2 + m^2}} \right)^2}{b^2} = 1$$

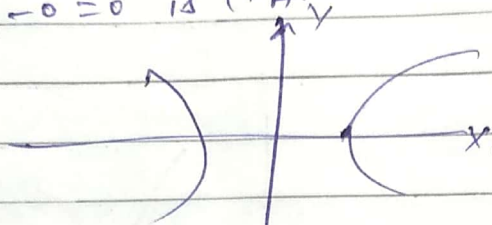
Similar type of eq<sup>n</sup> for hyperbola.

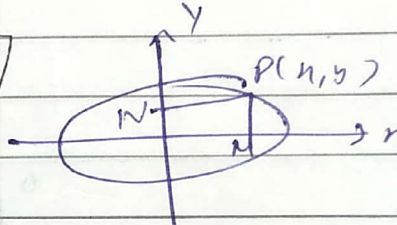


42

$$\Rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$x - 0 = 0$  is C.A. y

$$\frac{(x-0)^2}{a^2} - \frac{(y-0)^2}{b^2} = 1$$


$$\boxed{\frac{(x-\alpha)^2}{a^2} - \frac{(y-\beta)^2}{b^2} = 1}$$


$$L(TA) = 2a$$

eq<sup>n</sup> of TA is  $x - \alpha = 0 \Rightarrow x = \alpha$ .

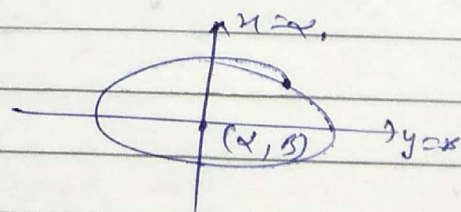
$$L(CA) = 2b$$

eq<sup>n</sup> of CA is  $y - \beta = 0 \Rightarrow y = \beta$

and centre of hyp. is  $x - \alpha = 0$  &  $y - \beta = 0$   
i.e.  $(\alpha, \beta)$ .

$$\ast \frac{(x-\alpha)^2}{a^2} + \frac{(y-\beta)^2}{b^2} = 1$$

$a > b$



\* eq<sup>n</sup> of minor axis is  $x - \alpha = 0 \Rightarrow x = \alpha$   
eq<sup>n</sup> of Major axis is  $y - \beta = 0 \Rightarrow y = \beta$ .



e for hyp:  $e = \sqrt{1 + \frac{b^2}{a^2}}$

Que: find everything.

$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$

$a=5$   
 $b=4$

$a > b$

L (Major axis) = 10

L (Minor axis) = 8

L (L.R) =  $\frac{2b^2}{a} = \frac{32}{5}$

$e = \frac{c}{a} = \frac{3}{5}$

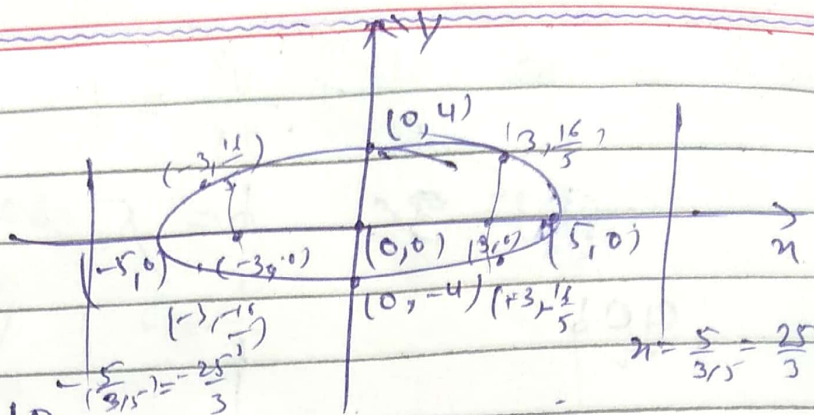
foci =  $(\pm 3, 0) = (ae, 0)$

$b^2 = a^2(1-e^2) \Rightarrow 16 = 25(1-e^2)$

$e = \frac{3}{5}$

A.A =  $y=0$

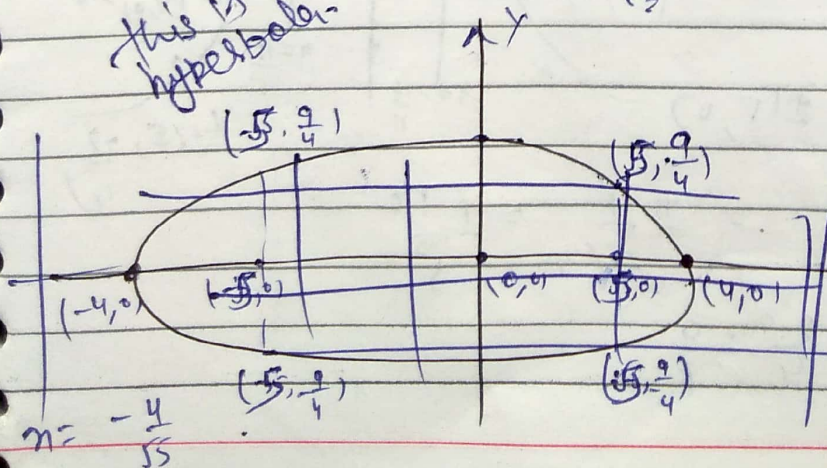
A =  $x=0$



$c^2 = a^2 - b^2$   
 $c = \sqrt{25 - 16}$   
 $c = \sqrt{9}$   
 $c = 3$   
 $e = \sqrt{1 - \frac{b^2}{a^2}}$   
 $= \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$

Que: find  $\frac{x^2}{16} + \frac{y^2}{9} = 1$

this is hyperbola



$a=4, b=3$   $c^2 = a^2 + b^2$

L (L.R) =  $\frac{16}{4} = 4$

$e = \frac{5}{4}$

$e = \sqrt{1 + \frac{b^2}{a^2}}$

$a = \frac{4}{5}$



$$x^2 + \frac{y^2}{25} = 1$$

9/3/

$$b \equiv s$$

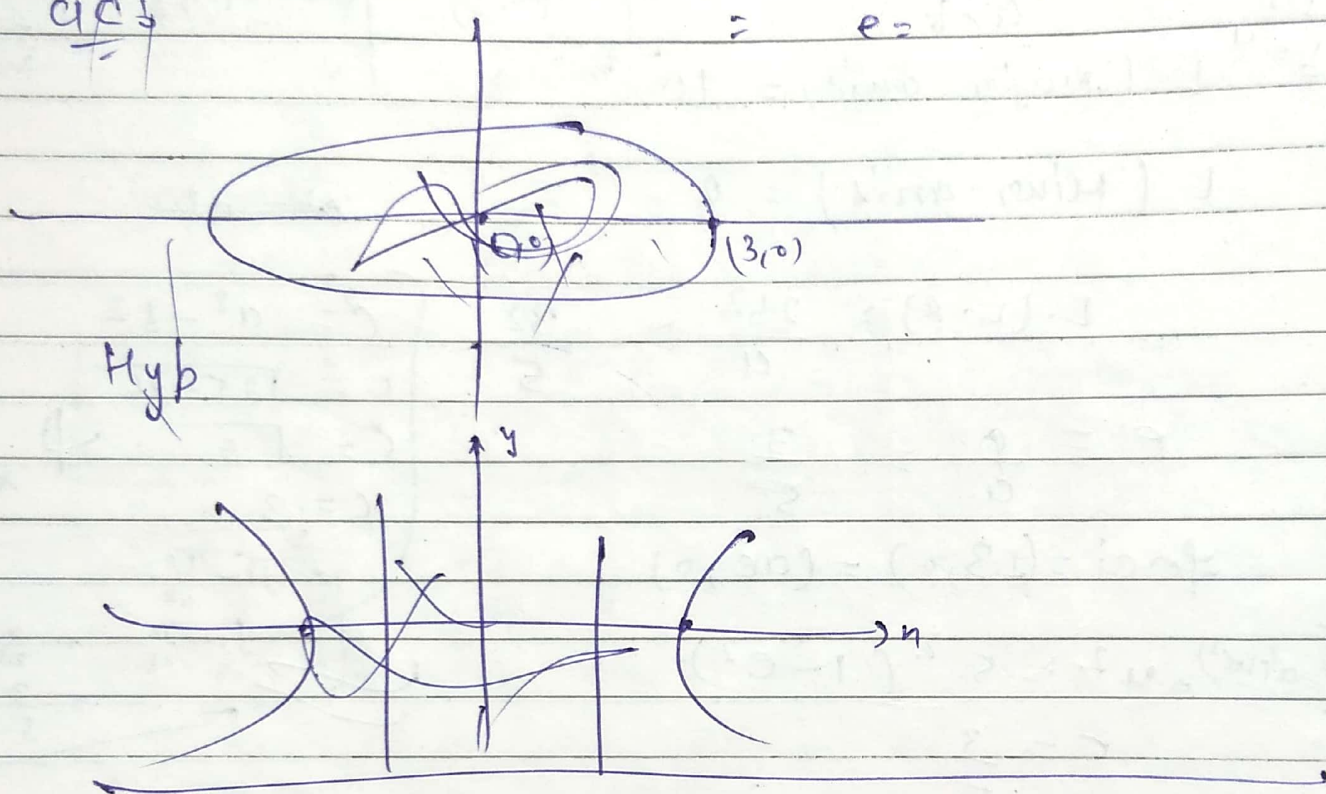
$$L(M, A) = 110$$

$$L(\mathbb{R}, A) = 16.$$

key

903

79 e2



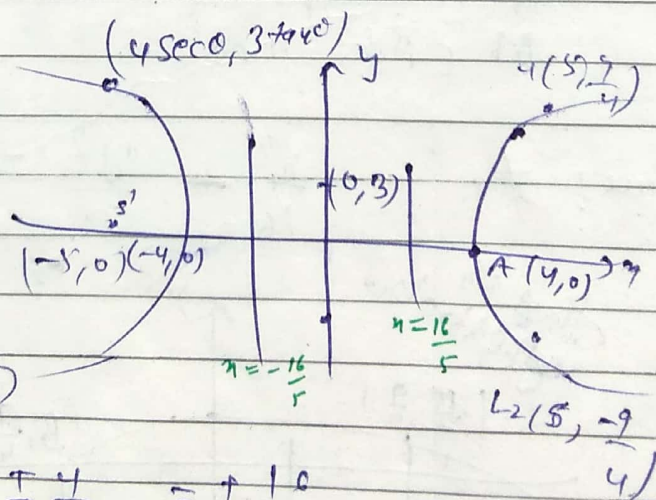
1

$$a = 4$$

$$b = 3$$

$$e = \int 1 + \frac{b^2}{a^2}$$

$$S(\pm a\rho, 0) \cong \pm(r, 0)$$



$$\eta = \frac{1}{e} \frac{q}{\sqrt{4}} = \frac{1}{5}$$

$$C. (L^R) = \frac{2 \cdot 2}{1} = \frac{2 \cdot 9}{4} = \frac{9}{2}$$



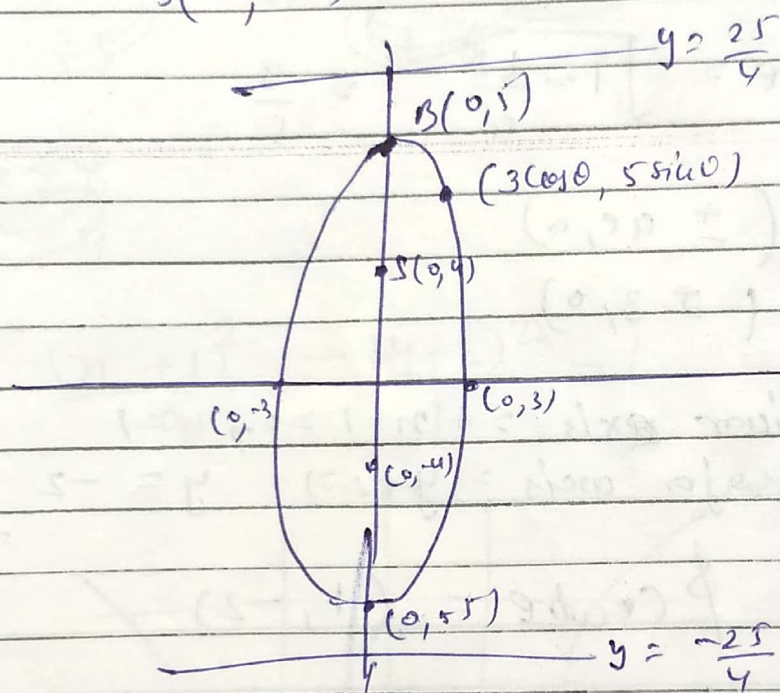
$$\textcircled{2} \quad \frac{x^2}{9} + \frac{y^2}{25} = 1$$

$$a = 3, b = 5, a < b$$

$$a^2 = 25(1 - e^2) \Rightarrow e = \frac{4}{5}$$

$$S(0, \pm b)$$

$$S(0, \pm 5)$$



$$L = (1 - e)$$

$$= \frac{2a^2}{b}$$

$$= \frac{2 \cdot 9}{5}$$

$$= \frac{18}{5}$$

$$\text{eqn of Dir} = y \pm \frac{b}{e}$$

$$\pm \frac{5}{4/5} = \pm \frac{25}{4}$$



minor 2 B/B

major = small 8

Ques! find everything.

$$\frac{(x-1)^2}{25} + \frac{(y+2)^2}{1} = 1$$

$a = 5$

$$b = 4$$

a, b

$$e = \sqrt{1 - \frac{b^2}{a^2}} = \frac{3}{5}$$

$$S(\pm ae, 0)$$

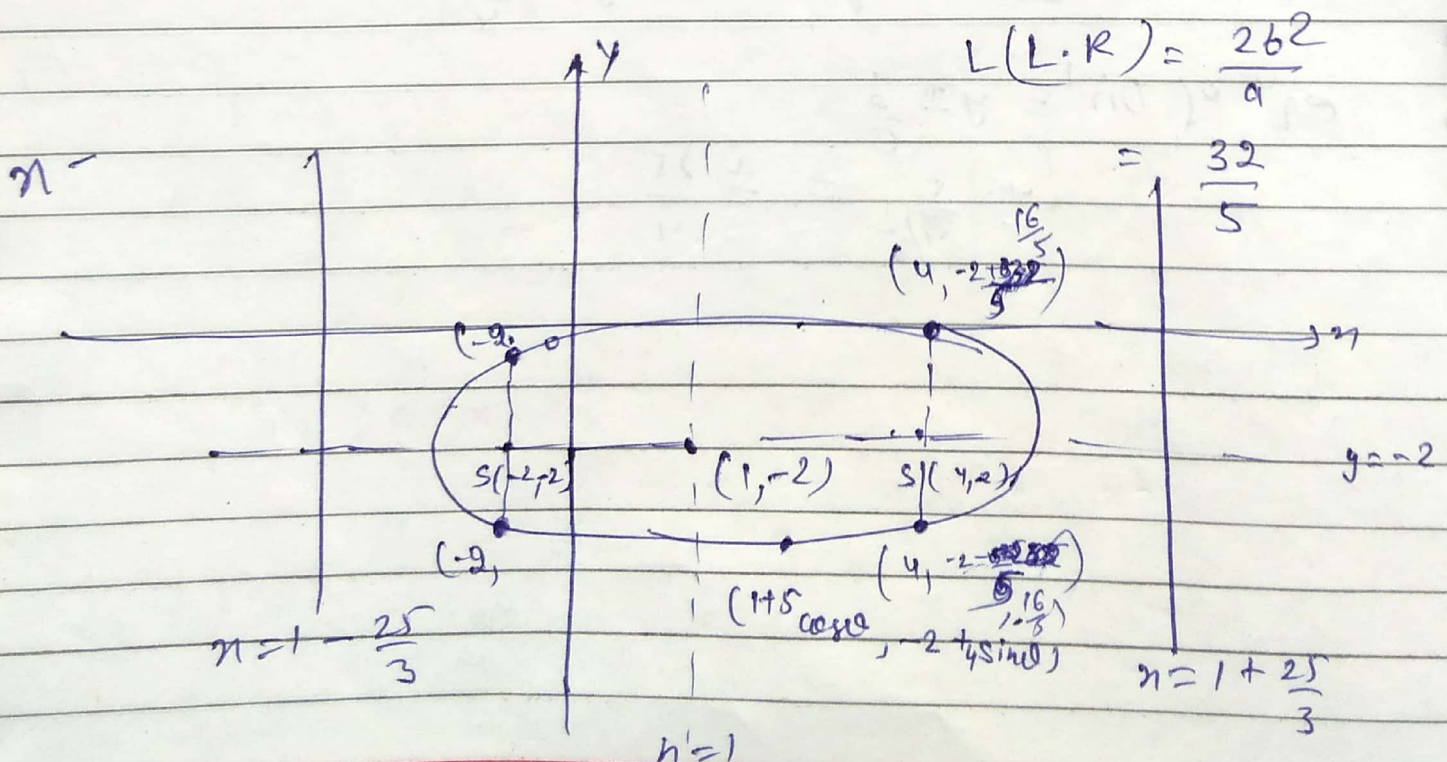
$$(\pm 3, 0)$$

$$(\pm 3, 0)$$

eg<sup>4</sup> of minor axis =  $n-1 \geq 0, n \geq 1$

Major axis =  $y+1 \Rightarrow y = -2$

\$centre = (1, -2)\$





$y \parallel$  to axis? direction  $\left( -n = \frac{a}{e} \right)$

eq<sup>n</sup> of dir  $n = \frac{a}{e} = \frac{5}{3/5} = \frac{25}{3}$

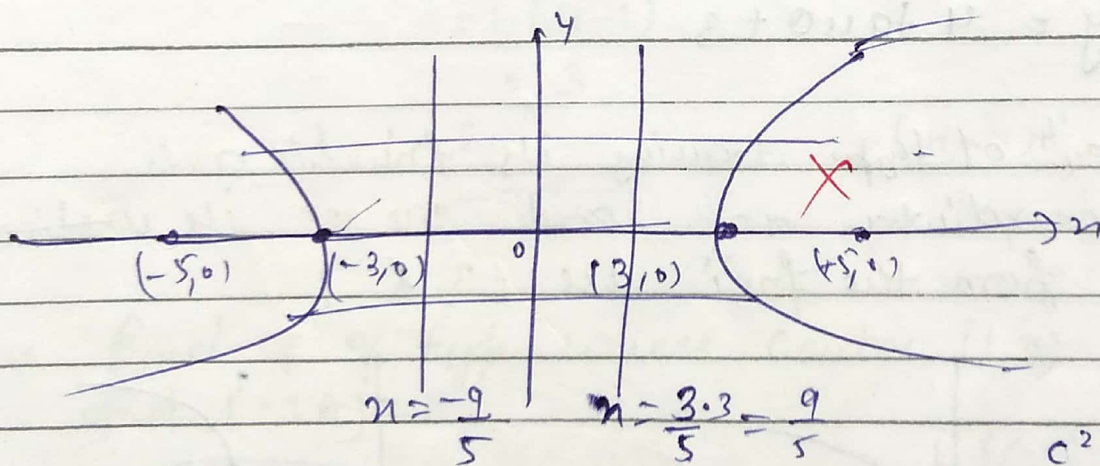
$n-1 = 5 \cos \theta$

$y+2 = 4 \sin \theta$

Q.  $\frac{(x-1)^2}{25} + \frac{(y+2)^2}{16} = 1$

$a=5$   
 $b=4$

Que!  $\frac{(x+1)^2}{9} - \frac{(y-3)^2}{16} = 1$   $a=3$   $b=4$



$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{16}{9}}$

$e = \frac{5}{3}$   $(\pm 5, 0)$

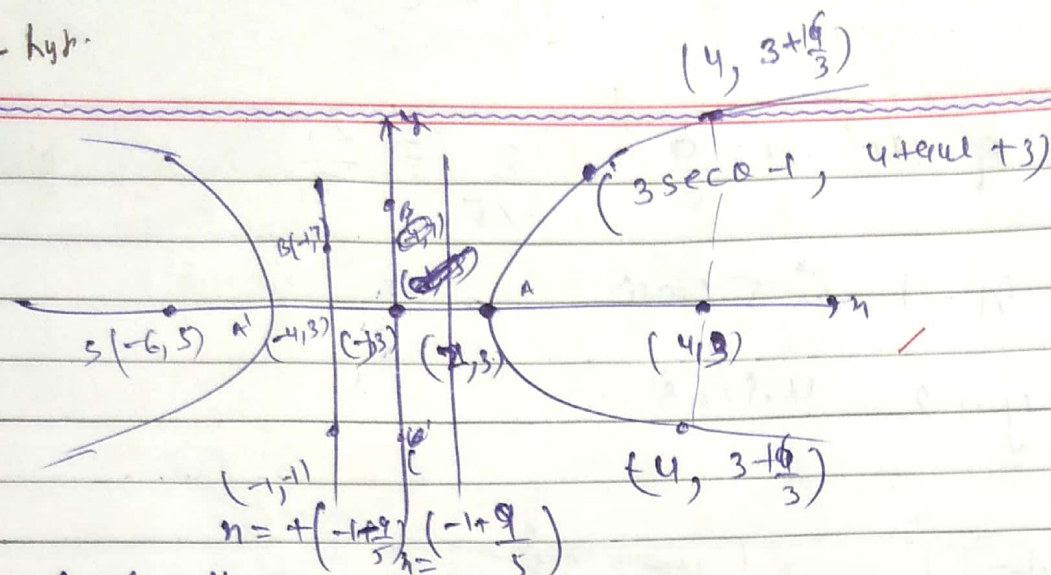
$c^2 = a^2 + b^2$   
 $c^2 = 9 + 16$   
 $c = 5$

TA.  
eq<sup>n</sup> of ~~minor~~ axis =  $y-3=0$   $y=3$   
eq<sup>n</sup> of ~~major~~ axis =  $x+1=0$   $x=-1$   
CA  
 $(-1, 3)$



- hydr.

Q.  $\frac{0-1}{1}$  : hyp. 1,  
ellips = 3,



eq<sup>n</sup> of d'

$$C \cdot (L \cdot R) = \frac{262}{9}$$

$$\begin{array}{r} 2 \quad 32 \\ \hline 9 \end{array}$$

$$n+1 = 3 \text{ sec}$$

$$y - 3 = 4 \tan \theta$$

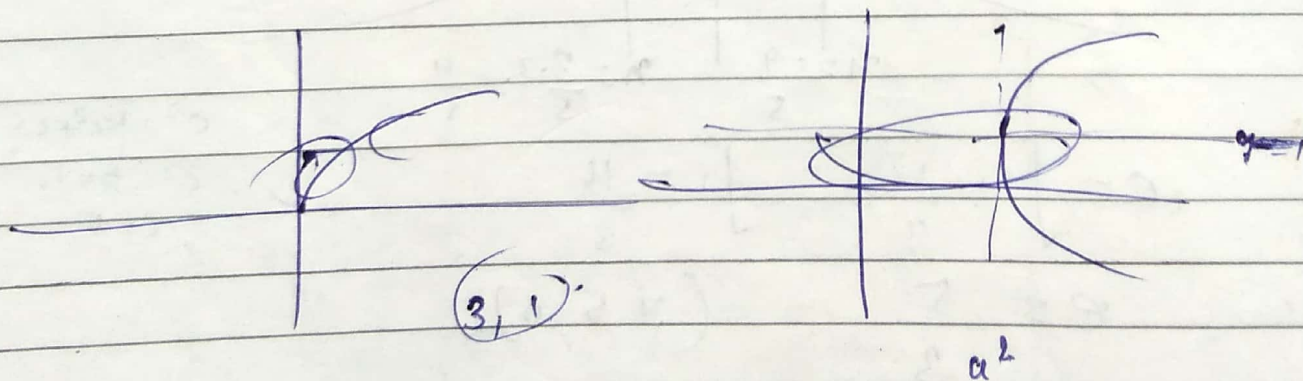
$$TA = 6$$

$$n = 3 \sec \theta - 1$$

$$CA = 8$$

$$y = 4 \tan \theta + 3$$

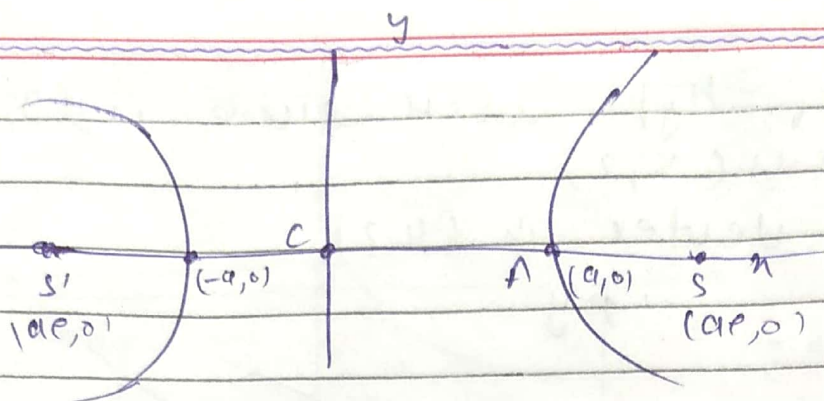
Ques: find eq<sup>n</sup> of hyp. having its principle axis  
its coordinate axis and one of its vertices  
Vertex from the foci are  $(3, \pm 1)$ .



5.  $\log_2 8 = 3$  (23)



Sol:



$$\begin{aligned} AS &= 1 = ae - a & \text{--- (i)} \\ AS' &= 3 = a + ae & \text{--- (ii)} \end{aligned}$$

$$2ae = 4$$

$$ae = 2$$

$$e^2 = 2 - 1$$

$$2a = 2$$

$$a = 1$$

$$b^2 = a^2(e^2 - 1)$$

$$= 1(4 - 1)$$

$$= 3$$

$$\frac{x^2}{1} - \frac{y^2}{3} = 1$$

$$\text{or } \frac{(x-0)^2}{1} - \frac{(y-0)^2}{3} = 1$$

Der find eq<sup>n</sup> of hyp. whose centre (1, 0) focus (6, 0) and L(A) = 6

$$a = 3$$

$$c = 6$$

$$c^2 = a^2 + b^2$$

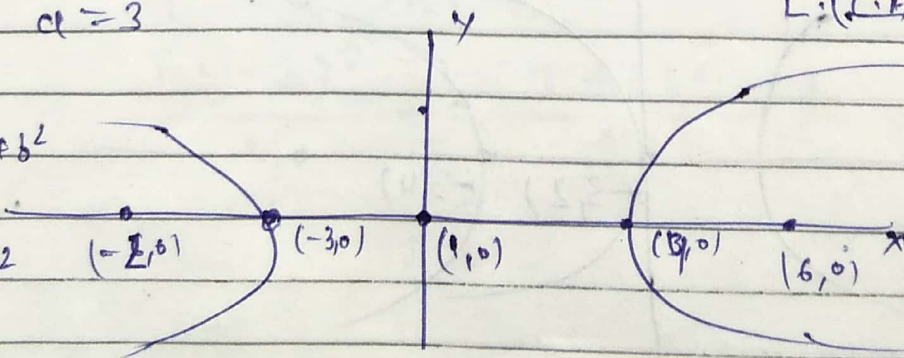
$$a +$$

$$6 - a = 12$$

$$b^2 = 12$$

$$b = 2\sqrt{3}$$

$$36 - 16$$



$$L(A) = 2\sqrt{27}$$

$$ae = \frac{5}{3}$$

$$e = \frac{5}{3}$$

$$\frac{(x-1)^2}{9} - \frac{(y-0)^2}{12} = 1$$

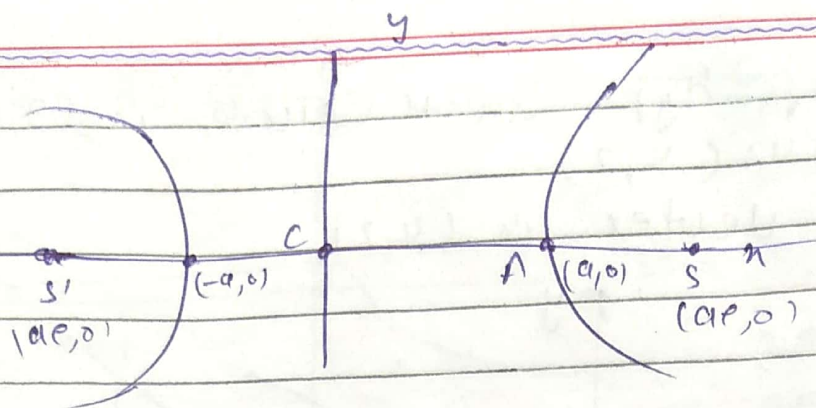
$$\frac{(x-1)^2}{9} - \frac{(y-0)^2}{16} = 1$$

$$cs = ae = 5$$

$$b^2 = a^2(e^2 - 1) = 9\left(\frac{25}{9} - 1\right) = 16$$



Q. 10/19



$$AS = 1 = ae - a \quad \text{--- (i)}$$

$$AS' = 3 = a + ae \quad \text{--- (ii)}$$

$$2ae = 4$$

$$ae = 2$$

$$e^2 = 2 - 1$$

$$2a = 2$$

$$a = 1$$

$$b^2 = a^2(e^2 - 1)$$

$$= 1(4 - 1)$$

$$= 3$$

$$\frac{x^2}{1} - \frac{y^2}{3} = 1$$

$$\text{or } \frac{(x-0)^2}{1} - \frac{(y-0)^2}{3} = 1$$

Ques. Find eq<sup>n</sup> of hyp. whose centre (1, 0) focus (6, 0)  
and L(A) = 6

$$a = 3$$

$$L(A) = 2\sqrt{27}$$

$$ae = \frac{5}{3}$$

$$e = \frac{5}{3}$$

$$c = 6$$

$$c^2 = a^2 + b^2$$

$$36 = a^2 + b^2$$

$$36 - 9 = 27$$

$$b^2 = 27$$

$$b = 2\sqrt{3}$$

$$36 - 16$$

$$\frac{(x-1)^2}{9} - \frac{(y-0)^2}{27} = 1$$

$$\frac{(x-1)^2}{9} - \frac{(y-0)^2}{16} = 1$$

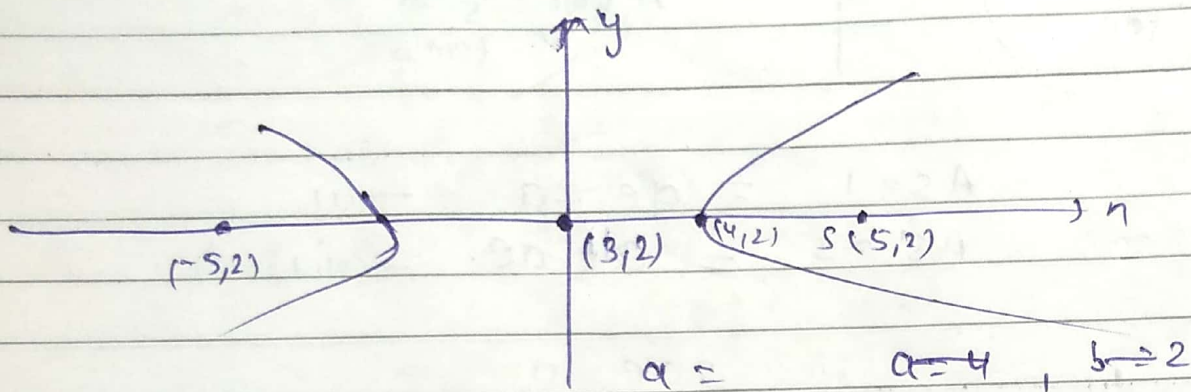
$$b^2 = a^2(e^2 - 1)$$

$$= 9\left(\frac{25}{9} - 1\right) = 16$$

$$cS = ae = 5$$



Q. Find eq<sup>n</sup> of Hyp. whose Centre is  $(3, 2)$   
 focus  $(5, 2)$   
 vertex is  $(4, 2)$



$$\frac{(x-3)^2}{1} - \frac{(y-2)^2}{9} = 1$$

$$e = 2$$

$$b^2 = a^2(e^2 - 1)$$

$$= 3$$

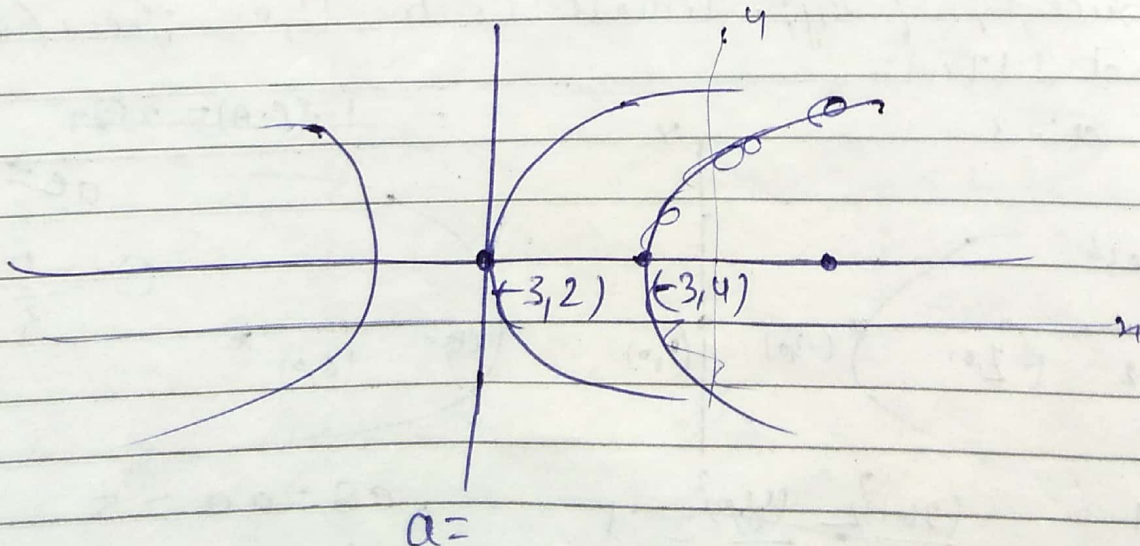
$$CA = a = 1$$

$$CS = ae = 2$$

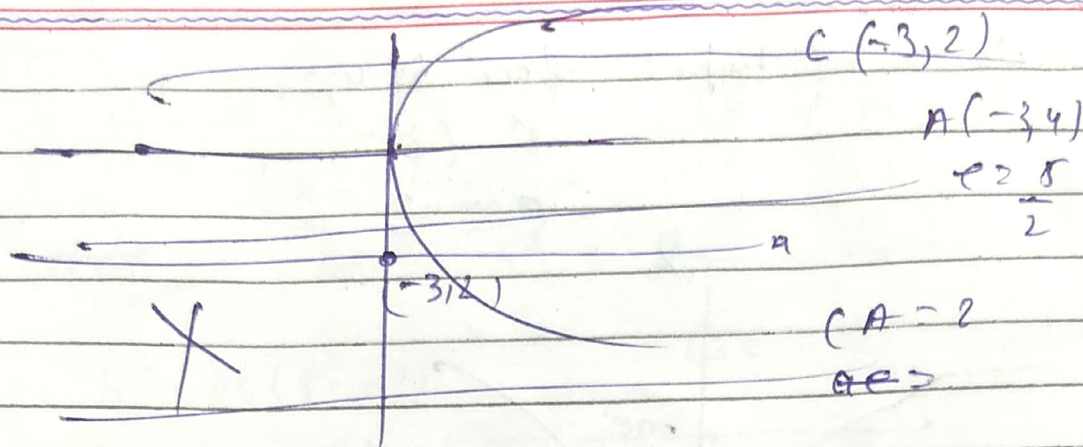
$$e = 2$$

Ques:

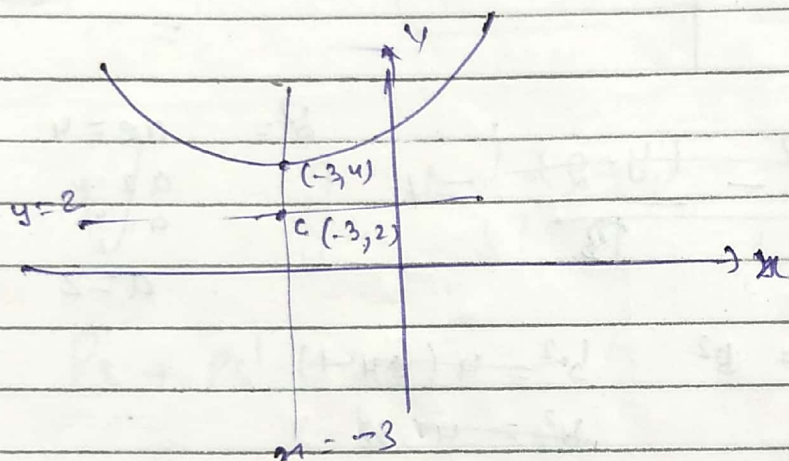
Find the eq<sup>n</sup> of hyp. centre  $(-3, 2)$   
 vertex  $(-3, 4)$   $e = \frac{5}{2}$







$$CA = 2$$



$$CA = 2 = a$$

$$C = (-3, 2)$$

$$A = (-3, 4)$$

$$e = \frac{5}{2}$$

$$b^2 = a^2(e^2 - 1)$$

$$b^2 = 21$$

$$\frac{y^2}{4} - \frac{x^2}{21} = 1$$

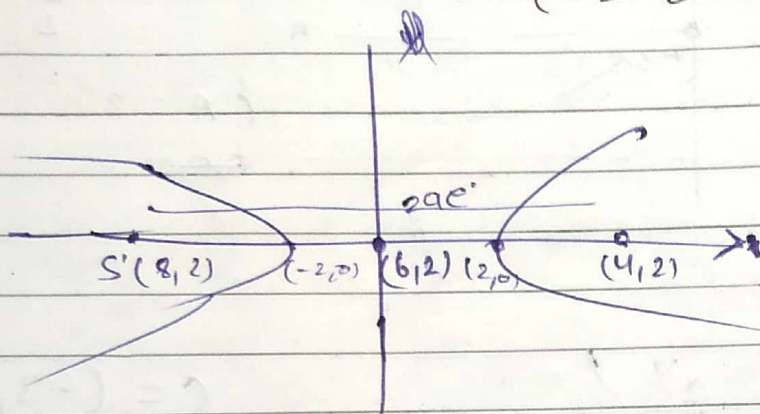
$$\frac{(y-2)^2}{4} - \frac{(x+3)^2}{21} = 1$$



Q find eq<sup>n</sup> of Hyp. foci: S(4,2)

$$S'(-8,2)$$

$$e = 2$$



$$ae =$$

$$=$$

$$\frac{(x-6)^2}{4} - \frac{(y-2)^2}{12} = 1$$

$$b^2 = ae = 4$$

$$a^2 = 4$$

$$a = \frac{4}{2}$$

$$a = 2$$

$$a^2 = b^2$$

$$b^2 = 4(4-1)$$

$$b^2 = 4(3)$$

$$b^2 = 12$$

$$SS' = 2ae = 4 \Rightarrow ae = 2 \quad C(6,2)$$

$$a = 1$$

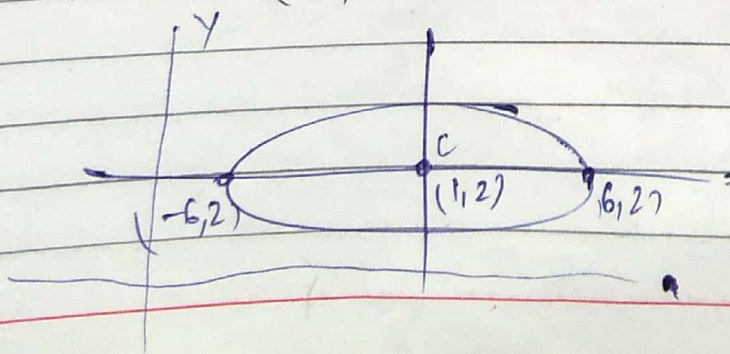
$$b^2 = 3$$

$$\frac{(x-6)^2}{1} - \frac{(y-2)^2}{3} = 1$$

Ques: find eq<sup>n</sup> of ellipse with its centre (1,2) and passes through (4,6)

$$S(6,2)$$

and passes through (4,6)



$$\frac{16}{a^2} + \frac{36}{b^2} = 1$$



$$ae = 6$$

$$\frac{(x-1)^2}{a^2} + \frac{(y-2)^2}{b^2} = 1$$

$$\frac{(4-1)^2}{a^2} + \frac{(6-2)^2}{b^2} = 1$$

$$ae = 5 \quad \text{--- (i)}$$

$$b^2 = 25(1 - e^2)$$

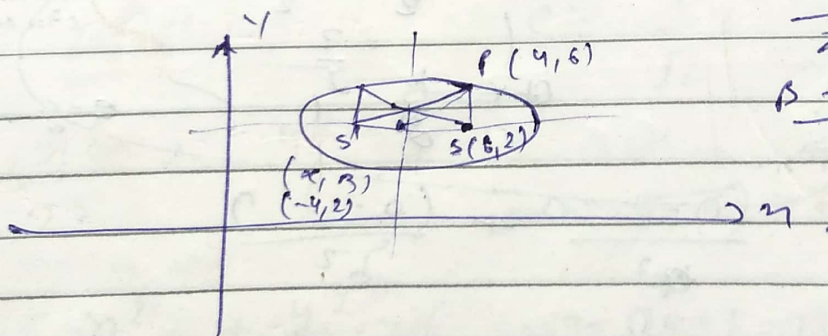
$$a^2 - b^2 = 25 \quad \text{--- (ii)}$$

$$e = 1 \quad \text{--- (iii)}$$

$$\frac{(4-1)^2}{a^2} + \frac{(6-2)^2}{b^2} = 1$$

$$\frac{9}{a^2} + \frac{16}{b^2} = 1$$

M-2  $PS + PS' = 2a$



$$\frac{x+6}{2} = 1 \Rightarrow x = -4$$

$$\frac{y+2}{2} = 2 \Rightarrow y = 2$$

$$PS + PS' = 2a$$

$$\sqrt{64+16} + \sqrt{16+4} = 2a$$

$$\sqrt{80} + \sqrt{20} = 2a$$

$$4\sqrt{5} + 2\sqrt{5} = 2a$$

$$a = 3\sqrt{5}$$

$$2S + 2S' = 2a$$

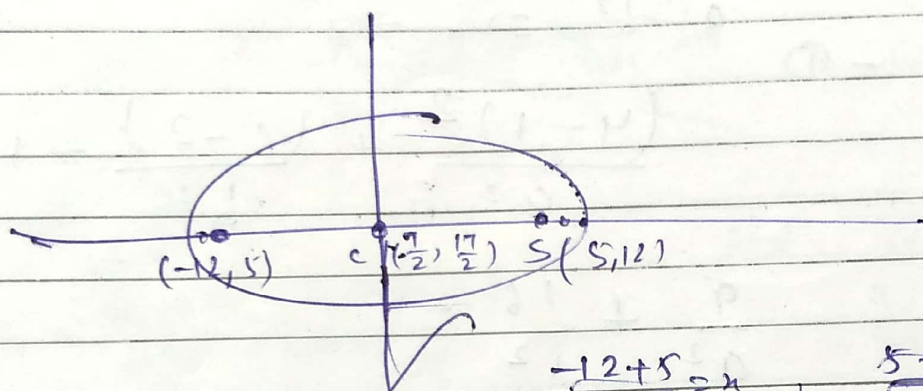
$$\Rightarrow \sqrt{(x+4)^2 + (y-2)^2} + \sqrt{(x-6)^2 + (y-2)^2} = 6\sqrt{5}$$



Q. Let  $S(5, 12)$

&  $S'(-12, 5)$

are the foci of an ellipse which passes through origin. find its  $e$ .



$$\begin{aligned} 2ae &= 17 \\ ae &= \frac{17}{2} \end{aligned}$$

$$\frac{-12+5}{2} = x, \quad \frac{5+12}{2} = y$$

$$\frac{7}{2} = x$$

$$\frac{17}{2} = y$$

$$ae = 7 \cdot \frac{5-7}{2}$$

$$ae = \frac{3}{2}$$

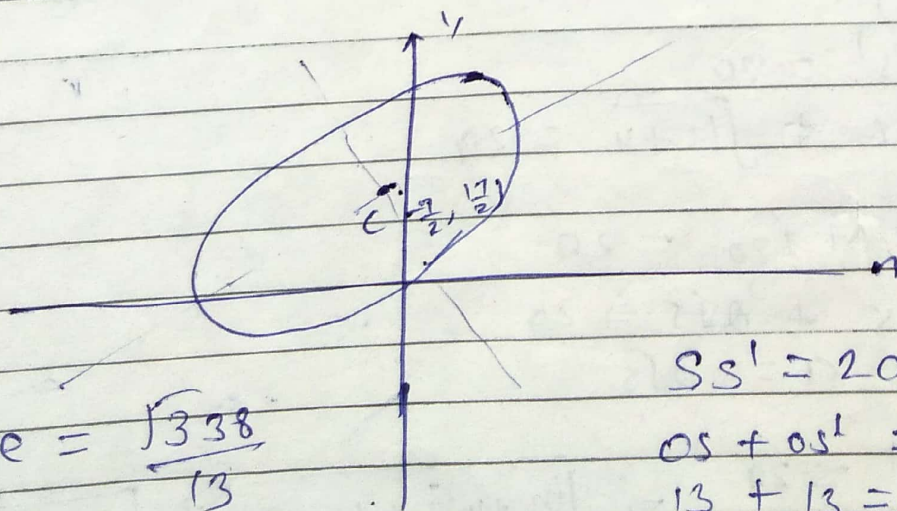
$$c^2 = a^2 - b^2$$

$$e = \frac{c}{a}$$

$$a = \frac{3}{2} + x$$

$$\frac{a^2}{a^2} + \frac{(a-x)^2}{b^2}$$

Ans



$$2e = \frac{\sqrt{338}}{13}$$

$$e = \frac{13\sqrt{2}}{262} = \frac{1}{\sqrt{2}}$$

$$SS' = 2ae = \sqrt{338} = 10$$

$$OS + OS' = 2a$$

$$13 + 13 = 2a$$

$$a = 13$$



Ques find  $e$  of the ellipse if the length of minor axis is equal to its focal length

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$b =$$

$$\text{let } c = a \quad a = \frac{2}{2}$$

$$c^2 = a^2 - b^2$$

$$2ae = 4b$$

$$e = \frac{2b}{2a}$$

$$e = \frac{b}{a}$$

$$a^2 = b^2(1 - e^2)$$

$$b^2 = a^2 - a^2 e^2$$

$$= a^2 - b^2$$

$$2b^2 = a^2$$

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$e = \frac{1}{\sqrt{2}}$$

$$b^2 = a^2 - a^2 e^2$$

$$b^2 = a^2 - b^2$$

Ques:  $\frac{x^2}{25} + \frac{y^2}{16} = 1$

$$a = 5, b = 4$$

$$e = \frac{3}{5}$$

$$e^2 = \frac{x^2}{25} + \frac{y^2}{16} = 1$$

$$a = \sqrt{24}, b = 7$$

$$e =$$

$$A = 4 \left( \frac{1}{2} \cdot 3 \cdot 5 \right)$$

find eqn of  $b$

find area of the fig

formed by the joining foci of these two ellipses

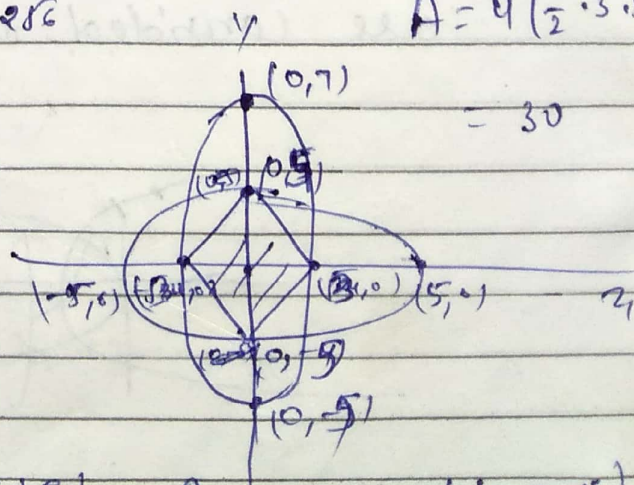
$$\sqrt{24} =$$

$$24 = 49(1 - e^2)$$

$$e = \frac{5}{7}$$

$$S_1(0, \pm 3e)$$

$$(0, \pm 5)$$



$$\text{Area} = 4 \left( \frac{1}{2} \cdot 3 \cdot 5 \right)$$

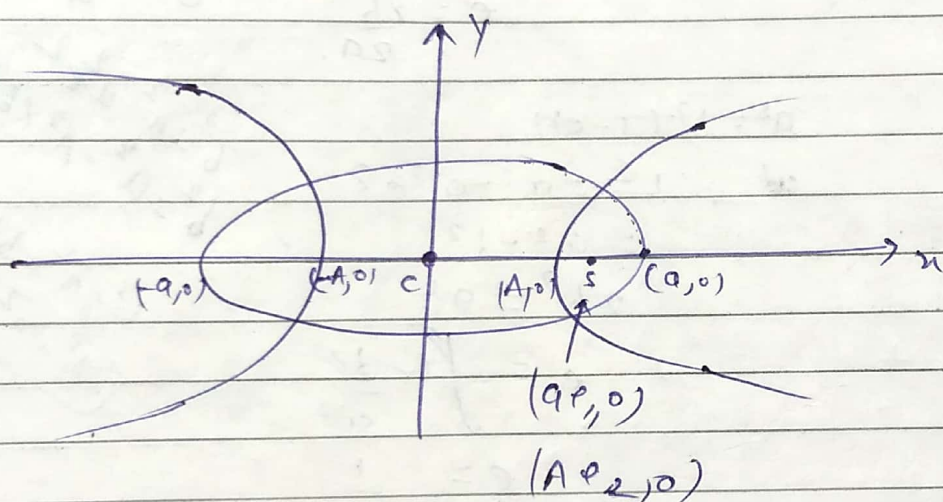
$$\frac{12}{6} \quad \frac{2 \times 24}{2 \times 2} \quad 2 \times 2$$



Note 1 \* An E and an Hgp. have same foci'  
(con focal)

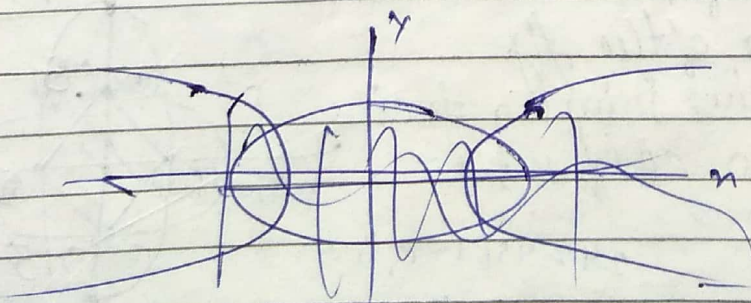
$$E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad b^2 = a^2(1 - e^2)$$

$$H: \frac{x^2}{A^2} - \frac{y^2}{B^2} = 1 \quad B^2 = A^2(e^2 - 1)$$

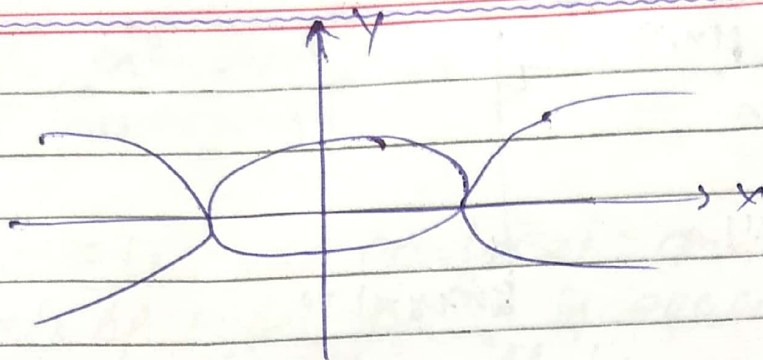


$$Ae_2 = ae_1$$

\* E and H are such that % their axes are considered and  $L(TA) = L(MA)$  length







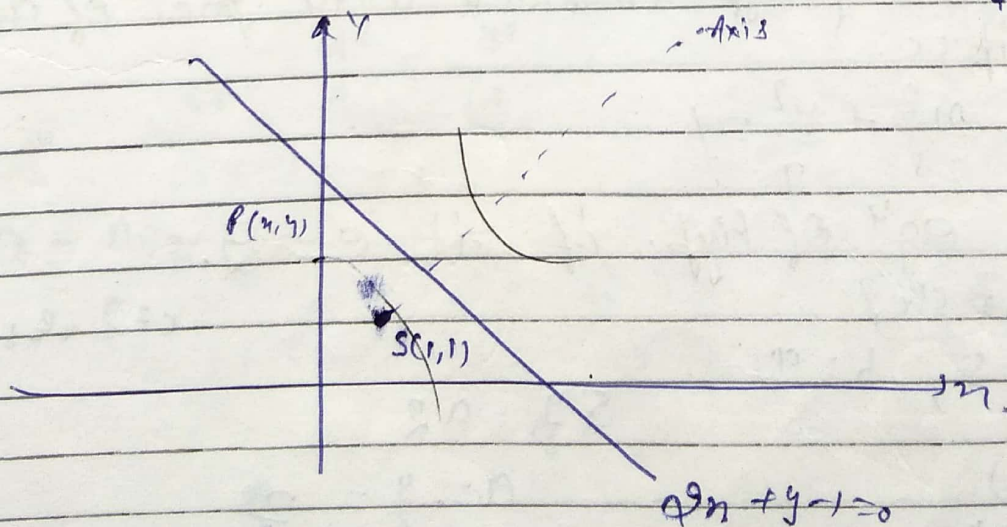
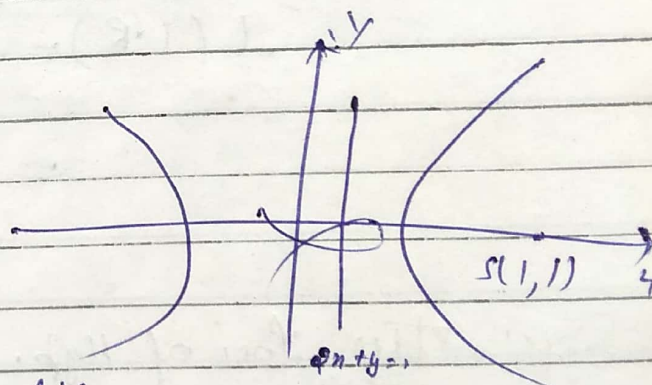
$$E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$H: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

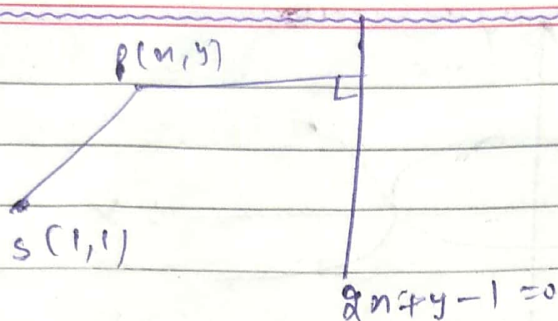
Ques! find eq<sup>n</sup> of hyp. whose direct<sup>n</sup> =  $2x + y = 1$   
 $S(1,1)$   $e = \sqrt{3}$

dir<sup>n</sup> =

$$2- = \sqrt{3} \left( \frac{2x + y - 1}{\sqrt{4 + 1}} \right)$$







$$PS^2 = e^2 PM^2$$

$$(x-1)^2 + (y-1)^2 = 3 \left( \frac{2x+y-1}{\sqrt{5}} \right)^2$$

∴ find eccn :

L (L.R) of H.

$$= \frac{2b^2}{a}$$

$$\frac{2b^2}{a}$$

$$L (L.R) = 2e(\text{dist b/w f and O})$$

$$= 2\sqrt{3} \cdot \frac{2}{\sqrt{5}} = \frac{4\sqrt{3}}{\sqrt{5}}$$

Solve The foci of Hyp. Considered with foci of an ellipse.

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

find eq<sup>n</sup> of Hyp. if it  $e = 2$ .

$$Ae = ae$$

$$c^2 = a^2 - b^2$$

$$a = 5, b = 3$$

$$5 \cdot \frac{y}{x} = a^2$$

$$a = \frac{y}{x} = 2$$

$$S(\pm 4, 0)$$

$$e = \frac{5}{3}$$

$$\text{eq<sup>n</sup>, } b^2 = 4(4-1)$$

$$b^2 = 4(3) = 12$$



$a, b, e$

elliptic = 1, 2, 3, 4, 5

$J-K = 1, 2, 3, 4$

$Hyp = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$

$J-H = 1, 2$

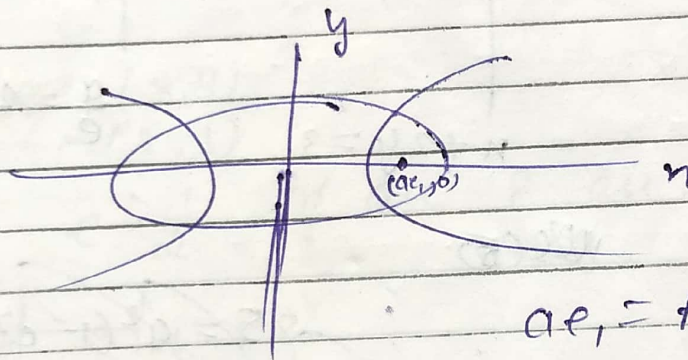
$$\frac{x^2}{4} - \frac{y^2}{12} = 1$$

Ques: An  $E/H$  are confocal and conjugate  
axes of hyperbola is equal to the  
minor axis of the ellipse.  
if  $e_1$  and  $e_2$  are the  $e$  of the  $E$  and  
H.

then prove that

$$\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1 = 2. \text{ Ans}$$

foci =



$$ae_1 = Ae_2$$

MA = Min A.

$$e_1 = \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$e_2 = \frac{x^2}{A^2} - \frac{y^2}{b^2} = 1$$

$$b^2 = a^2(1 - e_1^2)$$

$$b^2 = A^2(e_2^2 - 1)$$

divide

$$a^4(1 - e_1^2) = A^2(e_2^2 - 1)$$

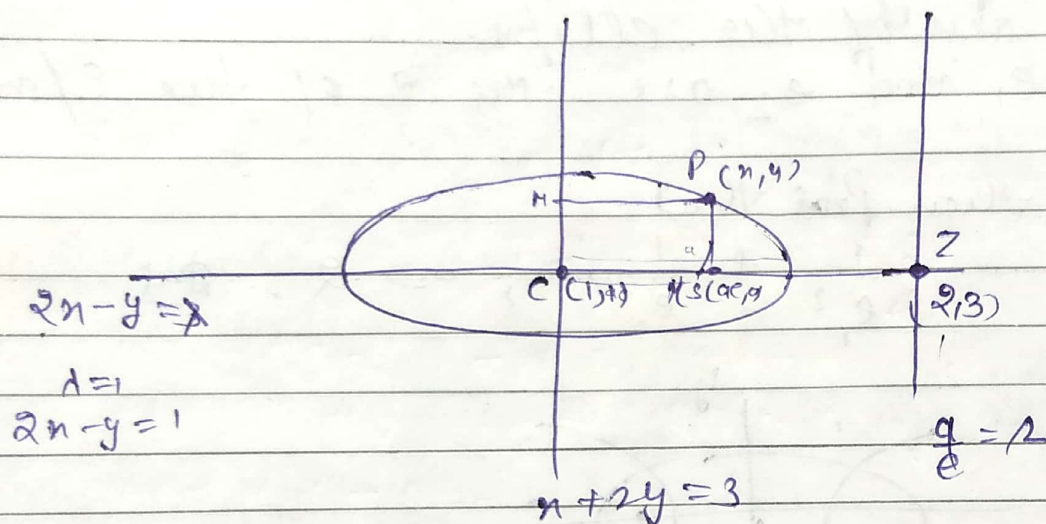
$$a = \frac{Ae_2}{e_1}$$

$$\frac{A^2 e_2^2}{e_1^2} (1 - e_1^2) = A^2 (e_2^2 - 1)$$



Q. eq<sup>n</sup> of minor axis of an ellipse is  $x + 2y = 3$   
 l (Minor axis) =  $\sqrt{5}$   
 foot of directrix =  $(2, 3)$

Major axis, minor axis, centre.



~~$b = \sqrt{5}$~~   ~~$100\% \text{ correct}$~~

$2b = \sqrt{5}$

$cz = \frac{a}{e}$

$\sqrt{4+1} = \frac{a}{e}$

$\sqrt{5} = \frac{a}{e}$

$b = \frac{\sqrt{5}}{2}$

$\sqrt{5} = \frac{a}{e}$

$\sqrt{5} = \sqrt{2}a$

$a = \frac{\sqrt{5}}{\sqrt{2}}$

~~$5 = a^2(1 - e^2)$~~   
 ~~$5 = a^2 - a^2e^2$~~   
 ~~$\frac{5}{a^2} = 1 - e^2$~~   $a^2 = \frac{5}{1 - e^2}$

$b^2 = a^2 - a^2e^2$

$5 = 5e^2(1 - e^2)$

$\frac{1}{4} = e^2 - e^4$

$4e^4 - 4e^2 + 1 = 0$

$e = \frac{1}{\sqrt{2}}$



$$\frac{2(x-1)^2}{5} + \frac{4(y-1)^2}{5}$$

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$$2(x-1)^2 + 4(y-1)^2 = 5$$

$$\frac{PM^2}{a^2} + \frac{PN^2}{b^2} = 1$$

$$\left( \frac{x+2y-3}{\sqrt{5}} \right)^2 + \frac{\left( \frac{2x-y-1}{\sqrt{5}} \right)^2}{5/4} = 1$$

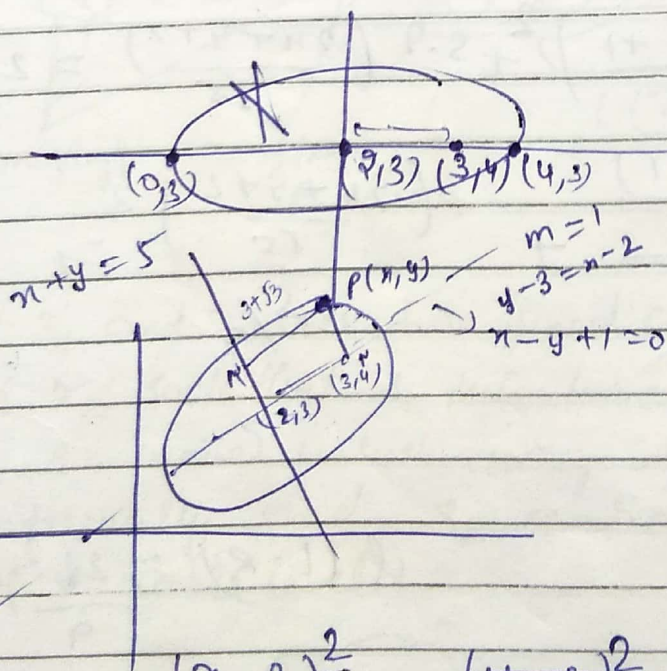
Ques:

if focus (3, 4)

C (2, 3)

$e = \frac{1}{2}$  of an ellipse respectively

the find ellipse



$$ae = \sqrt{2}$$

$$a \frac{1}{2} = \sqrt{2}$$

$$a = 2\sqrt{2}$$

$$b^2 = 8 \left( 1 - \frac{1}{4} \right)$$

$$b^2 = 8 \left( \frac{3}{4} \right)$$

$$b^2 = 6$$

$$\frac{(x-2)^2}{4} + \frac{(y-3)^2}{6} = 1$$



$$\frac{PM^2}{a^2} + \frac{PN^2}{b^2} = 1$$

$$\frac{\left(\frac{n+y-5}{\sqrt{2}}\right)^2}{8} + \frac{\left(\frac{n-y+1}{\sqrt{2}}\right)^2}{6} = 1$$

Que! find Length of (L.R) of ellipse.

$$4(n-2y+1)^2 + 9(2n+y+2)^2 = 25$$

$$4(n-2y)^2 + 1 + 2 \cdot 4(n-2y) + 9(2n+y)^2 + 4 + 2 \cdot 2 \cdot (2n+y) = 25$$

$$4(n^2 + 4y^2 - 4xy + 1 + 2n - 4y) + 9(4n^2 + y^2 + 4ny + 4 + 8n + 4y) = 25$$

$$4n^2 + 16y^2 - 16xy -$$

$$4n^2 + 16y^2 - 16xy + 4 + 8n - 16y + 36n^2$$

$$5 \cdot 4 \left(\frac{n-2y+1}{\sqrt{5}}\right)^2 + 5 \cdot 9 \left(\frac{2n+y+2}{\sqrt{5}}\right)^2 = 25$$

$$\frac{\left(\frac{n-2y+1}{\sqrt{5}}\right)^2}{5/4} + \frac{\left(\frac{2n+y+2}{\sqrt{5}}\right)^2}{5/9} = 1$$

$$a^2 = -\frac{5}{4}$$

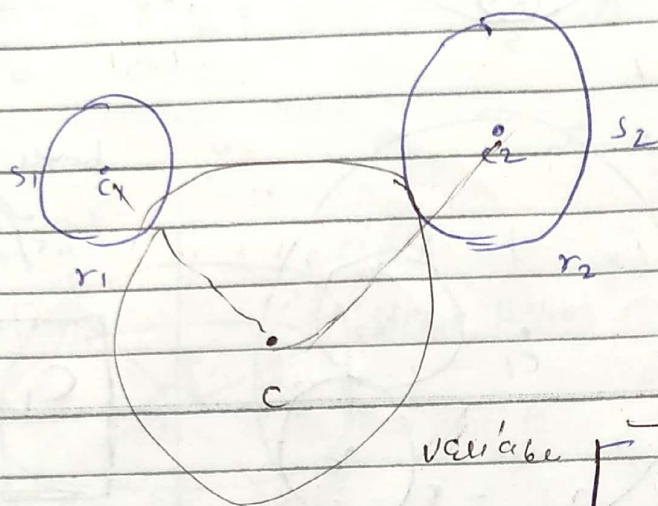
$$b^2 = \frac{5}{9}$$

$$d(L.R) = \frac{2a^2}{9}$$



dist diff.  $r_1 - r_2 = \text{const}$

Ques: Show that locus of centre of variable circle which touches to given circles is a hyp.



$$CC_1 = r + r_1 \quad (1)$$

$$CC_2 = r + r_2 \quad (2)$$

$$(1) - (2)$$

$$CC_1 - CC_2 = (r_1 - r_2)$$

$\swarrow$  moving pt.       $\swarrow$  fixed pt. (to S)

$L(TA)$

Note 1

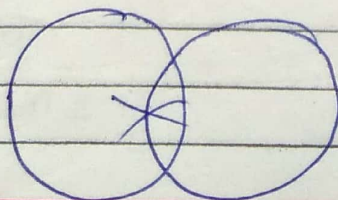
$$r_1 - r_2 = L(MA)$$

$$C_1 C_2 = 2ae$$

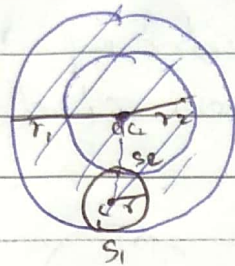
$$e = \frac{c}{a}$$

Imp:

Ques: Let  $S_1$  and  $S_2$  are two fixed circles with radii  $r_1$  &  $r_2$  such that  $S_2$  is contained in  $S_1$ . A variable circle  $S$  moves in such a way that it touches  $S_1$  internally and  $S_2$  externally. Find locus of centre of variable circle.





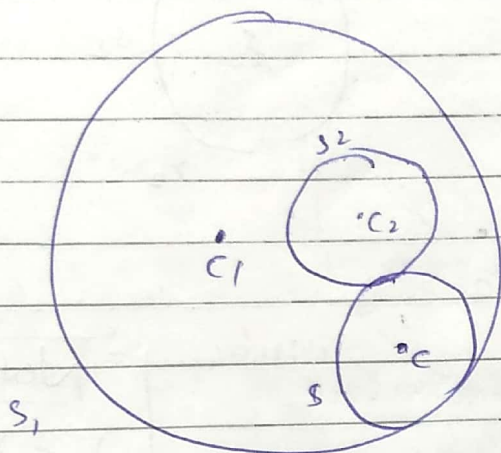


$C_1$

$$CC_1 = r_1 - r_2$$

$C$

Imp:  
Dist



both Centre  
focus.

$C_1$  and  $C_2$   
are focus

$C_1$  and  $C_2$

$$\text{Dist} = 2ae$$

$$r_1 + r_2 = 2a$$

$S_1, S_2$  touches ext

$$CC_2 = r_1 + r_2 \quad \text{--- (i)}$$

$S_1, S_2$  touches int

$$CC_1 = r_1 - r_2$$

(i) + (ii)

$$CC_1 + CC_2 = r_1 + r_2$$

focus



Que: dist of a Pt on the ellipse from its centre is 2. find  $\theta$  & angle of that point

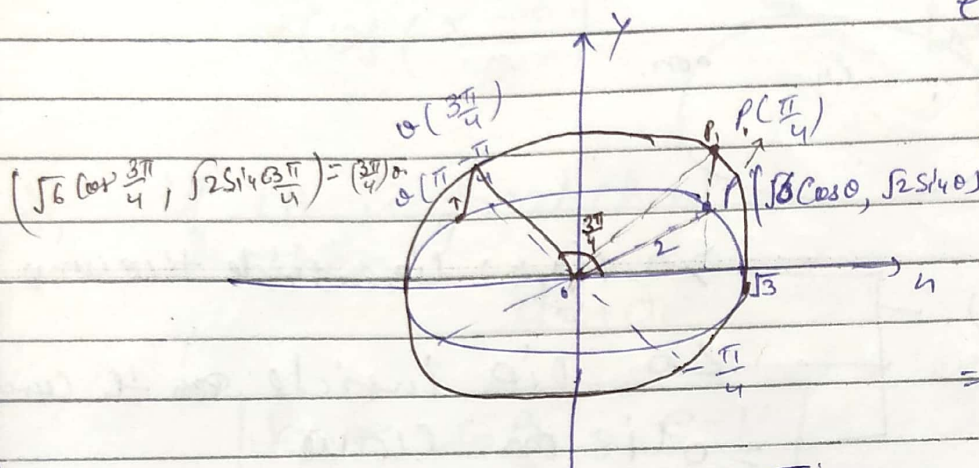
$$\frac{x^2}{6} + \frac{y^2}{2} = 1$$

$$a = \sqrt{6}, b = \sqrt{2}$$

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$e = \sqrt{1 - \frac{2}{6}}$$

$$e = \frac{2}{\sqrt{3}}$$



$$\sqrt{6} \cos \theta =$$

$\pi$  diff

$$CP^2 = 4$$

$$(\sqrt{6} \cos \theta - 0)^2 + (\sqrt{2} \sin \theta - 0)^2 = 4$$

$$6 \cos^2 \theta + 2 \sin^2 \theta = 4$$

$$4 \cos^2 \theta + 2 \sin^2 \theta = 4$$

$$4 \cos^2 \theta + 2 = 4$$

$$4 \cos^2 \theta = 2$$

$$\cos^2 \theta = \left(\frac{1}{\sqrt{2}}\right)^2 = \cos^2 \frac{\pi}{4}$$

$$\theta = n\pi \pm \frac{\pi}{4}$$

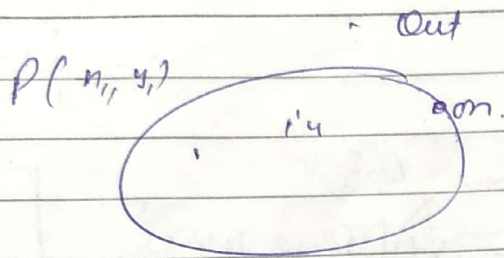
$$n=0, \theta = +\frac{\pi}{4}$$

$$n=1, \theta = \pi \pm \frac{\pi}{4}$$



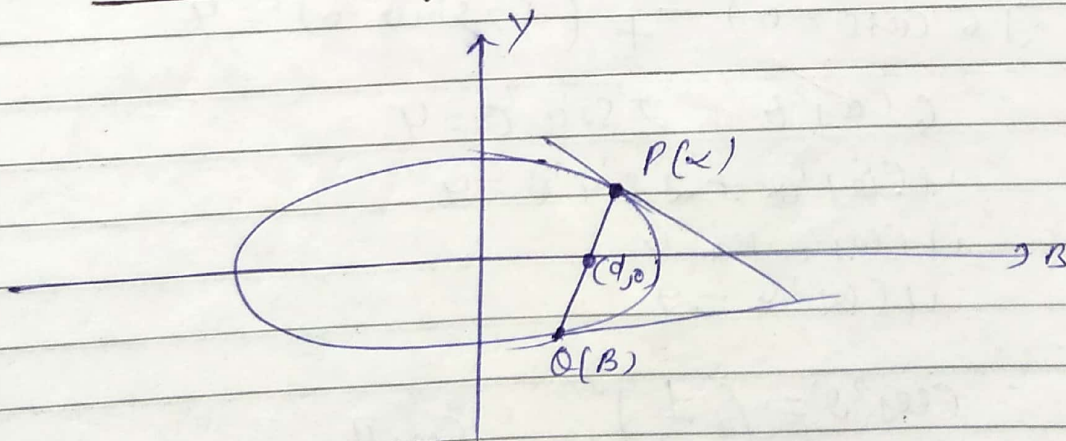
\* Position of Pt w.r.t an ellipse:

$$S(x, y) = \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



$S(x_1, y_1) \begin{cases} > 0 & \text{lie outside the curve} \\ < 0 & \text{lie inside the curve} \\ = 0 & \text{lie on curve} \end{cases}$

\* Line @ joining Chords:



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Line joining chord PQ =

$$\frac{x}{a} \cos\left(\frac{\alpha + \beta}{2}\right) + \frac{y}{b} \sin\left(\frac{\alpha + \beta}{2}\right) = \cos\left(\frac{\alpha - \beta}{2}\right)$$



If chord PQ passes through  $(d, 0)$

$$\frac{\cos \frac{(\alpha + \beta)}{2}}{\cos \left( \frac{\alpha - \beta}{2} \right)} = \frac{a}{d} ; \quad \boxed{\tan \frac{\alpha}{2} \tan \frac{\beta}{2} = \frac{d-a}{d+a}}$$

Remember,

\* if PQ is focal chord  
then  $d = \pm ae$

$$\boxed{\tan \frac{\alpha}{2} \tan \frac{\beta}{2} = \frac{e-1}{e+1}}$$

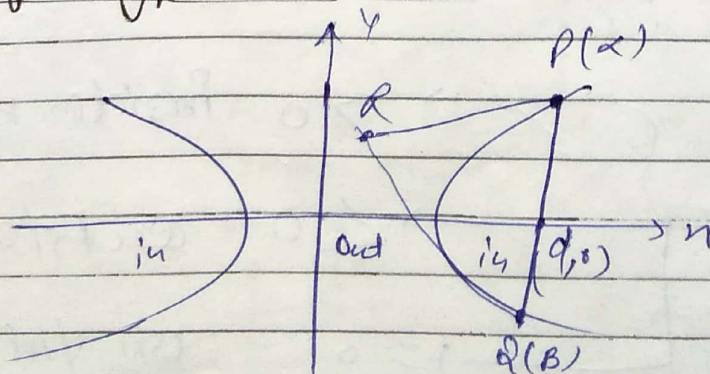
\* tangent at Pt. P and Q meet at Pt R

$$R \left( \frac{a \cos \frac{\alpha + \beta}{2}}{\cos \frac{\alpha - \beta}{2}} , b \frac{\sinh \left( \frac{\alpha + \beta}{2} \right)}{\cosh \left( \frac{\alpha - \beta}{2} \right)} \right)$$

Verify  
sheet

\* Hyper

\* Chord of Hyperbola:





$$E \div 0-13, 125, 8, 9, 12 \div JM \Rightarrow 6, 7, 9, \\ S \div 1, 4, JA \Rightarrow 1, 8,$$

$$\frac{m}{a} \cos \frac{\alpha - \beta}{2} - \frac{y}{b} \sin \frac{\alpha + \beta}{2} = \cos \frac{\alpha + \beta}{2}$$

If chord passes through  $(d, 0)$

$$\boxed{\tan \frac{\alpha}{2} \tan \frac{\beta}{2} = \frac{a-d}{a+d}}$$

If  $PQ$  is focal chord

then  $d = \pm ae$

\* Tangent at  $P$  and  $Q$  meet at pt.  $R$

$$R \left( a \frac{\cos(\frac{\alpha - \beta}{2})}{\cos(\frac{\alpha + \beta}{2})}, b \frac{\sin(\frac{\alpha - \beta}{2})}{\cos(\frac{\alpha + \beta}{2})} \right)$$

\* Position of pt. w.r.t. hyp.!

$$S(x, y) \equiv \frac{x^2}{a^2} - \frac{y^2}{b^2} - 1$$

$$P(x, y)$$

$$S(x, y) \begin{cases} > 0 = \text{inside hyp.} \\ < 0 = \text{outside hyp.} \\ \geq 0 = \text{on the hyp.} \end{cases}$$



## \* Tangent

Find condition when line  $y = mx + c$  touches E.

$$y = \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad y = mx + c$$

$$\frac{x^2}{a^2} + \frac{(mx+c)^2}{b^2} = 1$$

$mx+c$

$$\frac{x^2}{a^2} + \frac{m^2x^2 + c^2 + 2mxc}{b^2} = 1$$

$$x^2b^2 + a^2m^2x^2 + a^2c^2 + 2amx = a^2b^2$$

$$\frac{x^2}{a^2} + \frac{m^2x^2 + 2mxc + c^2}{b^2} = 1$$

$$x^2b^2 + a^2m^2x^2 + a^22mxc + a^2c^2 = a^2b^2$$

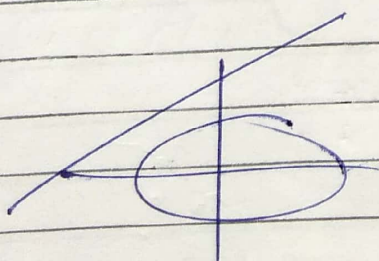
$$(m^2a^2 + b^2)x^2 + 2mca^2x + (c^2 - b^2)a^2 = 0 \quad \begin{matrix} x_1 \\ x_2 \end{matrix}$$

$$D < 0 \quad 4m^2c^2a^4 = 4(m^2a^2 + b^2)(c^2 - b^2)a^4$$

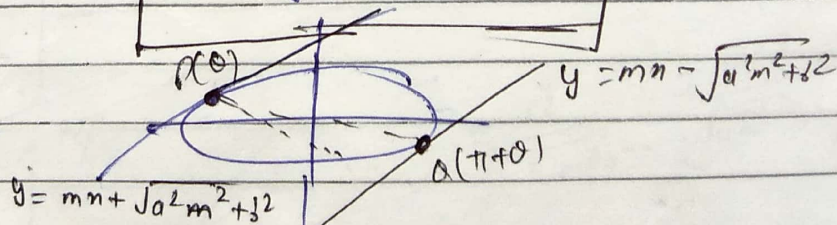
$$D = 0$$

$$c^2 = a^2m^2 + b^2$$

$$c = \pm \sqrt{a^2m^2 + b^2}$$

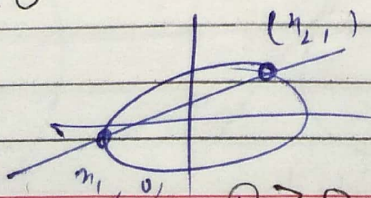


$D < 0$



$D = 0$

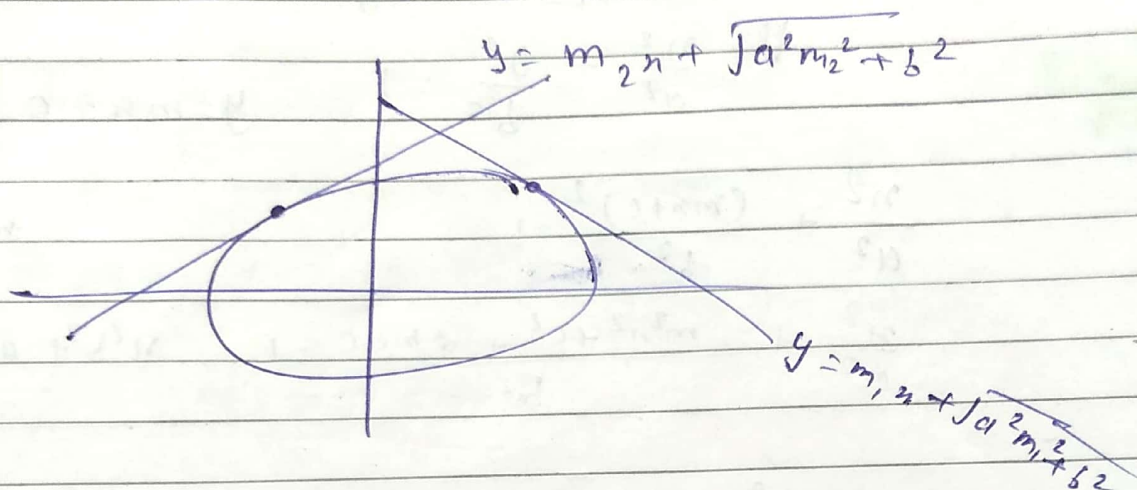
P and Q are directly opp.



$D > 0$



Hence line  $y = mx \pm \sqrt{a^2m^2 + b^2}$  touches above ellipse for all values of  $m$



\* Similarly

eq<sup>n</sup> of tangent to hyp. is

$$\frac{x^2}{a^2} + \frac{y^2}{-b^2} = 1$$

$$y = mx \pm \sqrt{a^2m^2 - b^2}$$

i.e  $b^2$  is replaced by  $-b^2$



Curve

$$x^2 + y^2 = a^2$$

$$y^2 = 4ax$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{(x-\alpha)^2}{a^2} + \frac{(y-\beta)^2}{b^2} = 1$$

$$\frac{(x-\alpha)^2}{a^2} - \frac{(y-\beta)^2}{b^2} = 1$$

Tangent

$$y = mx \pm a\sqrt{1+m^2}$$

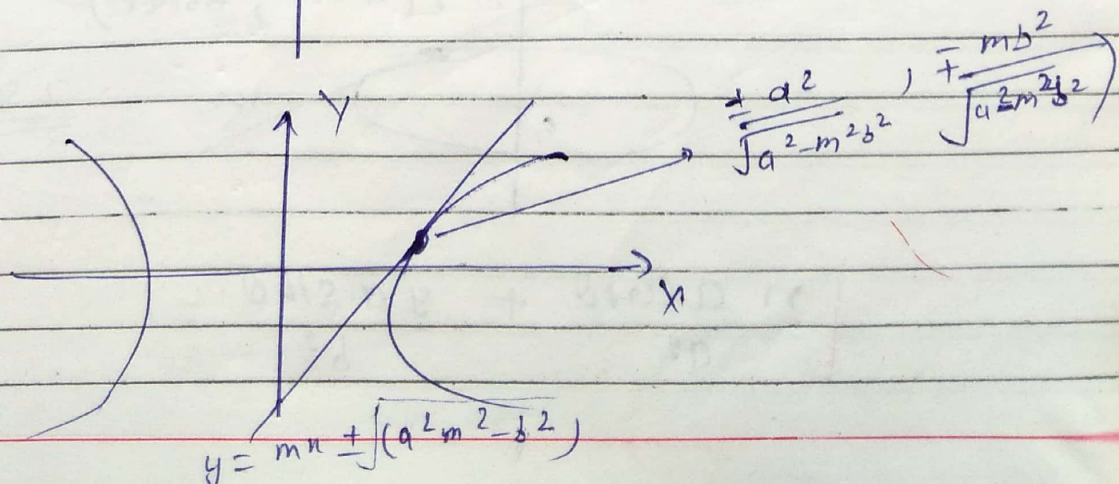
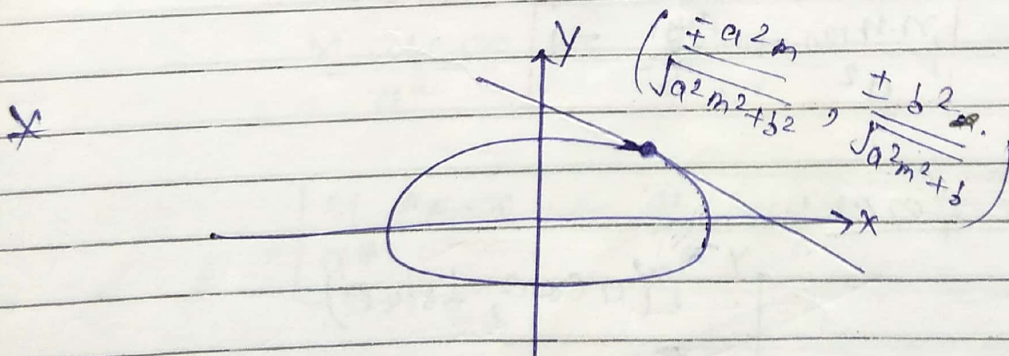
$$y = mx \pm \frac{a}{m}$$

$$y = mx \pm \sqrt{a^2 m^2 + b^2}$$

$$y = mx \pm \sqrt{a^2 m^2 - b^2}$$

$$(y-\beta) = m(x-\alpha) \pm \sqrt{a^2 m^2 + b^2}$$

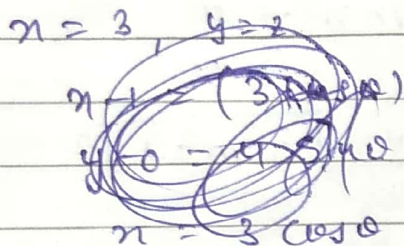
$$y - \beta = m(x - \alpha) \pm \sqrt{a^2 m^2 - b^2}$$





\* any tangent to the ellipse

$$\frac{(m-1)^2}{9} + \frac{y^2}{4} = 1$$

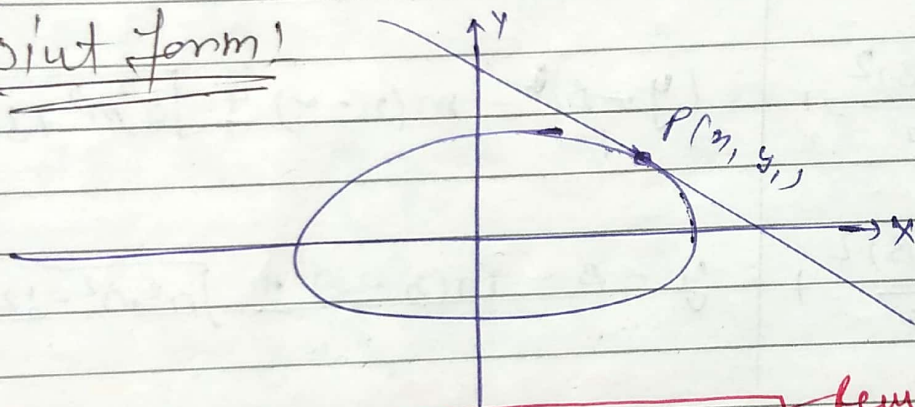


$$y = m(x-1) \pm \sqrt{4m^2+4}$$

**IMP.**

\* Other form of tangent!

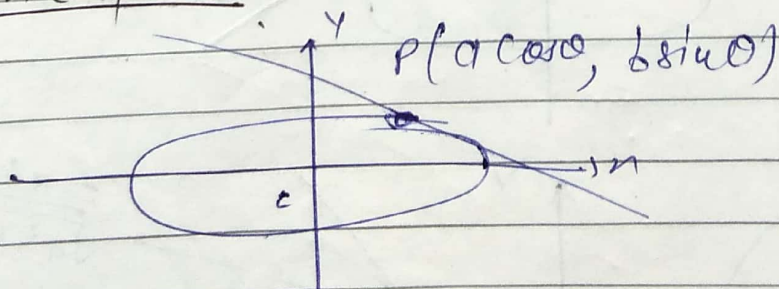
Point form!



$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$$

Remember

3) Parametric form!

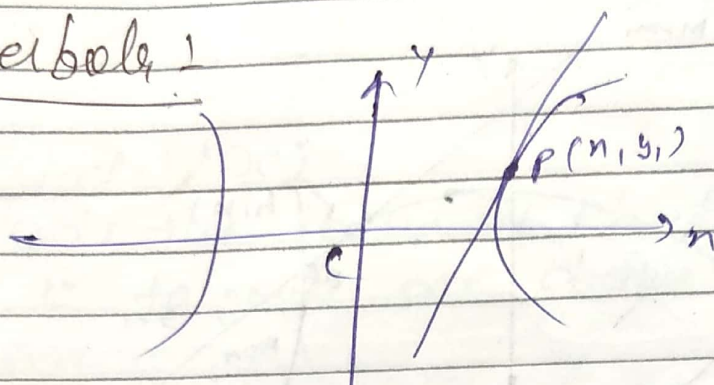


$$\frac{x a \cos \theta}{a^2} + \frac{y b \sin \theta}{b^2} = 1$$



$$\boxed{\frac{x \cos \theta}{a} + \frac{y \sin \theta}{b} = 1} \quad \text{Remember}$$

For Hyperbola:



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

(1) pt form  $P(x_1, y_1)$

$$\boxed{\frac{x x_1}{a^2} - \frac{y y_1}{b^2} = 1} \quad \text{Remember}$$

(2) Parametric form:  $P(a \sec \theta, b \tan \theta)$

$$\frac{x \sec \theta}{a} - \frac{y \tan \theta}{b} = 1$$

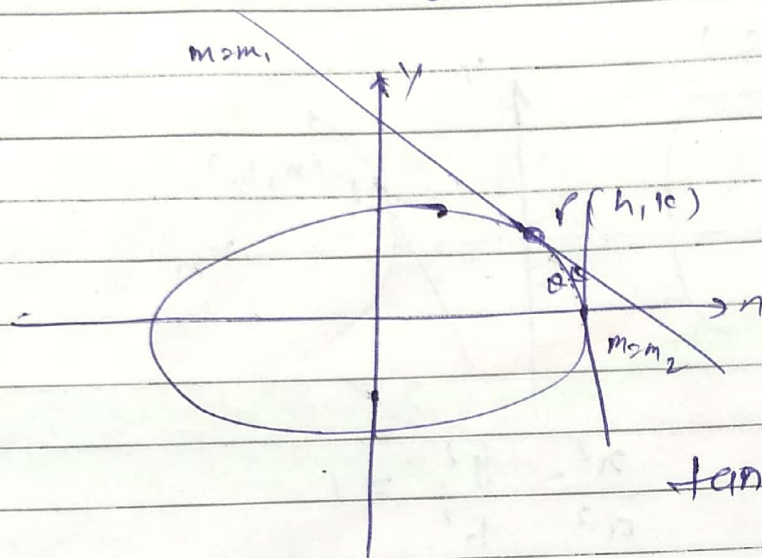
$$\boxed{\frac{x \sec \theta}{a} - \frac{y \tan \theta}{b} = 1} \quad \text{Remember}$$

ellipse:



## Ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



tangent to the ellipse

$$y = mx + \sqrt{a^2m^2 + b^2}$$

Passes through Pt (h, k)

$$k = mh + \sqrt{a^2m^2 + b^2}$$

$$(k - mh)^2 = a^2m^2 + b^2$$

$$m^2(h^2 - a^2) - 2mhk + (k^2 - b^2) = 0$$

$\swarrow m_1$   
 $\swarrow m_2$

$$m_1 + m_2 = \frac{2hk}{h^2 - a^2}$$

$$m_1 m_2 = \frac{k^2 - b^2}{h^2 - a^2}$$

Hence Angle b/w pair of T drawn from Pt P(h, k) as shown in the fig.



$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| = \frac{\sqrt{(m_1 + m_2)^2 - 4m_1 m_2}}{1 + m_1 m_2}$$

### \* Director Circle (DC)

DC is the locus of pt. from which pair of  $1^{\text{st}}$  tangents are drawn to the ellipse

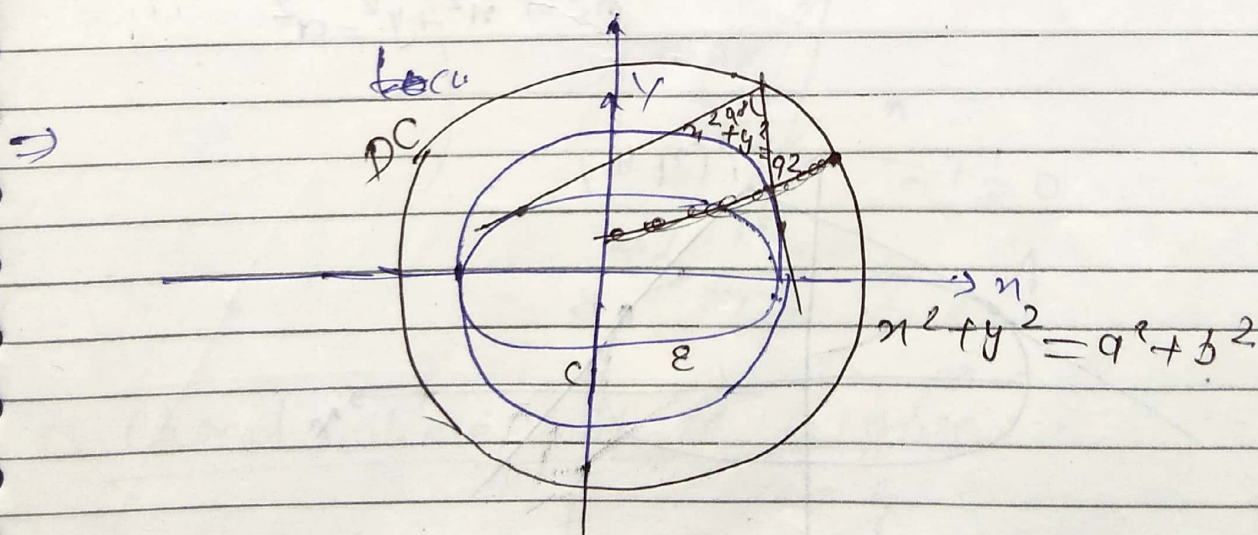
$$\angle C = 90^\circ$$

$$\frac{1}{0} = \tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$m_1 m_2 = -1$$

$$h^2 - b^2 = -k^2 + a^2$$

$$h^2 + k^2 = a^2 + b^2$$





of hyp. is

$$x^2 + y^2 = a^2 - b^2$$

i.e  $b^2$  is replaced by  $-b^2$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

no 16 da possible

$\downarrow a < b$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$x^2 - y^2 = 9$$

$a = 3$

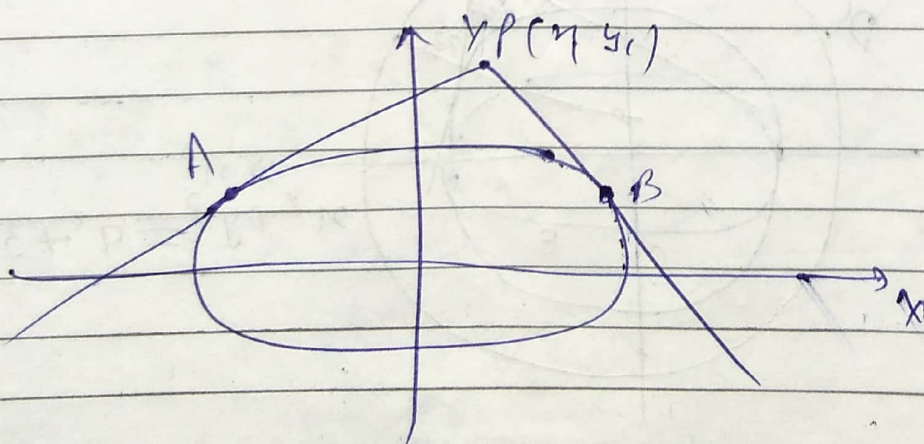
$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

$$213 \quad OC \rightarrow x^2 + y^2$$

$r = 16 - 9 = 7$

$$OC \Rightarrow x^2 + y^2 = a^2$$



① eq<sup>n</sup> of pair of tangent in any conic is

$$SS_1 = T^2$$

$\leftarrow$   $I \perp$  is joint eq<sup>n</sup> of line  $AB$  &  $OC$



Hence S is  $S \equiv \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1$

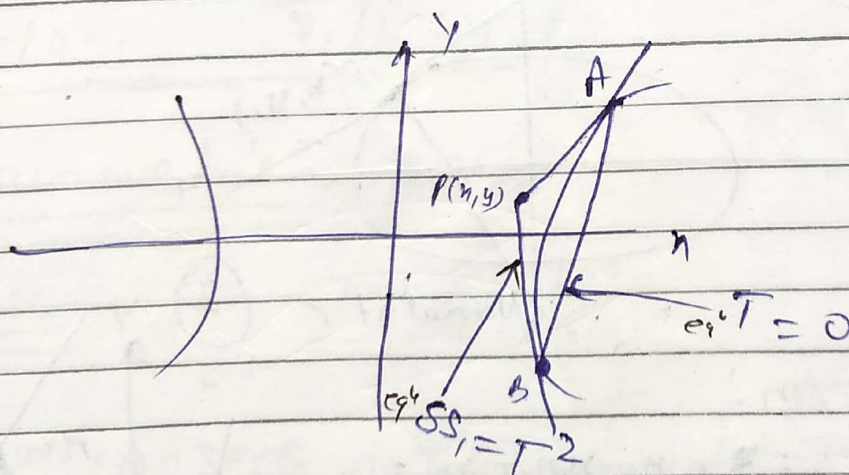
$S_{11} \equiv \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} - 1$

$T \equiv \frac{xx_1}{a^2} + \frac{yy_1}{b^2} - 1$

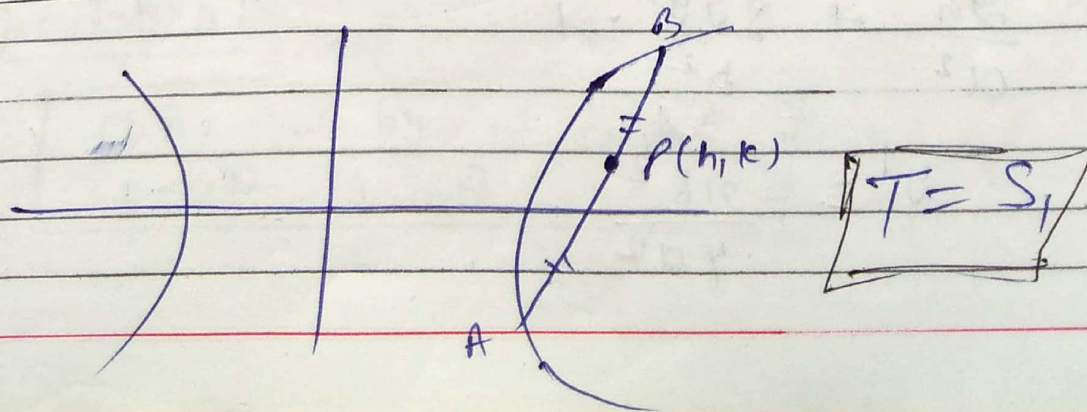
② Chord of Contact

line joining AB is called C.O.C. to the ellipse w.r.t Pt P.

condn is  $T = 0$

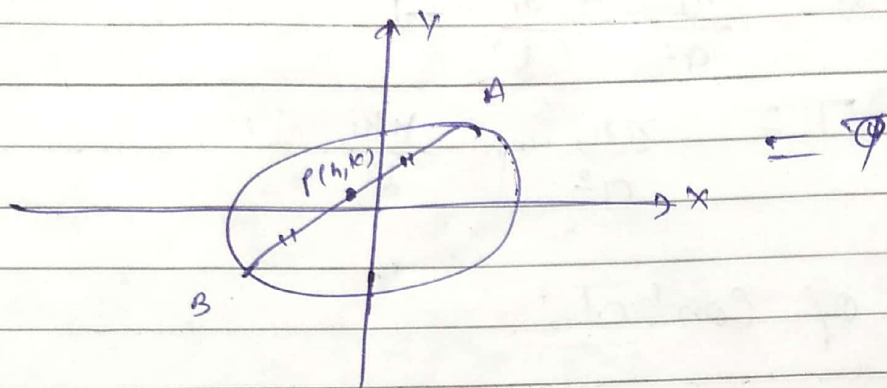


③ Chord whose mid Pt is given





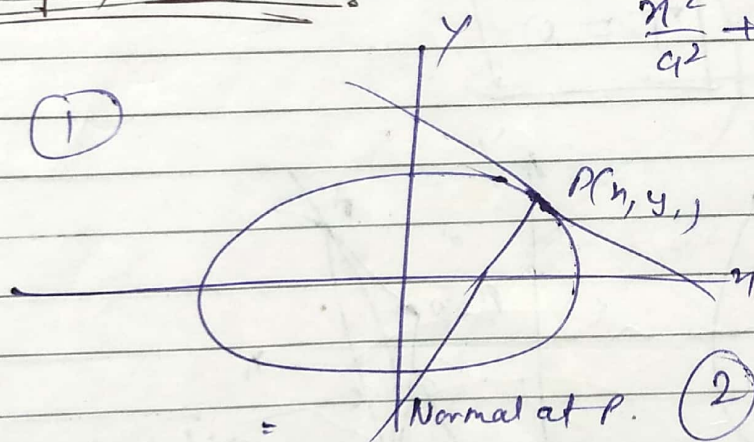
$$T = S_1$$



\* Normal

eq<sup>n</sup> of Normal

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



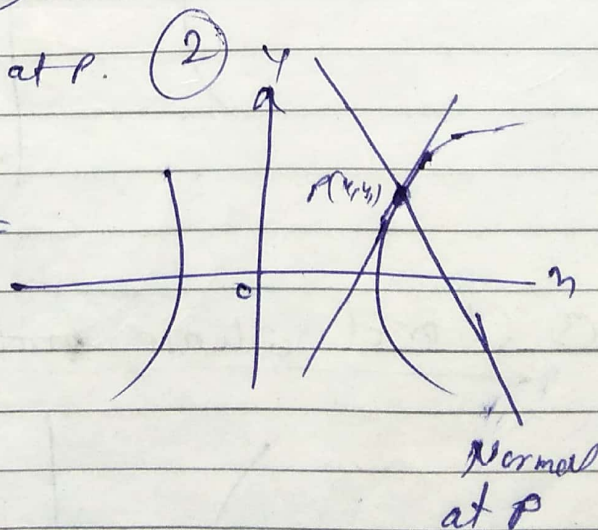
\* Point form:

$$x = x_1, \quad y = y_1$$

(1)

$$\frac{2x}{a^2} + \frac{2yy'}{b^2} = 1$$

$$y' = -\frac{nb^2}{ya^2}$$





$$(y')_P = -\frac{x_1 b^2}{y_1 a^2}$$

eq<sup>n</sup> of Normal is  $\boxed{(y - y_1) = \frac{y_1 a^2}{x_1 b^2} (x - x_1)}$

$$\boxed{\frac{x - x_1}{x_1 / a^2} = \frac{y - y_1}{y_1 / b^2}}$$

① Hyp.

$$\boxed{\frac{x - x_1}{x_1 / a^2} = -\frac{y - y_1}{y_1 / b^2}}$$

$$\boxed{\frac{x - x_1}{x_1 / a^2} = \frac{y - y_1}{y_1 / (-b^2)}}$$

② Parametric form!

ellipse  $x_1 \rightarrow a \cos \theta$   
 $y_1 \rightarrow b \sin \theta$

$$\boxed{a \sec \theta - b \csc \theta = a^2 - b^2}$$

③ Hyperbola!

$$\boxed{\frac{a x}{\sec \theta} + \frac{b y}{\tan \theta} = a^2 e^2}$$



## ② Slope form:

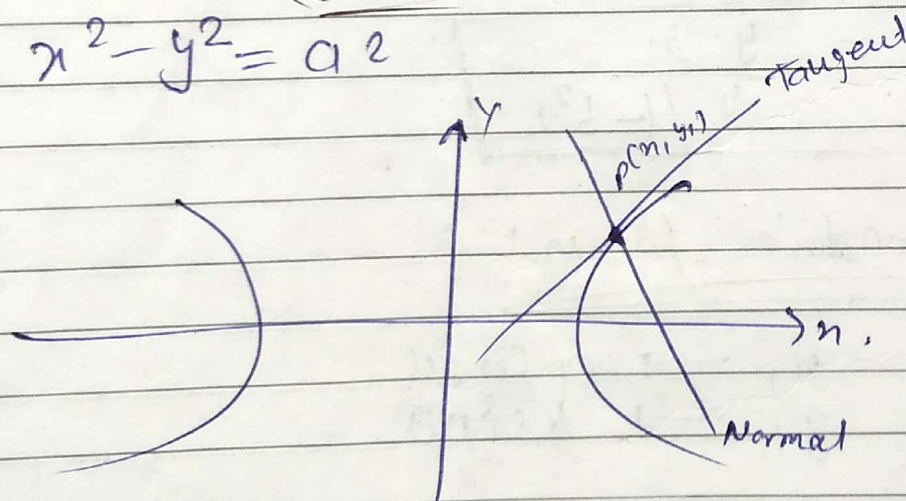
Ellipse  $y = mx \pm \frac{(a^2 - b^2)m}{\sqrt{a^2 + b^2 m^2}}$

Hyp :

$$y = mx \pm \frac{m(a^2 + b^2)}{\sqrt{a^2 - m^2 b^2}}$$

\* eq<sup>n</sup> of Normal to rectangular hyp.  
( $a = b$ )

$$x^2 - y^2 = a^2$$



(i) Point form (Cartesian form) :

$$\frac{x}{x_1} + \frac{y}{y_1} = 2$$

(ii) Parametric form :  $\frac{x}{\sec \theta} + \frac{y}{\tan \theta} = 2a$



## \* asymptote of hyperbola

if a line touches hyp.  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

at infinity then it is called asymptote

if  $y = mx + c$  is asymptote of hyp. then

$$x^2 b^2 - (mx + c)^2 a^2 = a^2 b^2$$

$$x^2 (b^2 - m^2 a^2) - 2mc a^2 x - a^2 (c^2 + b^2) = 0 \quad \begin{matrix} \alpha \\ \beta \end{matrix}$$

for asymptote both roots must be  $\infty$ .

∴ coeff. of  $x^2 = 0$  i.e.  $b^2 - m^2 a^2 = 0$ .

$$\boxed{m = \pm \frac{b}{a}}$$

∴ coeff. of  $x = 0 \Rightarrow$

$$2mc a^2 = 0$$

$$\boxed{c = 0}$$



Ques: Find tangent to the E  
 $3x^2 + 4y^2 = 12$

(i) || to  $y = 2x + 4$

(ii)  $\perp$  to  $2y + 3x = 5$

$$3x^2 + 4y^2 = 12$$

$$\frac{x^2}{4} + \frac{y^2}{3} = 1$$

hence  $a = 2, b = \sqrt{3}$ .

(i)  $y = 2x + 4$   
 $m = 2$

$$= 2x + \sqrt{2^2 \cdot 2^2 + 3}$$

$$= 2x + \sqrt{16 + 3}$$

$$y = 2x + \sqrt{19}$$

(ii)  $2y = 3x + 5$   
 $y = \frac{3x + 5}{2}$

$$y = \frac{3}{2}x + \sqrt{4\left(\frac{9}{4}\right) + 3}$$

$$= \frac{3}{2}x + \sqrt{4\left(\frac{9}{4}\right) + 3}$$

$$= \frac{3}{2}x + \sqrt{12}$$



Q.2 find position of  $(4, -3)$  w.r.t. an ellipse

$$5x^2 + 7y^2 = 140$$

$$\frac{x^2}{28} + \frac{y^2}{20} = 1$$

~~Q.2.8~~

$$OP = 28, \quad y = 20$$

$$\frac{16}{28} + \frac{9}{20} =$$

$$\frac{16}{28} + \frac{9}{20}$$

$$S(4, -3) = 5(4)^2 + 7(-3)^2 - 140$$

$$= 5 \cdot 16 + 7 \cdot 9 - 140 = +ve \text{ outside}$$

Q.ue find  $\lambda$  for which line  $y = 2x + \lambda$  touches

$$16x^2 - 9y^2 = 144$$

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

$$a^2 = 9, \quad b^2 = 16$$

$$\lambda = \sqrt{a^2 - b^2} = 16$$

$$\lambda = \sqrt{36 - 16}$$

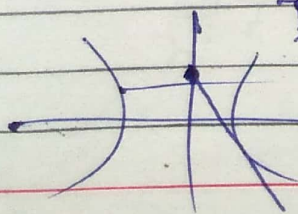
$$\lambda = \sqrt{20}$$

$$y = 2x \pm \sqrt{20}$$

Q.ue: Tangent are drawn to figt.  $3x^2 - 2y^2 = 25$  from  $(0, 5/2)$  find their eq<sup>n</sup>.

$$\frac{x^2}{25/3} - \frac{y^2}{25/2} = 1$$

$$\frac{x^2}{25/3} - \frac{y^2}{25/2} = 1$$

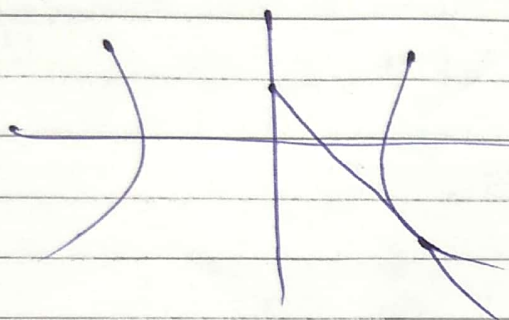


$$\sec \theta = \frac{a}{b} \text{ by } \sec \theta = \frac{a^2 + b^2}{a^2 - b^2}$$

$$= \frac{25 - 25}{3} \sec \theta = \frac{25 \sec \theta}{3} = \frac{(25/3)^2 + (25/2)^2}{(25/3)^2 - (25/2)^2}$$



$$y = mx \pm \sqrt{\frac{25}{3}m^2 - \frac{25}{3}}$$



$$0 = \frac{5}{2}m \pm \sqrt{\frac{25}{3}m^2 - \frac{25}{3}}$$

$$\frac{5}{2} = 0 \pm \sqrt{\frac{25}{3}m^2 - \frac{25}{3}}$$

$$\frac{25}{4} = \frac{25}{3}m^2 - \frac{25}{3}$$

$$\frac{75}{4} = \frac{25}{3}m^2$$

$$m = \pm \frac{3}{2}$$

$$y = mx \pm \sqrt{\frac{25}{3}m^2 - \frac{25}{3}}$$

Ques: a tangent having slope  $m = -\frac{4}{3}$  to the ellipse  $\frac{x^2}{18} + \frac{y^2}{32} = 1$

intersects  $x$  and  $y$  at  $A$  and  $B$  respectively. Find area of  $\triangle OAB$ .  $O \rightarrow$  origin.

$$\frac{x^2}{18} + \frac{y^2}{32} = 1$$

$$m = -\frac{3}{2}$$

$$\frac{64}{143}$$

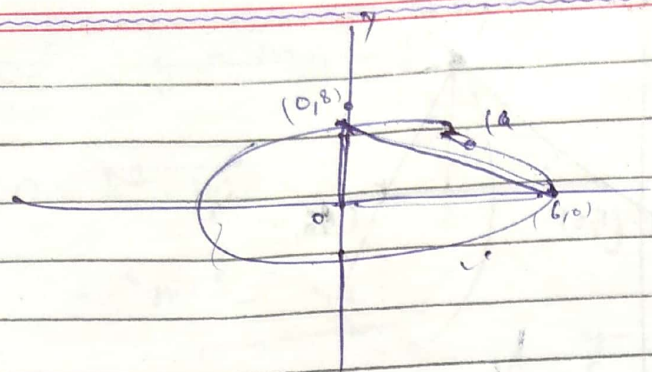
$$y = -\frac{4}{3}x \pm \sqrt{8 \cdot \frac{9}{4} + 32}$$

$$y = -\frac{4}{3}x \pm \sqrt{\frac{81}{2} + 32}$$

$$y = -\frac{3}{2}x +$$



Qmk



$$y = -\frac{3}{2}x + \sqrt{\frac{21}{2} + 32}$$

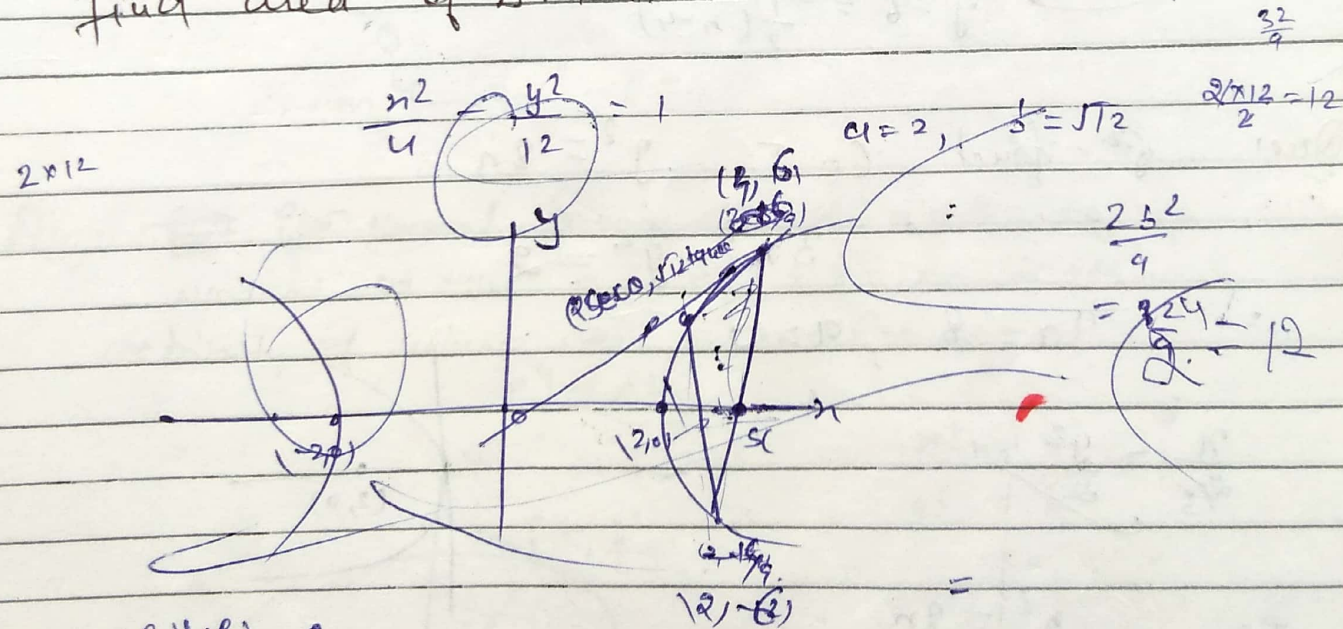
$$y = -\frac{4}{3}x + 8$$

$$\frac{1}{2} \times 6 \times 8 = 24$$

Q. Tangent and Normal are drawn at upper end  $(x_1, y_1)$  of a later section of hyp. intersecting transverse axis  $P$  and  $G$  respectively.

$P(x_1, y_1)$   $x_1 > 0, y_1 > 0$ ,  $\frac{x^2}{4} - \frac{y^2}{12} = 1$   $\frac{d}{dx} \left| \frac{x}{N} \right|$

Find area of  $\Delta PTG$ .

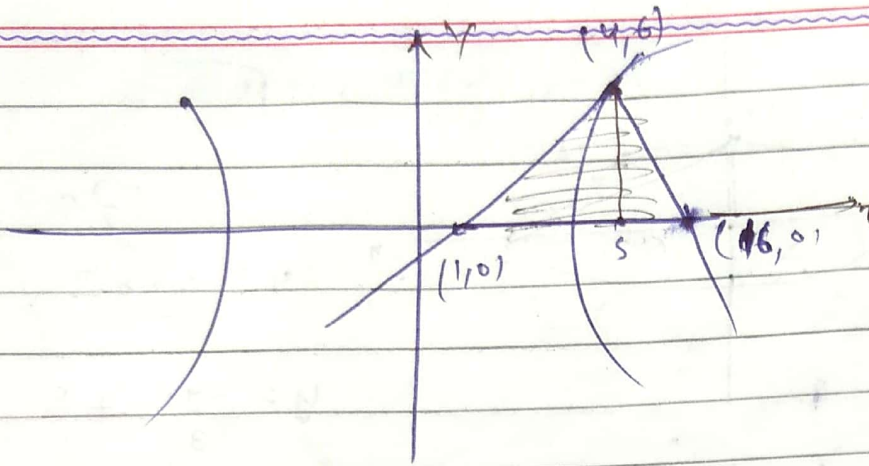


$\log P(K.R) = e$

y

$$b^2 = a^2(e^2 - 1) \quad 12 = 4(e^2 - 1) \quad e = 2$$





$$A = \frac{1}{2} \times 15 \times 6 = 45$$

$$e = 2 \quad L(L.R) = \frac{2bc}{a} = 2 \cdot \frac{12}{2} = 12$$

$$m = -\frac{1}{2}$$

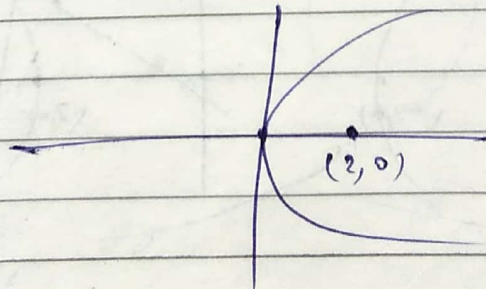
$$y - 6 = -\frac{1}{2}(x - 4)$$

Que!  $y^2 = 8x$  find C.T

$$3x^2 - y^2 = 2$$

$$4a = 8 \quad a = 2$$

$$\frac{x^2}{2/3} - \frac{y^2}{2} = 1$$



f =

$$y^2 = 8x$$

$$y = mx + \frac{1}{m}$$

$$y = mx + \frac{1}{m}$$

$$y = mx + \frac{1}{m}$$

f =



$$y = y - mx = \frac{2}{m}$$

$$c = -\frac{2}{m}$$

$$D: b^2 - 4ac = 0$$

$$y = mx + \sqrt{\quad}$$

$$m^2 - \frac{4 \cdot 2}{m}$$

$$m^2 - \frac{8}{m} = 0$$

Ans

$$y = mx + \frac{2}{m}$$

$$y = mx + \sqrt{\frac{2}{3}m^2 - 2}$$

> for C.T

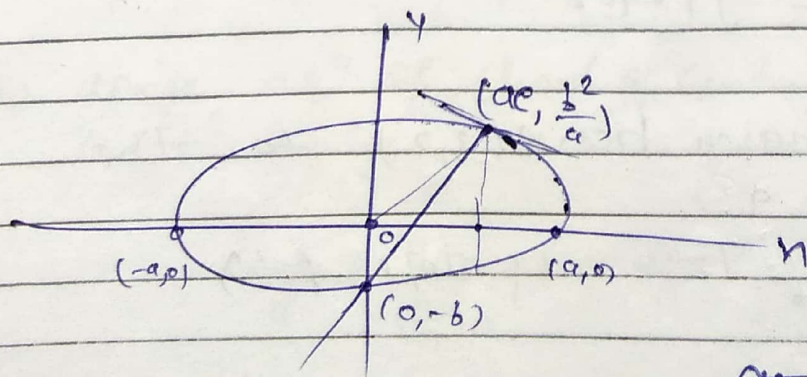
$$\frac{2}{m} = \sqrt{\frac{2}{3}m^2 - 2}$$

$$\frac{4}{m^2} = \frac{2}{3}m^2 - 2$$

$$\frac{2}{3}m^4 - 2m^2 = 4$$

$$m^4 - 3m^2 - 6 = 0$$

Ques: ~~If Normal passing through extremity of~~  
Normal at the end of l.c. passes through  
extremity of minor axis then P.T  
 $e^4 + e^2 = 1$



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

=

$$\frac{a}{2}$$

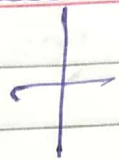
an

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\text{arccos } \cos \theta = \cos \theta = a^2/b^2$$



$$a \sec \theta = \frac{a^2}{b} \cos \theta \Rightarrow a^2 = b^2$$



$$a \sec \theta = \frac{b^2}{a} \cos \theta \Rightarrow a^2 = b^2$$

$$a^2 \sec \theta = \frac{b^3}{a} \cos \theta \Rightarrow a^3 = b^3$$

$$\frac{y - ae}{ae/a^2} = \frac{y - b^2}{b^2/a^2}$$

$$(y - ae) = e \left( y - \frac{b^2}{a} \right)$$

$$0 - ae = e \left( b - \frac{b^2}{a} \right)$$

$$a^2 = ab + b^2$$

$$b^2 = a^2(1 - e^2)$$

$$1 - \frac{b}{a} + \frac{b^2}{a^2}$$

$$1 = \sqrt{1 - e^2} + 1 - e^2$$

$$e^2 = \sqrt{1 - e^2}$$

Ques: Tangents are drawn from  $P(3, 2)$  to the ellipse  $x^2 + 4y^2 = 9$ .  
 Find eq<sup>n</sup> of T. (slope form)



(1,1) (3,2)

$$a = 3, b = \frac{3}{2}$$

$$x^2 + 4y^2 = 9$$

$$\frac{x^2}{9} + \frac{y^2}{9/4} = 1$$

$$y = mx + \sqrt{9m^2 + \frac{9}{4}}$$

$$\frac{3m + 2 \cdot \frac{3}{2}}{\frac{9}{4}} = 1$$

$$2 = 3m + 3 \sqrt{m^2 + \frac{1}{4}}$$

$$(2 - 3m)^2 = 9(m^2 + \frac{1}{4})$$

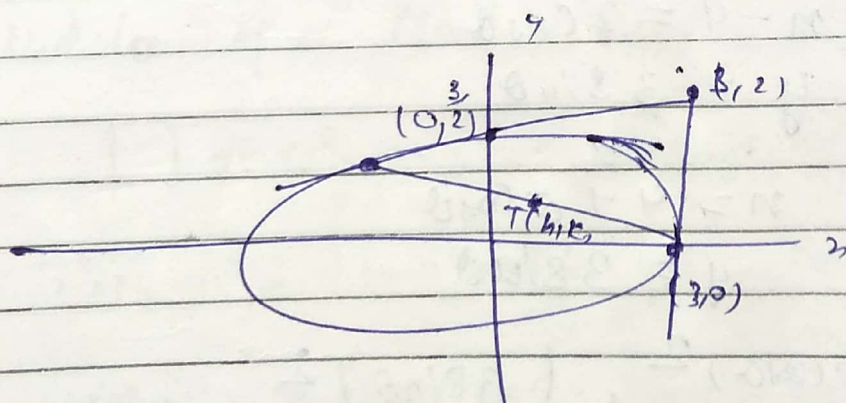
$$4 + 9m^2 - 12m = 9m^2 + \frac{9}{4}$$

$$12m = \frac{7}{4}$$

$$m = \frac{7}{48}$$

$$0 \cdot m^2 - 48m - 7 = 0$$

$$\frac{1}{48}$$



Q(ii) Write eq<sup>n</sup> of Chord of Contact of Pt P.  
w.r.t E.

$$T = 0$$

$$\frac{x^2}{9} + \frac{y^2}{9/4} - 1 = 0$$

$$\frac{3x}{9} + \frac{2y}{9/4} - 1 = 0$$

$$3x + 8y = 9$$



J-M = full - con ellipse  
J-M - hyp.

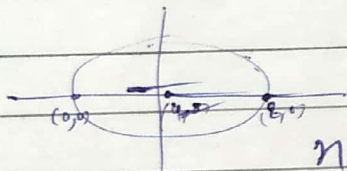
a find mid pt of C.O.C.  
T = S<sub>1</sub>

$$\frac{h}{9} + \frac{k}{9} = 5 \frac{h^2}{9} + \frac{k^2}{9/4}$$

and  $3x + 8y = 9$

$$\frac{h/9}{3} = \frac{k/9/4}{8} = \frac{h^2/9 + \frac{k^2}{9/4}}{9}$$

Ques: if  $(x, y)$  belong to R satisfy  $\frac{(x-4)^2}{4} + \frac{y^2}{9} = 1$   
then find max value of  $\frac{x^2}{8} + \frac{y^2}{18}$



$$x - 4 = 2 \cos \theta$$

$$y = 3 \sin \theta$$

$$x = 4 + 2 \cos \theta$$

$$y = 3 \sin \theta$$

$$= \frac{(4 + 2 \cos \theta)^2}{8} + \frac{(3 \sin \theta)^2}{18}$$

⇒ solve next page Solve

$$= \frac{1}{2} [5 + 4 \cos \theta]$$



$$\frac{(4+2\cos\theta)^2}{8} + \frac{(3\sin\theta)^2}{18}$$

$$\frac{16 + 4\cos^2\theta + 16\cos\theta + 9\sin^2\theta}{8}$$

$$\frac{4 + \cos^2\theta + 4\cos\theta + \sin^2\theta}{2}$$

$$\frac{4 + \cos^2\theta + 4\cos\theta + \sin^2\theta}{2}$$

$$\frac{4 + 4\cos\theta + 1}{2} = \frac{5 + 4\cos\theta}{2}$$

$$\frac{5 + 4\cos\theta}{2} = \frac{1}{2} [5 + 4\cos\theta]$$

Q. find locus of P(x, y) satisfy

$$\left| \sqrt{x^2 + y^2 + 8y + 16} - \sqrt{x^2 + y^2 - 6x + 9} \right| = 5$$

$$\left| \sqrt{x^2 + (y+4)^2} - \sqrt{(x-3)^2 + y^2} \right| = 5$$

$$\sqrt{x^2 + (y+4)^2} - \sqrt{(x-3)^2 + y^2} = 5$$

2

next page

$$\frac{\sqrt{x^2 + y^2 + 8y + 16}}{5} = \frac{\sqrt{(x-3)^2 + y^2}}{5} + 1$$

$$a^2 = 25, b^2 = 25$$

$$\sqrt{25\sec^2\theta} = \sqrt{25\tan^2\theta} + 5$$

$$\sqrt{(5\sec\theta)^2 + (5\tan\theta)^2} = 5(5 + \tan^2\theta)$$

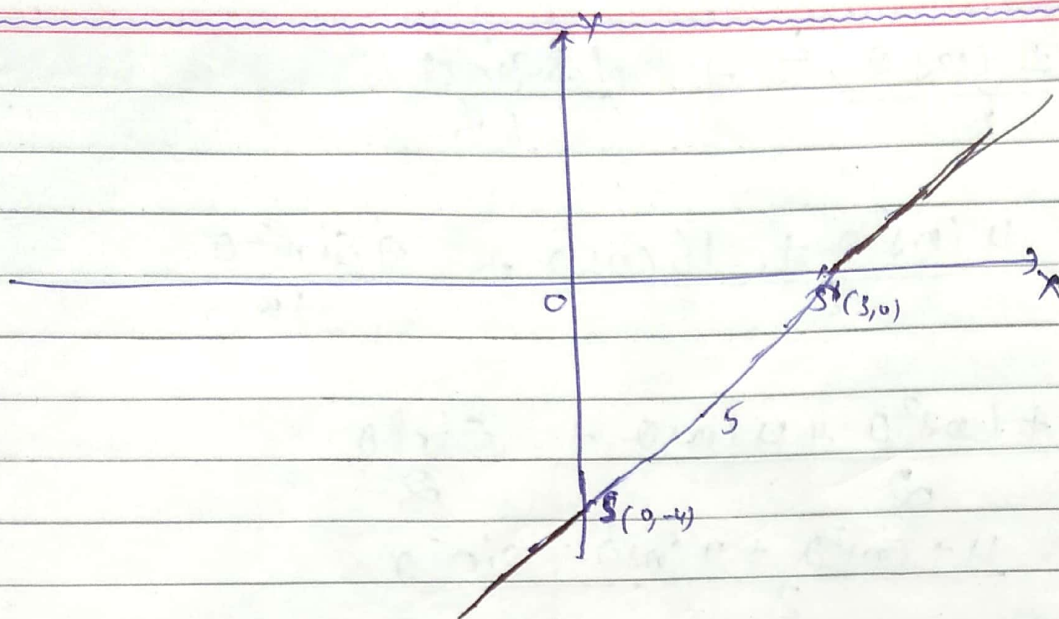
$$= \sqrt{(5\sec^2\theta + 5\tan^2\theta) - 5(5\sec^2\theta)} = 5$$

$$y = mx + \frac{a}{m}$$



$$PS - PS' = 2a.$$

$$0 < 2 < 3$$



$$P(x, y)$$

$$S(0, -4)$$

$$S'(3, 0)$$

$$|PS - PS'| = 5$$

$$|PS - PS'| = 4 \text{ (less than 5)}$$

locus of P is hyp.

$$|PS - PS'| = 6 \text{ (greater than 5)} \quad \emptyset$$

(no locus)

$$|PS - PS'| = 5 \text{ (two rays)}$$

$$|PS + PS'| = 5$$

$$|PS + PS'| = 14 \text{ (greater than 5)} \text{ Ellipse}$$

$$|PS + PS'| = 3 \text{ (less than 5)} \quad \emptyset$$



Que find min dis b/w Curve

$$y^2 = 4x \quad \&$$

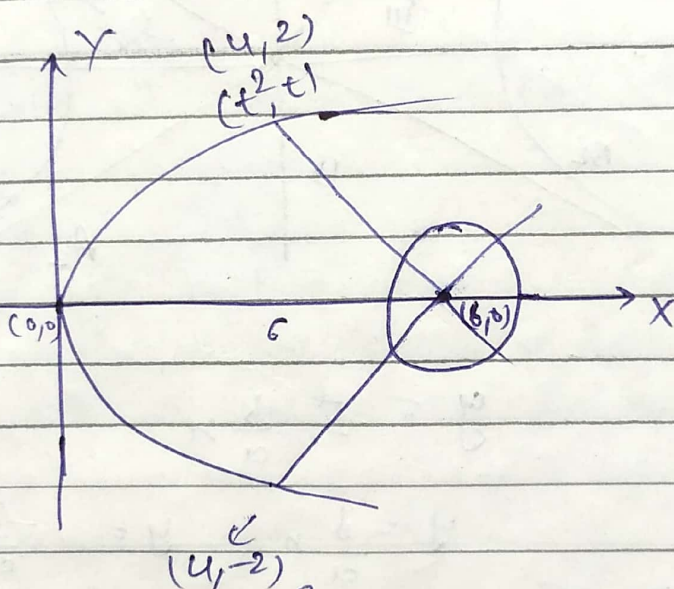
$$x^2 + y^2 - |2x + 3| = 0$$

$$a = 1$$

$$(6, 0)$$

$$x^2 - 12x + 36 + y^2 + 31 = 36$$

$$(x - 6)^2 + y^2 = 5$$



$$y + nt = 2at + at^3$$

$$y + nt = 2t + t^3$$

$$0 + 6t = 2 + t^3$$

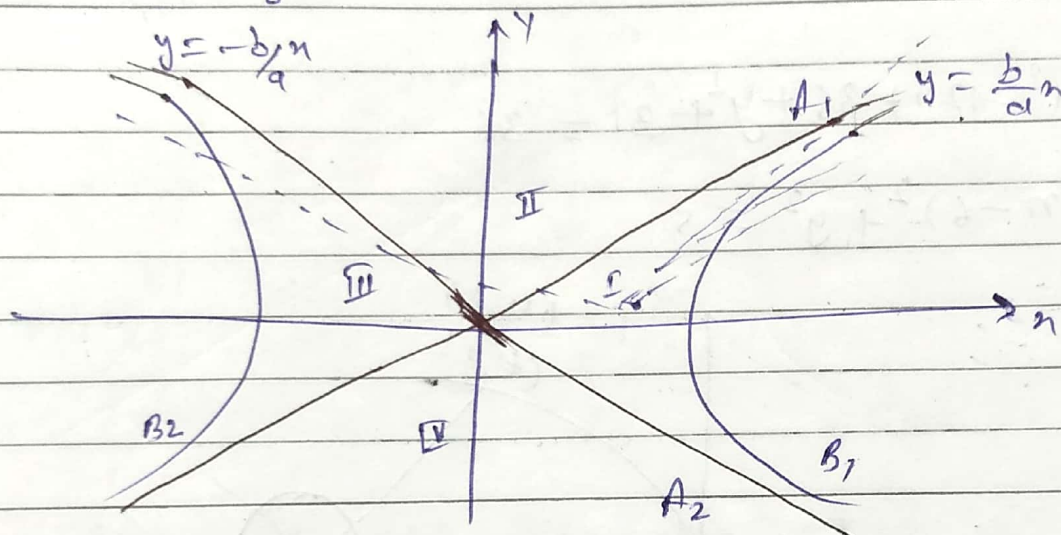
$$t^3 - 4t = 0$$

$$t = 0, t = 2, t = -2$$



\* More about asymptote of hyp!

$$H \equiv \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

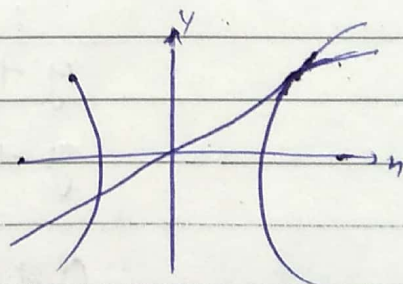


$$y = \pm \frac{b}{a} x$$

$$y = \frac{b}{a} x, \quad y = -\frac{b}{a} x$$

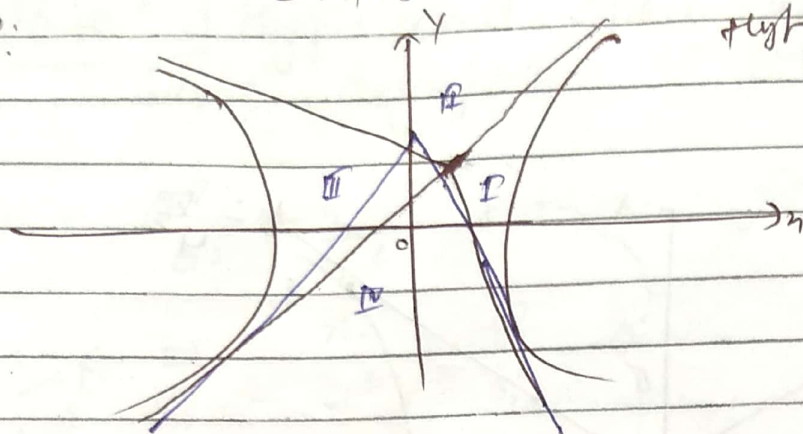
$$\frac{x}{a} - \frac{y}{b} = 0 \quad \text{or} \quad \frac{x}{a} + \frac{y}{b} = 0$$

$$A \equiv \frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$$





Dec. whose <sup>two</sup> tangents are drawn at the point  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$



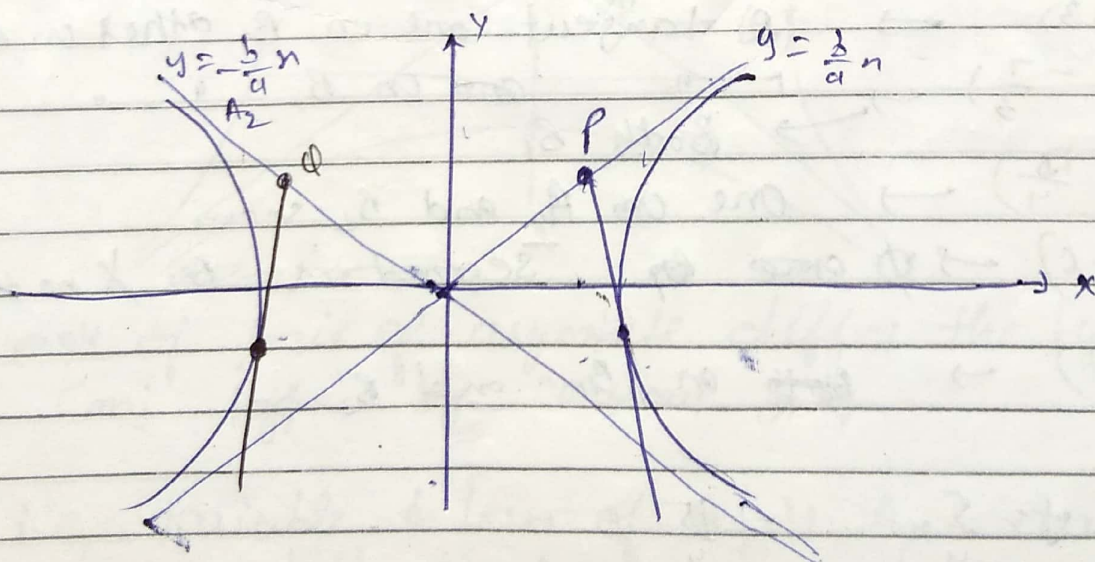
Both asymptote in different regions

Pt lie in Region I  $\Rightarrow$  two tangents to the branch  $B_1$

" " " " II  $\Rightarrow$  two tangents to the branch  $B_2$

" " " " III  $\Rightarrow$  one tangent to  $B_1$  and one to  $B_2$

(IV)  $\Rightarrow$  one " " "  $B_1$  and other  $B_2$



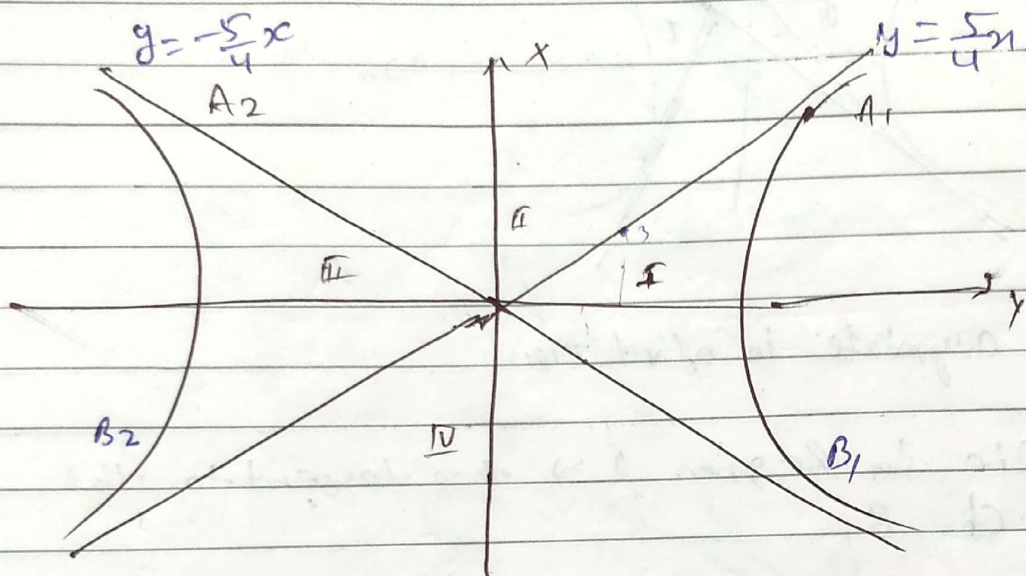
Pt lie on  $A_1$ , then one tangent will be  $A_1$  itself and other will be on the branch  $B_1$

\* If Pt lie on  $A_2$  then one tangent on  $A_2$  itself and other on the branch  $B_2$



Q.

$$\frac{x^2}{16} - \frac{y^2}{25} = 1$$



find which a tangent are possible or not to the hyp. shown. If yes then identify it is on the same branch of the following pt

- (i) (1, 3) → (Q) tangent one on B<sub>1</sub> other on B<sub>2</sub>
- (ii) (3, -7/3) → one on B<sub>1</sub> " "
- (iii) (-3, 15/4) → Both B<sub>1</sub>
- (iv) (7, -6) → one on A<sub>2</sub> and B<sub>2</sub> secant
- (v) (-2, -4) → second is B<sub>2</sub> X no tangent
- (vi) (-2, -4) → Both on B<sub>1</sub> and B<sub>2</sub>

②  $y = \frac{5}{4}x = \frac{15}{4} = 3$

LO  $y = \frac{5(-3)}{4} = -\frac{15}{4} = 3$

out  
→ outside  
+ inside

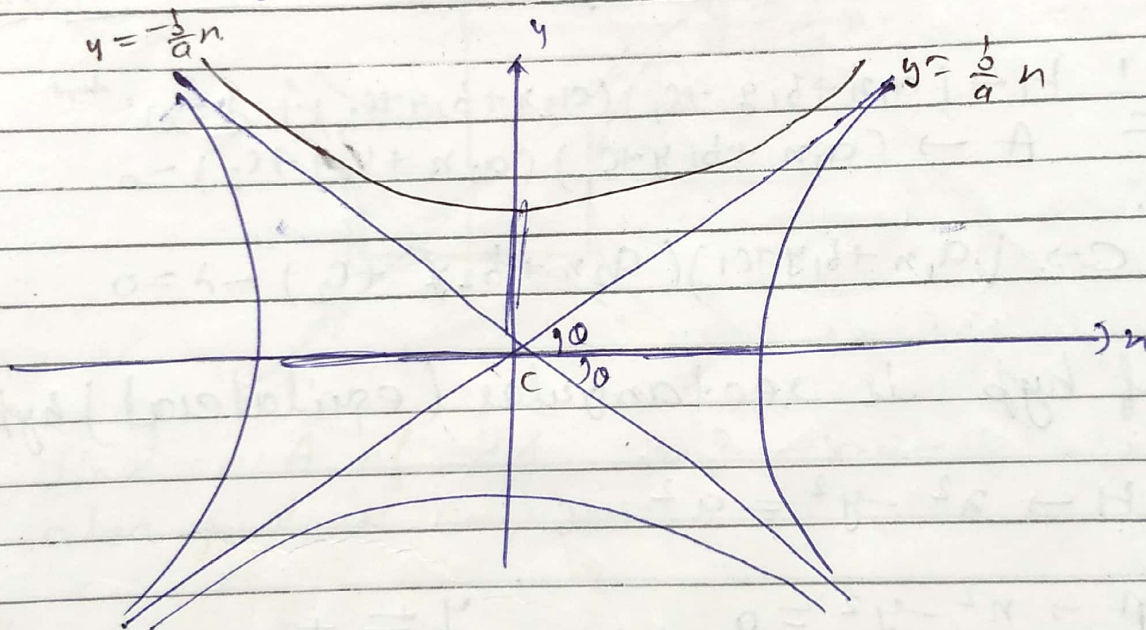


\* Note - If hyp. and their conjugate have same asymptote.

$$H \equiv \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Represent  $\rightarrow A \Rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$   
Pair of straight lines.

$$C \equiv \frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$



eq<sup>n</sup> of pair of asymptote differs the hyp and Conj. hyp. by constant only

i.e variable & term of  $x$ ,  $y$ ,  $H$ ,  $A$ ,  $C$  are same with diff. const. term and their const. term are in AP.

$$2A = H + C$$



$$* H \rightarrow ax^2 + 2hxy + by^2 + 2gx + 2fy + c_1 = 0$$

$$A \rightarrow ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

$$C \rightarrow ax^2 + 2hxy + by^2 + 2gx + 2fy + c_2 = 0$$

$$\S \quad 2c = c_1 + c_2$$

and Const. term are in AP. because A is represent pair of straight lines

$$\text{their } \Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

Remainder

Note  $H \rightarrow (a_1x + b_1y + c_1)(a_2x + b_2y + c_2) + \lambda = 0$  <sup>Asst.</sup>

$$A \rightarrow (a_1x + b_1y + c_1)(a_2x + b_2y + c_2) = 0$$

$$C \rightarrow (a_1x + b_1y + c_1)(a_2x + b_2y + c_2) - \lambda = 0$$

(2) if hyp. is rectangular (equilateral) hyp.

$$H \rightarrow x^2 - y^2 = a^2$$

$$A \rightarrow x^2 - y^2 = 0$$

$$y = \pm x$$

$$C \rightarrow x^2 - y^2 = -a^2$$

$$e = \sqrt{2}$$

(3) Asy. always passes through centre of Hyp.  
and bisector of angle b/w A is axis of Hyp.

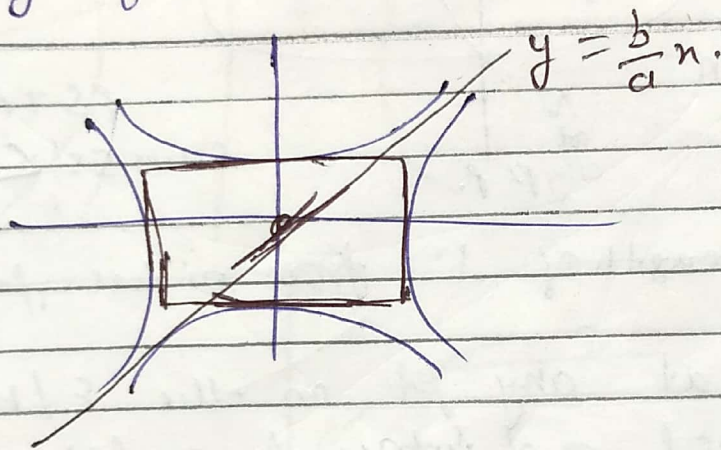


4) A are tangent to the hyp from centre

\* Angle b/w A =  $2\alpha$  then angle

$$e = \sec \alpha$$

5) A of hyp. are the diagonals of rectangle formed by lines drawn through the extremity of each axis parallel to other



Q. find A of hyp.  $3x^2 - 8xy - 2y^2 - 5x + 11y - 8 = 0$   
also find its conjugate

H

$$A \Rightarrow 3x^2 - 8xy - 2y^2 - 5x + 11y + 1 = 0 \rightarrow \text{represent a pair of str. line}$$

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

$$a = 3$$

$$h = -\frac{8}{2} \quad b = -2 \quad g = -\frac{5}{2} \quad f = \frac{11}{2}$$

$$c = 1$$

-4.

$$C \Rightarrow 3x^2 - 8xy - 2y^2 - 5x + 11y + 18 = 0$$



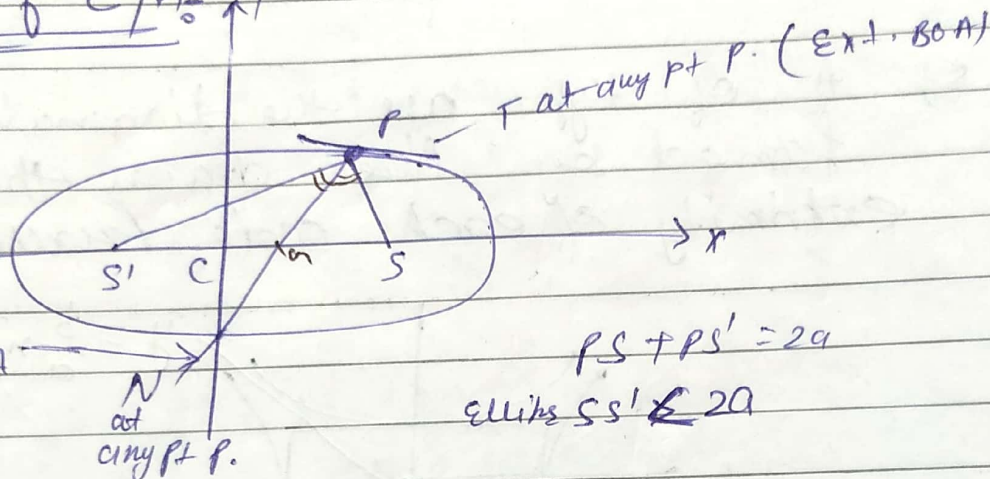
$$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

$$\lambda = -12$$

### \* Properties of E/H

$$\frac{PS}{PS'} = \frac{b'S}{b'S'}$$

This line is ind. B.O.A  
b/w PS & PS'



$$PS + PS' = 2a$$

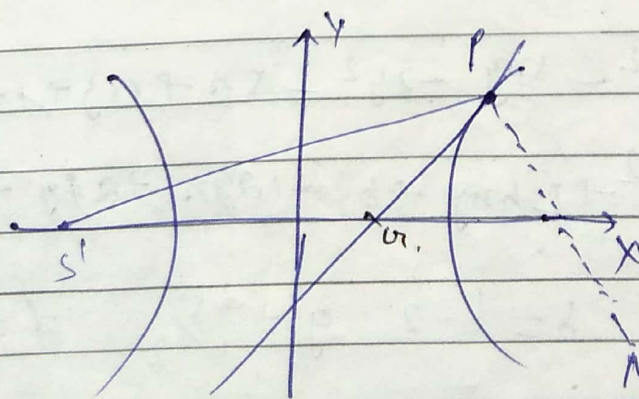
ellipse  $SS' < 2a$

### \* Product of length of L<sup>r</sup> from either foci

\* T and N at any pt on the E/H bisect the External and internal angles b/w the focal dist. of pt P. This refers to the well known properties E/H which state that rays from one focus are reflected to other focus and vice-versa.

$$|PS - PS'| = 2a$$

$b'S' > 2a$



$$\frac{PS}{PS'} = \frac{b'S}{b'S'}$$

Normal at  
any pt P.

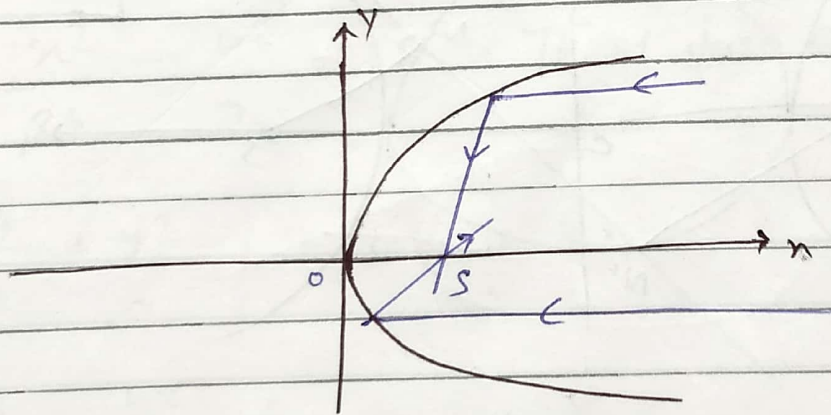
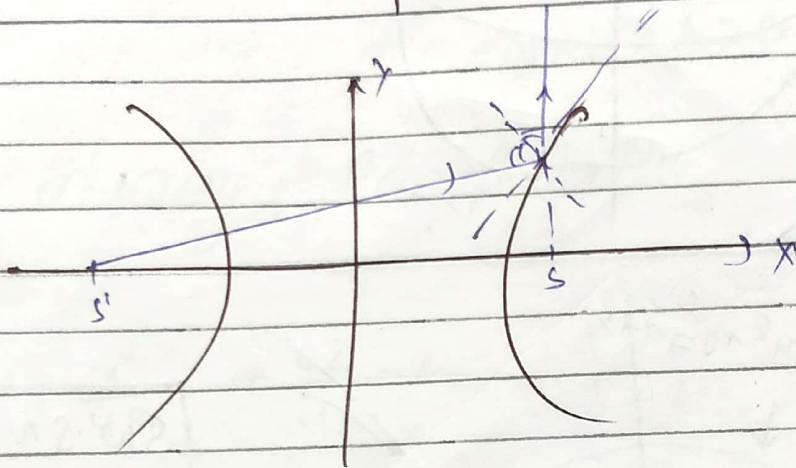
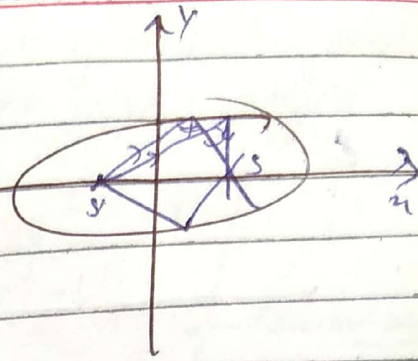
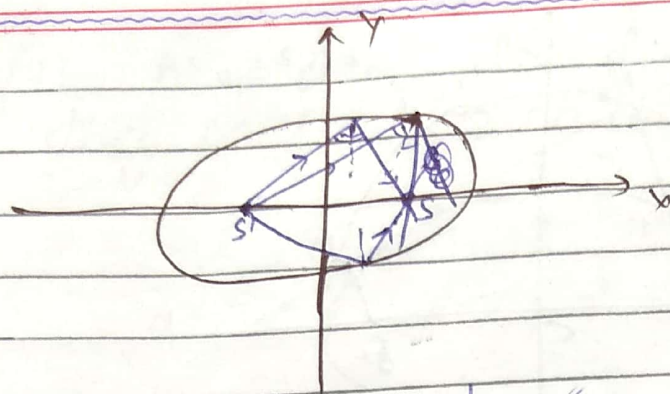
Ext. BOA

T at any

pt P is the BOA

int angle b/w PS & PS'



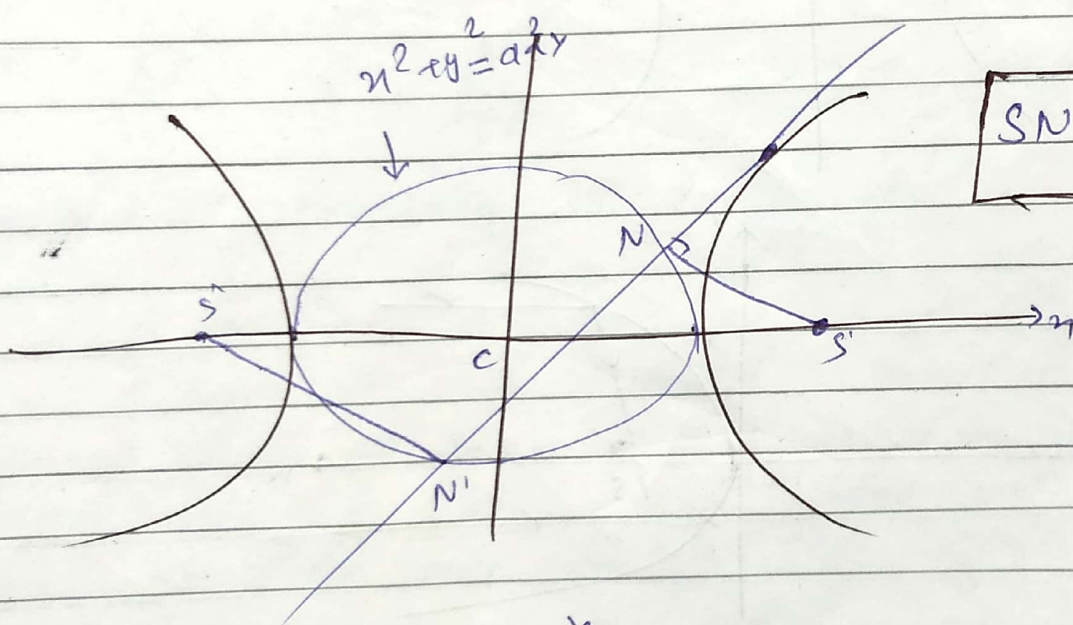
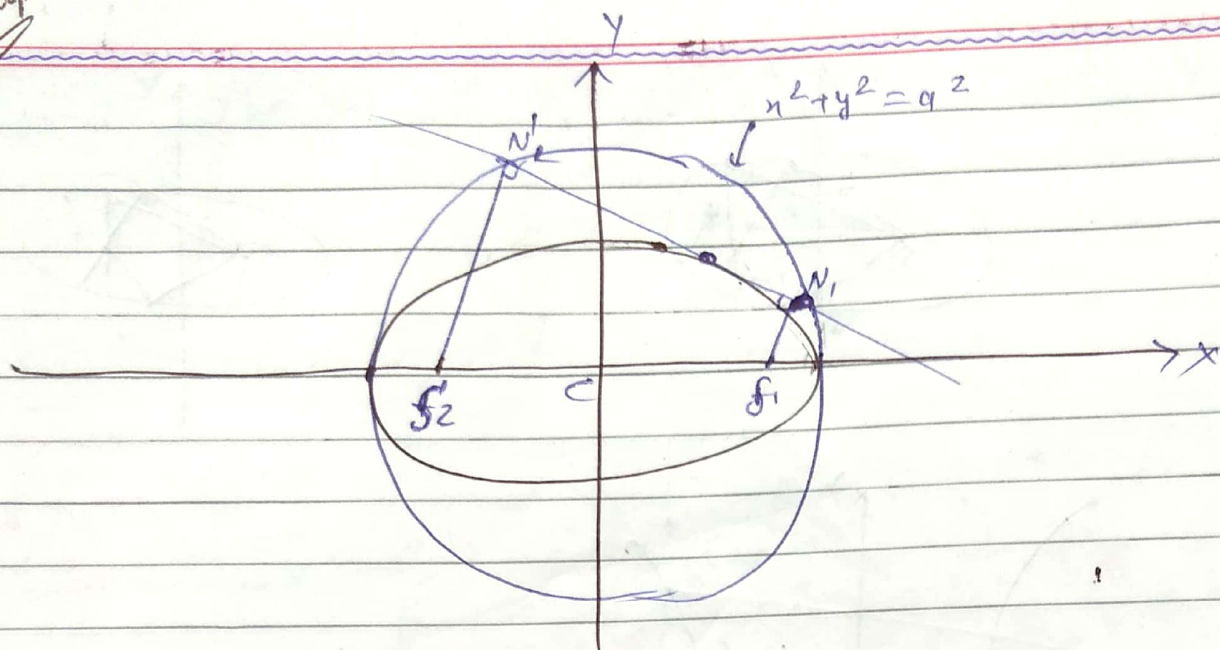


2) Product of Length of  $L^r$  from either foci on a variable tangent to E/H lies on its auxiliary circle is equal to  $(\text{semi-minor axis})^2 / (\text{semi-major axis})^2$

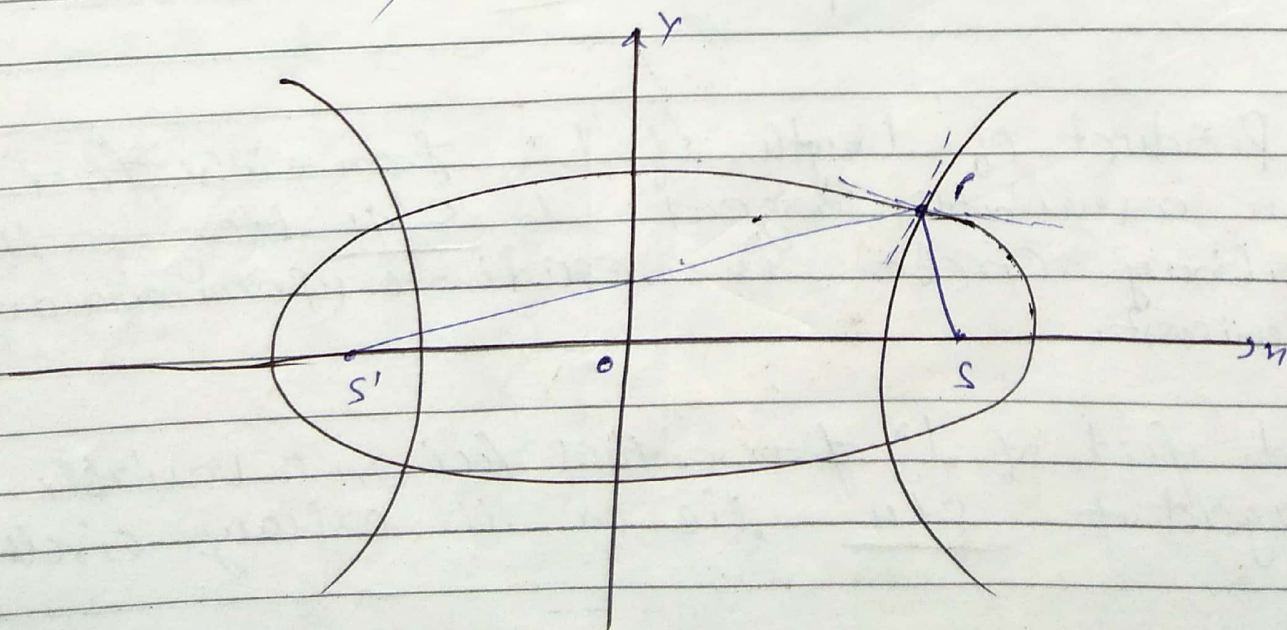
And feet of  $L^r$  from either foci on a variable tangent to E/H lie on its auxiliary circle



Prob.



$$SN \cdot S'N' = b^2$$





\* If E/H have same foci (i.e. they are Confocal) then they will cut each other at right angle.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \& \quad \frac{x^2}{a^2 - k^2} - \frac{y^2}{k^2 - b^2} = 1$$

Remember

$$a > k > b > 0$$

always Confocal.

ex:

$$\frac{x^2}{49} + \frac{y^2}{9} = 1$$

$$a = 7 \\ b = 3$$

$$\text{say } k = 4$$

$$\frac{x^2}{33} - \frac{y^2}{7} = 1$$

These two are confocal

$$\text{ex: } \frac{x^2}{25} + \frac{y^2}{9} = 1$$

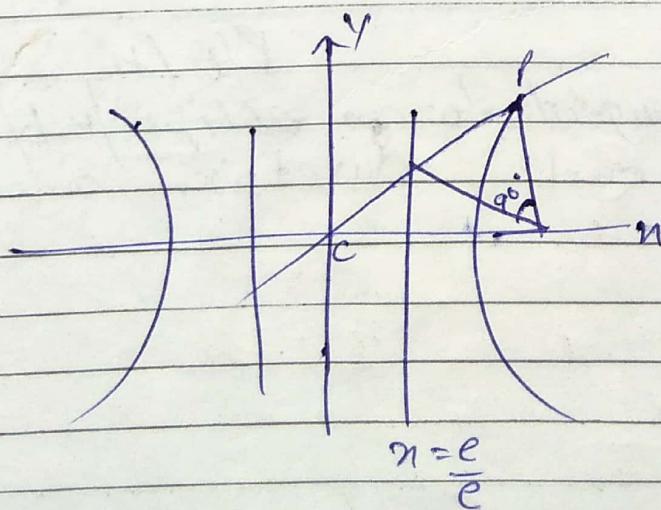
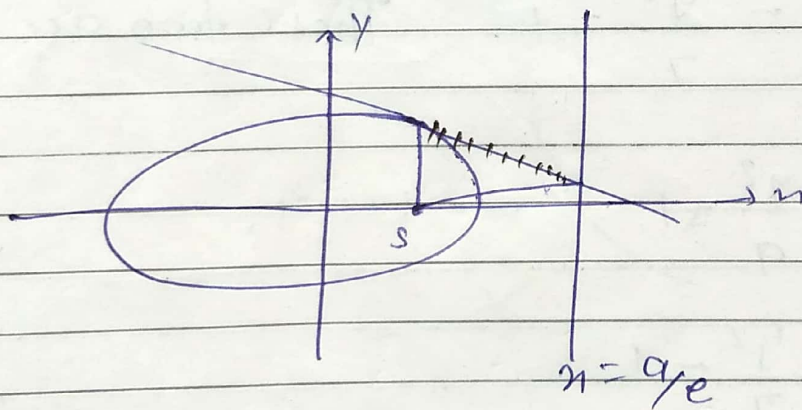
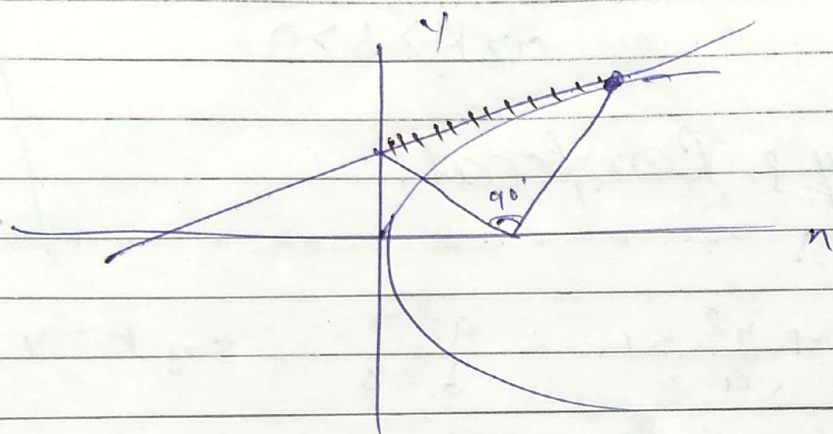
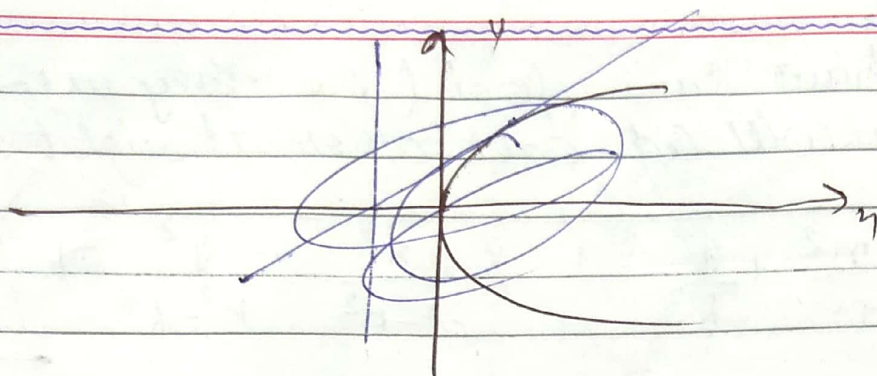
$$k = 4$$

$$\frac{x^2}{9} - \frac{y^2}{7} = 1$$

P/E/H

\* Portion of tangent from ellipse b/w the pt of contact and directrix and subtent right angle







Q. 1

\* For only Ellipse:

if Normal at any Pt. P with centre C meet major and minor axes in G and g respectively

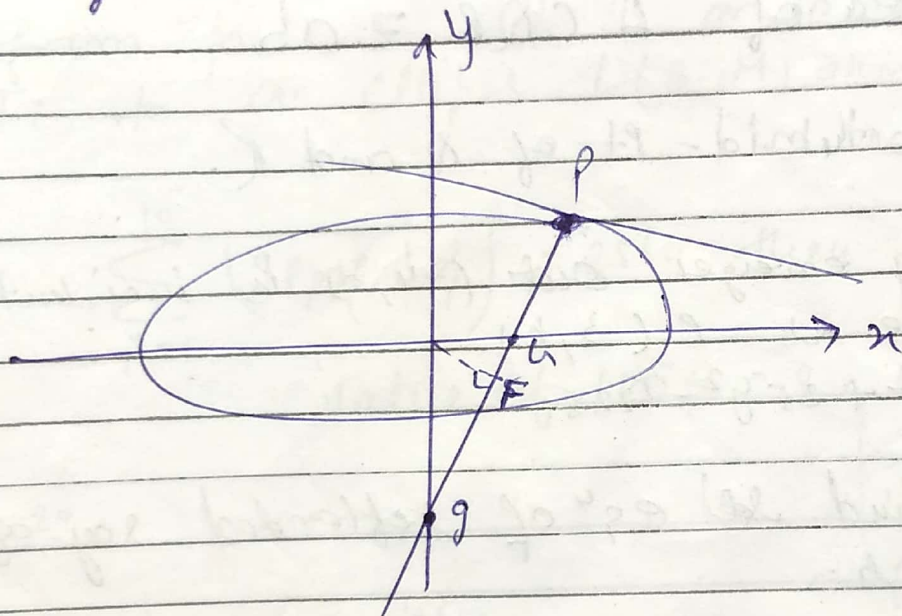
vely

normal then

(i)  $PF \cdot Pg = b^2$

(ii)  $Pf \cdot Pl = a^2$

(iii)  $Pl \cdot Pg = SP \cdot S'P$

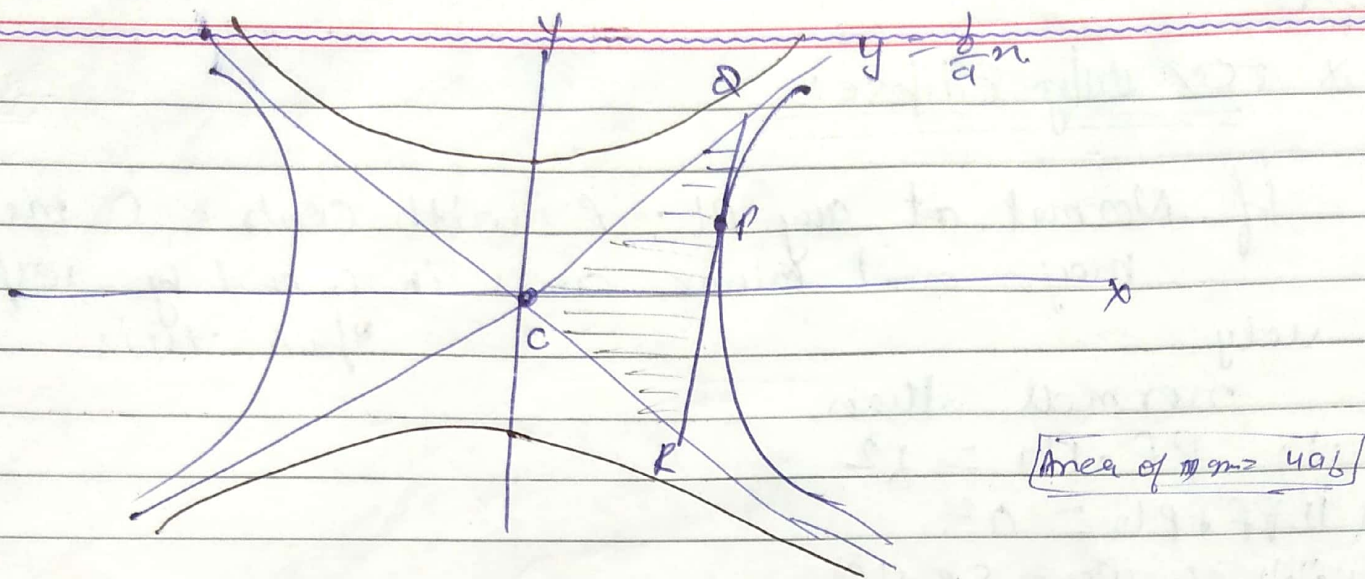


is the locus of mid pt of G and g is another E with same e as that of Original

\* for Hyperbola:

Tan. at any Pt P on a given hyp with centre C meet its Asymptote at Pt Q and R respectively.





then ii Area of  $\Delta CQR = ab$

(ii) P will be the mid-pt of Q and R.

Ques: A ray emerges out  $(-4,0)$  is incident on the ellipse at  $P(3,k)$

$$4x^2 + 25y^2 = 225$$

then find  $e$  of reflected ray after 1st reflection

$$4x^2 + 25y^2 = 225$$

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

$$a = 5, \quad b = 3$$

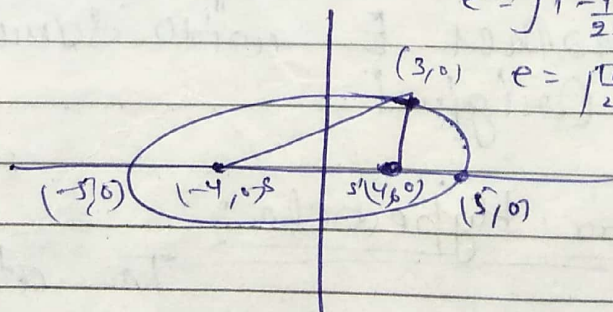
$$e = \frac{4}{5}$$

$$\text{foci} = (ae, 0) \\ (4, 0)$$

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$e = \sqrt{1 - \frac{9}{25}}$$

$$e = \sqrt{\frac{16}{25}} = \frac{4}{5}$$



$$y = mx \pm \sqrt{a^2m^2 + b^2} \\ k = 3m \pm \sqrt{25m^2 + 9}$$



$$= p = 3m$$

$$K = (3, \frac{12}{5})$$

$$\frac{9}{25} + \frac{K^2}{9} = 1$$

$$y - \frac{12}{5} = \frac{145}{3-4} (x-3)$$

Ques let  $P_i$  &  $P_i'$  be the feet of  $\perp$ s drawn from foci  $S$  and  $S'$  on a tangent to an ellipse  $L(S, M)$  and  $a = 20$ .

if  $\sum_{i=1}^{10} (SP_i)(S'P_i') = 2560$  then find

value of  $100e = ?$

$$a = 20$$

$$ae = 20$$

$$(SP_1 \cdot S'P_1') + (SP_2 \cdot S'P_2') + \dots = 2560$$

$$10b^2 = 2560$$

$$b^2 = 16$$

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$= \sqrt{1 - \left(\frac{4}{5}\right)^2} = \frac{3}{5}$$

$$= 100 \times \frac{3}{5} = 60$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{400} + \frac{y^2}{1600(e^2)} = 1$$

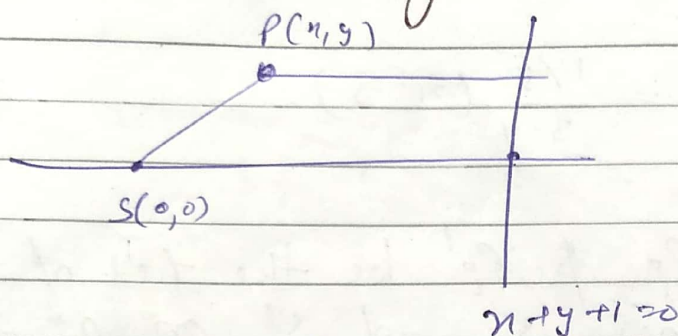
$$SP_1 =$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$= 2560$$



Ques A hyp. has one focus at origin,  $e = \sqrt{2}$  and one of the directrix  $x + y + 1 = 0$  find eq<sup>n</sup> of its asymptote.



$$e = \sqrt{2}$$

$$PS^2 = 2 PM^2$$

$$x^2 + y^2 = 2 \left( \frac{x+y+1}{\sqrt{2}} \right)^2$$

$$x^2 + y^2 = x^2 + y^2 + 1 + 2x + 2y + 2xy$$

$$2x + 2y + 2xy + 1 = 0$$

$$2x + 2y + 2xy + 1 = 0$$

$$(x+1)(y+1) = 0$$

$$h^2 > ab$$

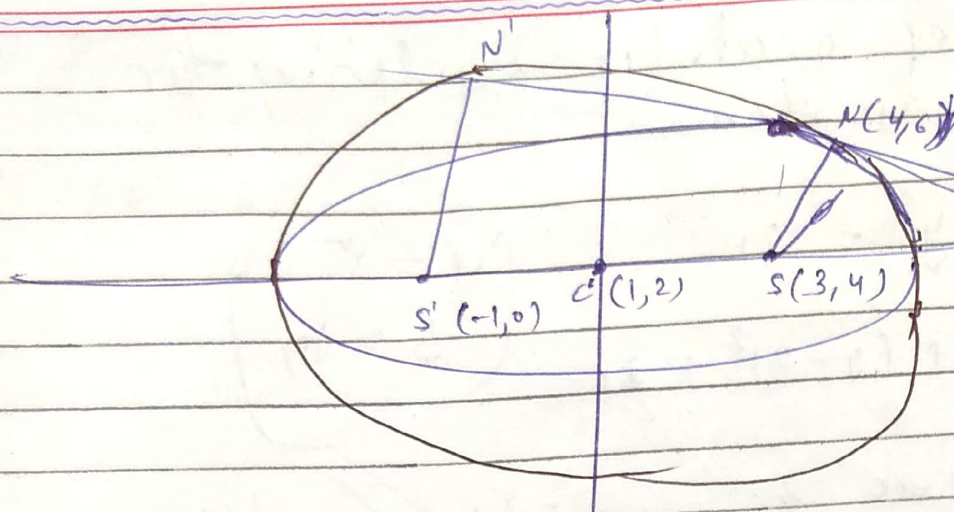
$$\Delta \neq 0$$

$$\Delta = 0$$

Ques Let two focus of an E  $S(-1,0)$  &  $S(3,4)$  and foot of  $\perp$  from focus  $(3,4)$  upon tangent to the Ellipse be  $P \in N(4,6)$

i) find length of semi minor axis





$$y-6 = -\frac{1}{2}(x-4)$$

$$ae = 2$$

$$e \geq \frac{2}{a}$$

$$ae = 2$$

qslope of SN =

$$y-6 = -\frac{1}{2}(x-4)$$

$$M_{SN} = \frac{6-4}{4-3} = 2$$

$$2y - 12 + x - 4 = 0$$

$$x + 2y - 16 = 0$$

$$S'N' = \frac{\sqrt{-1+0-16}}{\sqrt{5}} = \frac{17}{\sqrt{5}}$$

$$SN \cdot S'N' = b^2$$

$$\frac{17}{\sqrt{5}} = b^2$$

$$b^2 = 17$$

(i) find eq<sup>n</sup> of S'N'

$$y-6 = 2(x+1)$$

$$y-6 = -\frac{1}{2}(x-4)$$

$$y-6 = -\frac{1}{2}x + 2$$

$$y + \frac{1}{2}x = 2+6$$

$$y + \frac{1}{2}x = 8$$



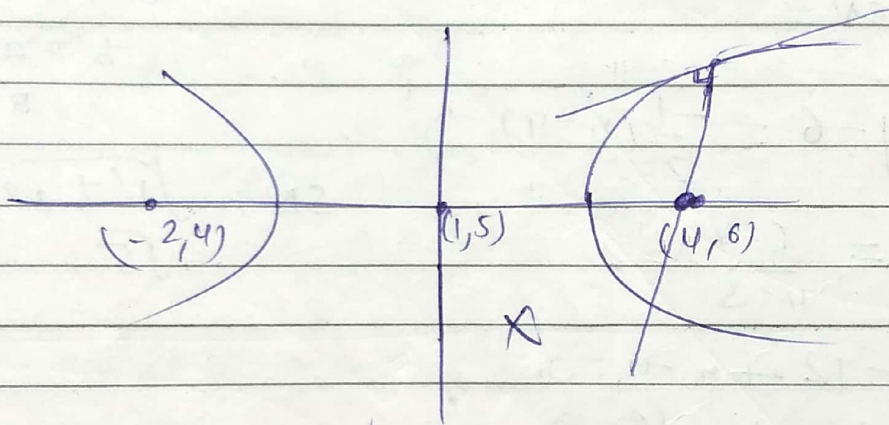
(iii) find eq<sup>n</sup> of auxiliary circle of the ellipse as shown

$$x^2 + y^2 = 21$$

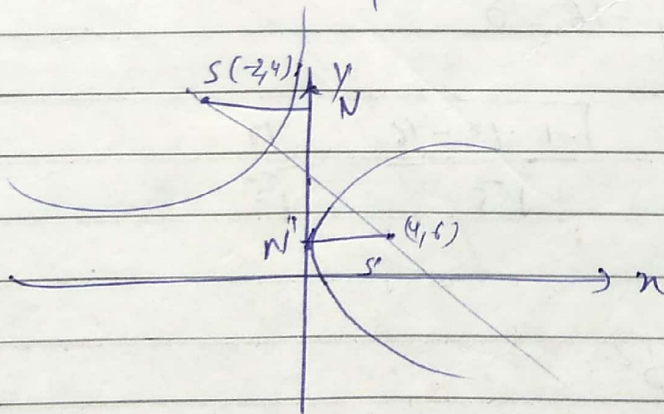
$$(x-1)^2 + (y-2)^2 = 25$$

$$\begin{pmatrix} a = 5 \\ b = 17 \end{pmatrix}$$

Ques! if Hyp. whose foci are  $S(-2, 4)$  &  $S'(4, 6)$  and touches y axis then find its equation.



$$\begin{aligned} &= \frac{a^2}{b^2} \cdot \frac{1}{10} \\ &= \frac{1}{10} \cdot 10 \\ &= 1 \end{aligned}$$



$$b^2 = 8$$

$$SS' = 2ae$$

$$\sqrt{4+36} = 2ae$$

$$2\sqrt{10} = 2ae$$

$$ae^2 = 10 + 1$$

$$SN \cdot S'N' = b^2$$

$$2 \cdot 4 = b^2$$

$$b^2 = a^2 - a^2e^2$$

$$8 + 10 = a^2$$

$$a^2 = 18$$



# \* Rectangular hyperbola:

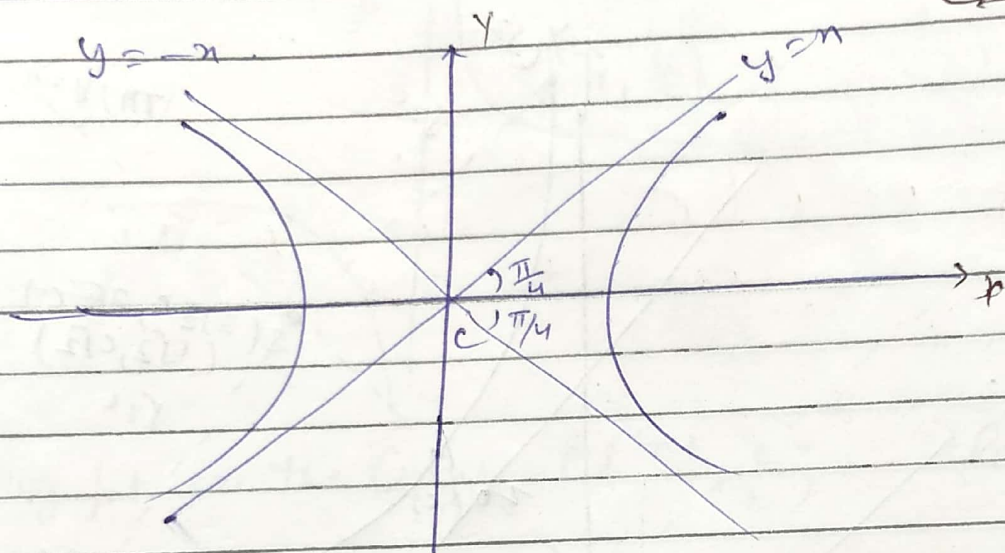
( $a=b$ )

If  $a=b$

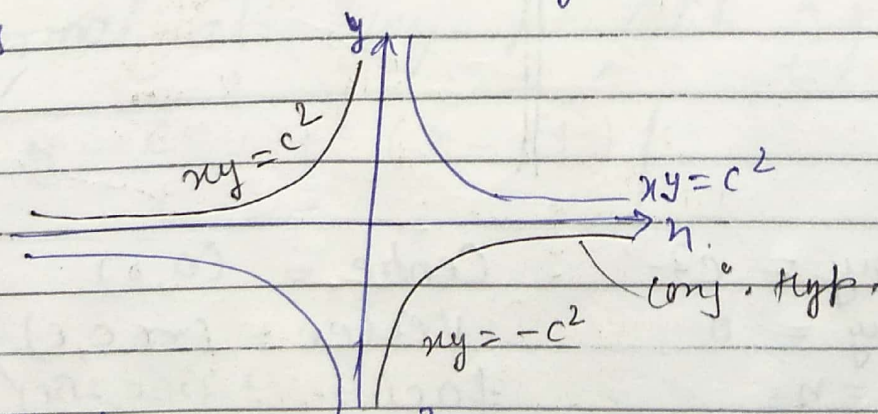
$$x^2 - y^2 = a^2 \quad \text{--- (1)}$$

$$y = \pm x$$

$$e = \sqrt{2}$$



\* Rotating the axes of above hyp. by  $\pm \frac{\pi}{4}$  about the origin the eq<sup>n</sup> of Rectangular hyp. given by eq<sup>n</sup> (1) Reduces to  $xy = \pm \frac{a^2}{2}$  or  $\pm c^2$  and its asymptote are coordinate axis



$$\text{Hyp} \Rightarrow xy = c^2$$

$$A \Rightarrow xy = 0$$

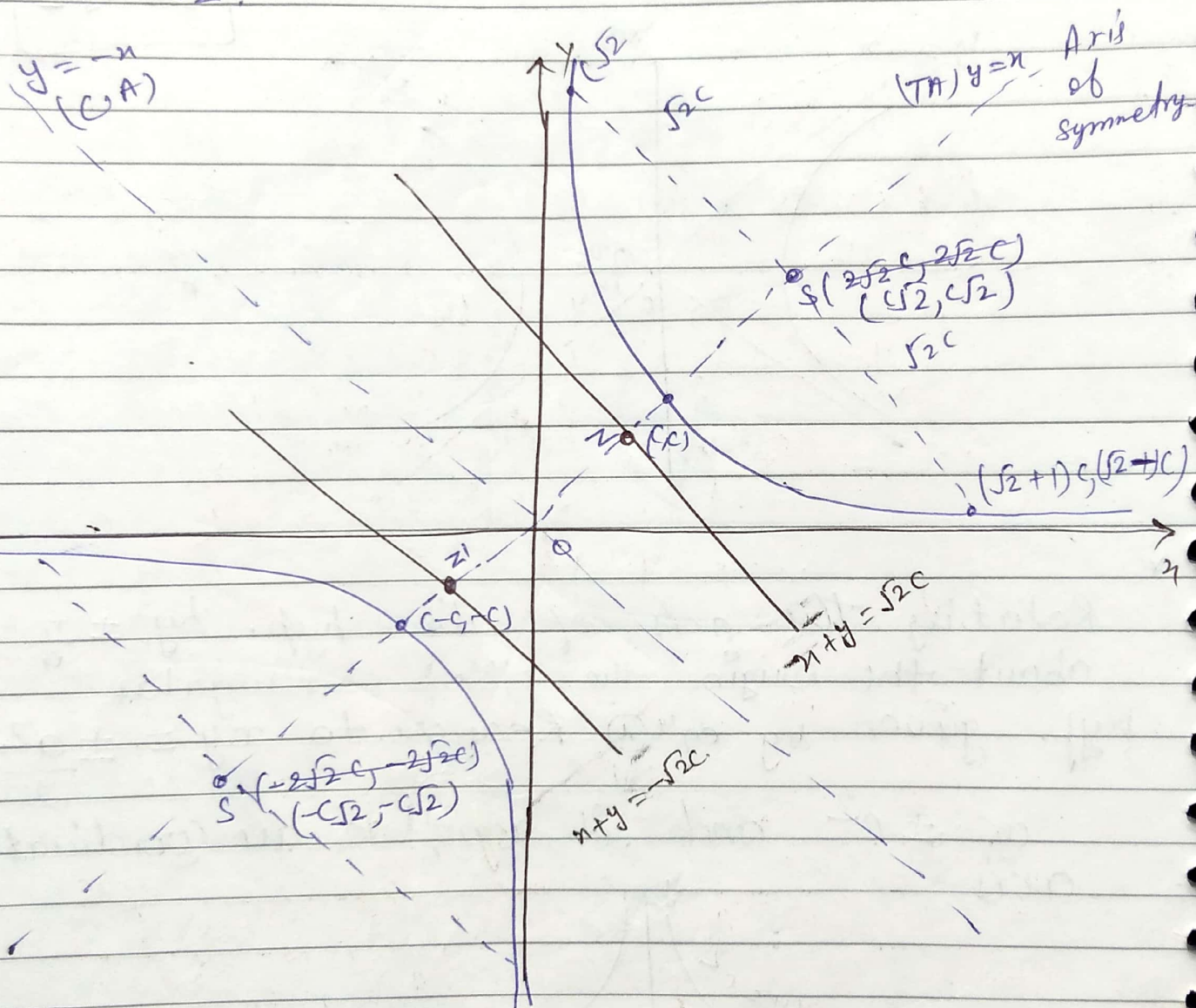
$$\text{Conj. Hyp} \Rightarrow xy = -c^2$$



Equilateral Hyp. means Rectangular Hyp.

\* Show everything for hyp.  $xy = c^2$

$e = \sqrt{2}$



Hyp  $\Rightarrow xy = c^2$

A  $\Rightarrow xy = 0$

TA  $= y = x$

CA  $\Rightarrow y = -x$

Centre = (0,0)

Vertex = (~~c~~ c, c) & (-c, -c)

foci =  $\pm (2\sqrt{2}c, 2\sqrt{2}c)$

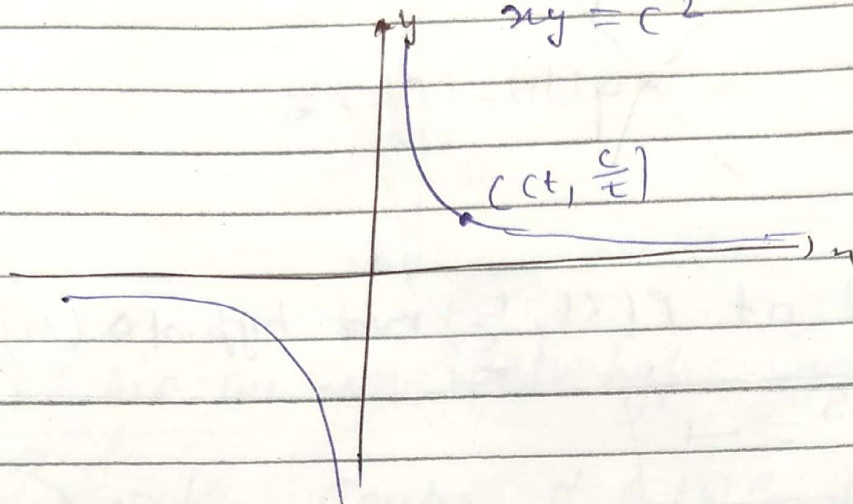
L(LA) =  $2\sqrt{2}$



eq<sup>n</sup> of rectangular circle is !

$$x^2 + y^2 = 2c^2$$

$$xy = c^2$$



\* Any pt on the Curve  $P(ct, \frac{c}{t})$  (Parametric)

for ex<sup>t</sup>  $x=4 \Rightarrow$  any pt  $(2t, \frac{2}{t})$

\* tangent  $(x\frac{c}{t} + yct) = c^2$

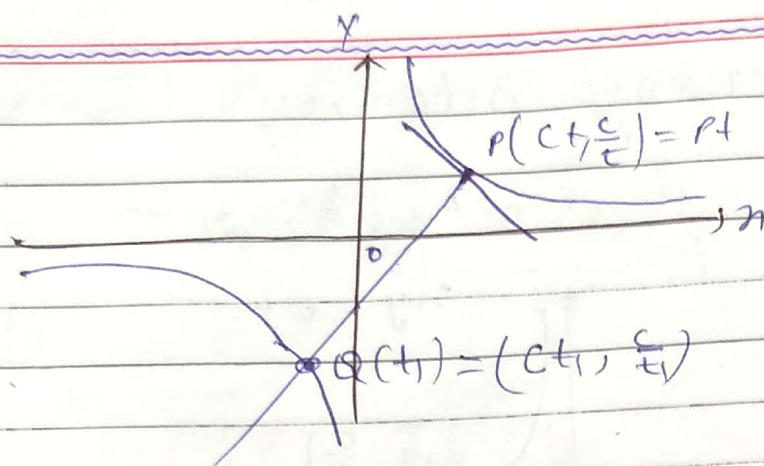
$$\boxed{\frac{x}{t} + yt = c} \quad \text{--- tangent at any pt } (ct, \frac{c}{t})$$

\* Normal at any pt.  $(ct, \frac{c}{t})$

$$\boxed{y - \frac{c}{t} = t^2(x - ct)}$$



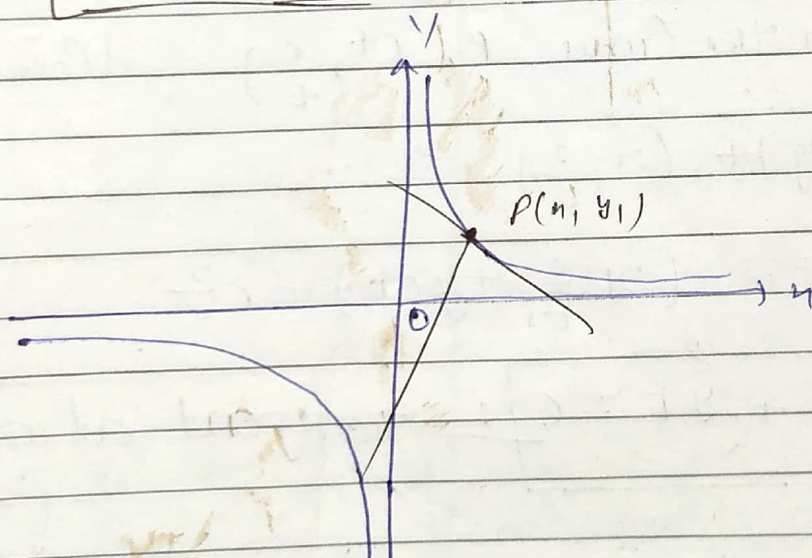
\*



\* Normal at  $P(ct, \frac{c}{t})$  meet hyp. at  $Q(t_1)$   
then

$$t, t^3 = -1$$

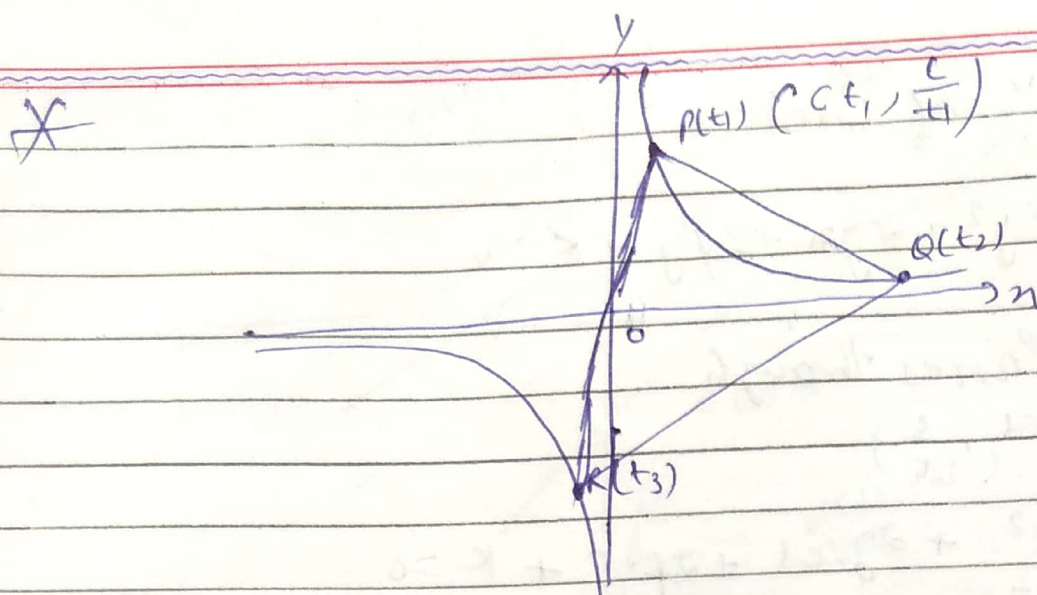
\*



\* eq<sup>n</sup> of tangent in Cartesian form,

$$\frac{x}{x_1} + \frac{y}{y_1} = 2$$





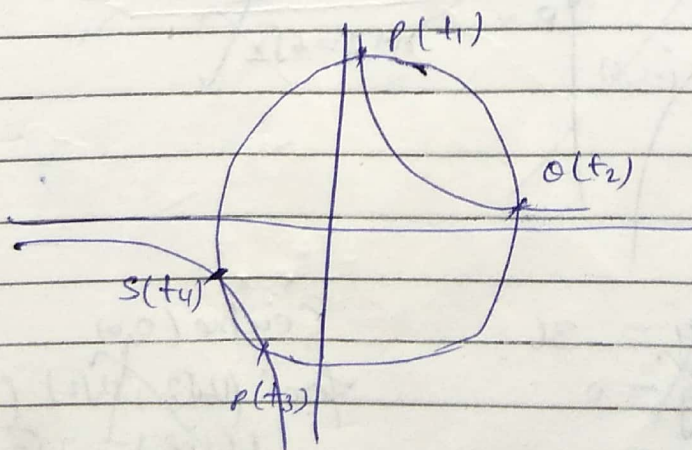
$P, Q, R$  lie on the rect. hyp  $xy = c^2$

then ortho centre of  $\Delta PQR$  lie on the hyp. itself and its coordinates is

$$\left( -\frac{c}{t_1 t_2 t_3}, -c t_1 t_2 t_3 \right)$$

Ques: if a circle and Rectang hyp  $xy = c^2$  meet in four points  $P(t_1), Q(t_2), R(t_3), S(t_4)$  then P.T

$$t_1 t_2 t_3 t_4 = 1$$





Suppos eq<sup>y</sup> of circle

$$x^2 + y^2 + 2gx + 2fy + K = 0$$

circle passes through  
 $(ct, \frac{c}{t})$

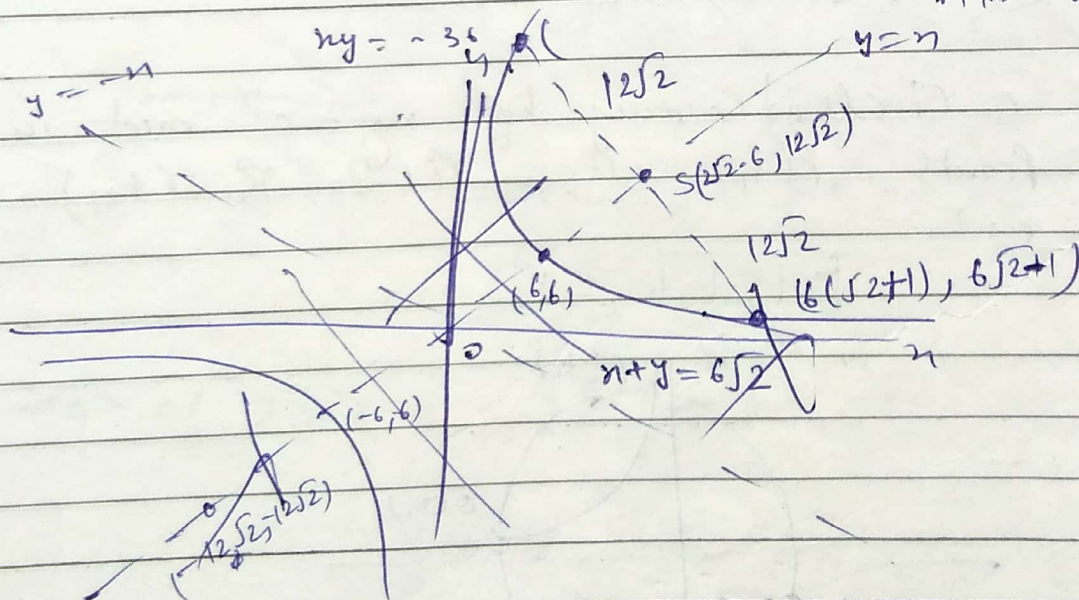
$$c^2 t^2 + \frac{c^2}{t^2} + 2g \cdot ct + 2f \cdot \frac{c}{t} + K = 0$$

$$c^2 t^4 + 2gct^3 + kt^2 + 2fct + c^2 = 0$$

$$t_1 t_2 t_3 t_4 = \frac{c^2}{c^2} = 1$$

6,

Que: find  $xy = -36$



$$xy = -36$$

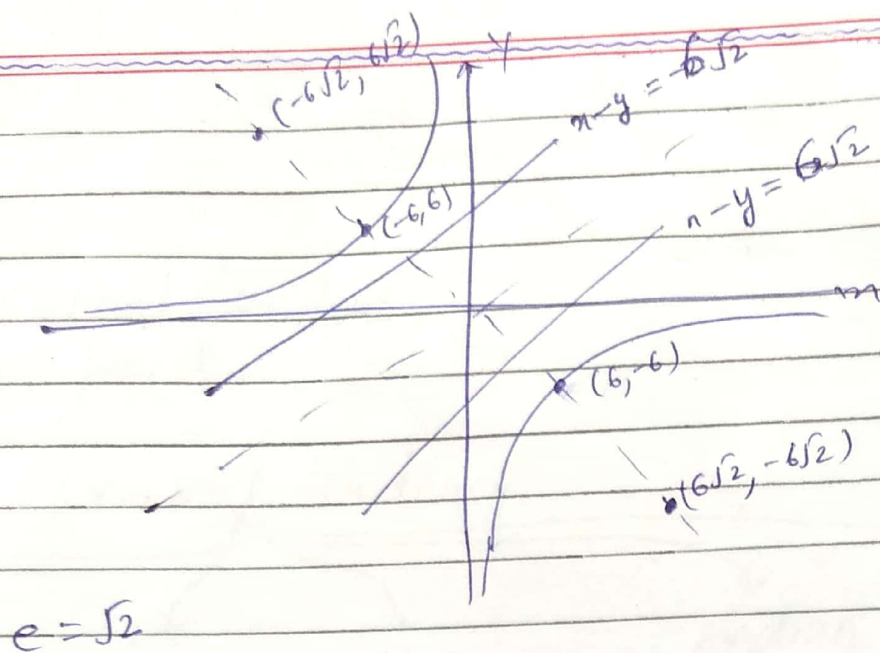
A  $\Rightarrow xy = 0$

Centre (0,0)

foci  $(12\sqrt{2}, 12\sqrt{2})$   $(-12\sqrt{2}, -12\sqrt{2})$

$4(a.e) = 2\sqrt{2}$





SBG STUDY