

SBG STUDY

03/08/17

chapter 10

Differential Equation:

* Diff. eq:

An eqⁿ involved independent variable (i.e. x) dependent variable (i.e. y) and their derivatives (differential coefficients i.e. y' , y'' , y''' - - - - -) is called differential equation.

For ex:

$$\frac{dy}{dx} + 4xy = 0 \quad \text{---} \quad * \quad \frac{du}{dx} + 2 \frac{du}{dy} = 0 \quad \text{---}$$

$$\frac{dy}{dx} + 2x = 0 \quad \text{---}$$

Partial diff. equation

$$\left(\frac{d^2y}{dx^2} \right) + 2x \left(\frac{dy}{dx} \right) + 3y = 0 \quad \checkmark$$

$$x + 3y = 0 \quad \times$$

$$\frac{dy}{dx} = x \quad \checkmark$$

* Order and degree of diff. equation!

Order:

Highest diff. coefficient occurs in it

Degree:

Exponent of Highest order diff. coefficient with the diff. eqⁿ with expressed as a polynomial in all the diff. coefficient.

$$f(x, y) \cdot \left(\frac{d^m y}{dx^m} \right)^p + g(x, y) \cdot \left(\frac{d^{m+1} y}{dx^{m+1}} \right)^q \dots = 0$$

Order = m

degree = p

($q, p \in \text{Natural No.}$)

(i) The exponent of all diff. coefficient should be from radical and fraction.

(ii) degree is always positive natural no.

(iii) the degree of diff. eqⁿ may or may not exist

Ques: $3ny \left(\frac{d^2y}{dn^2} \right)^4 - 4n \left(\frac{d^3y}{dn^3} \right)^2 + y = 0$

O = 3

D = 2

Q. $3ny \left(\frac{d^2y}{dn^2} \right)^5 - 4n \left(\frac{d^3y}{dn^3} \right) + y = 0$

O = 3

degree = 1.

Q. $\frac{d^2y}{dn^2} = e^{\frac{dy}{dn}}$

$(e^n = 1 + n + \frac{n^2}{1^2} + \dots + \infty)$

$= 1 + y' + \frac{(y')^2}{1^2} + \dots + \infty$

Order = 2

degree = N.A

Because diff. eqⁿ coeff. can not be expressed in form of polynomial.

∴ degree is not applicable

Q. $y = e^{\frac{dy}{dn}}$

Solⁿ: $\log y = \frac{dy}{dn}$

Order = 1 degree = 1

$$Q.1 \quad \int \frac{d^2 y}{dn^2} = \int \frac{dy}{dn} + 3$$

Order = 2

Degree = ~~N.B.~~

$$(8) \quad y = 1 + \frac{dy}{dn} + \frac{1}{2} \left(\frac{dy}{dn} \right)^2 + \frac{1}{6} \left(\frac{dy}{dn} \right)^3 - \dots$$

Order = 1 Degree = ~~N.B.~~ 1

$$y = e^{\frac{dy}{dn}}$$

$$Q.2 \quad \frac{dy}{dn} + y = \frac{dn}{dy} \left(\frac{dy}{dn} \right)^2 + y \left(\frac{dy}{dn} \right) = 1 \quad (3)$$

Order = 1

Degree = 2

$$e^{\frac{dy}{dn^2}} - n \frac{d^2 y}{dn^2} + y = 0$$

Order = 3 Degree = N.A

$$(5) \quad \sin^{-1} \left(\frac{dy}{dn} \right) = n + y \quad \frac{dy}{dn} = \sin(n+y) \quad (4) \quad \frac{d^2 y}{dn^2} = \left[y + \left(\frac{dy}{dn} \right)^6 \right]^{1/4}$$

Order = 1 Degree = ~~N.B.~~ 1

Order = 2 Degree = 4

$$(6) \quad \ln \left(\frac{dy}{dn} \right) = an + by$$

Order = 1 Degree = 1

$$\frac{dy}{dn} = e^{an+by}$$

$$(7) \quad \frac{d^4 y}{dn^4} - \sin \left(\frac{d^3 y}{dn^3} \right) = 0$$

Order = 4
Degree = N.E



Formation of diff. eqⁿ!

We have to find diff. equation whose solution is $f(n, y, c_1, c_2, \dots, c_n) = 0$ where $c_1, c_2, \dots, c_n$ are arbitrary independent constant. we have to eliminate constant for which we required $n+1$ equations.

DE for above eqⁿ is obtained as follows

(i) diff. given eqⁿ with 1. to n at many times as the no. of const. in it

(m+1)

(ii) eliminate arbitrary constant.

(iii) eliminant is the required diff. eqⁿ.

Note:

(i) DE always represent family of curve satisfying some common properties

(ii) No. of Independent Constant in the solⁿ of DE is equal to order of corresponding DE

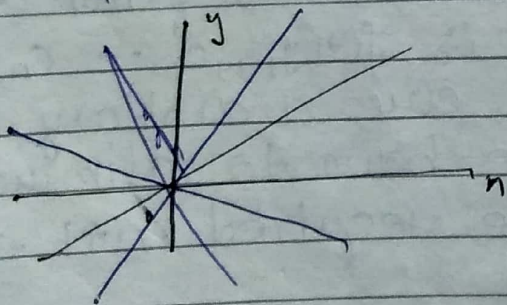
Ex: 1:

$$y = mx \quad \text{--- (1)}$$

$f(n, y, m) = 0$
no. of indep. constant = $n = 1$

$$\frac{dy}{dn} = m \quad \text{--- (2)}$$

$$\frac{dy}{dn} = \frac{y}{n}$$



Ex: 2: $y = an^2 + bn + c \quad \text{--- (1)}$

$$f(n, y, a, b, c) = 0$$

$$\frac{dy}{dn} = 2an + b \quad \text{--- (2)}$$

$$\frac{d^2y}{dn^2} = 2a \quad \text{--- (3)}$$

$$\frac{d^3y}{dn^3} = 0 \quad \text{--- (4)}$$

It is the require DE corresponding to the given eqⁿ.

line passes through parabola in a plane.



Solⁿ of DE:

It is relation b/w variables of eqⁿ or not containing the derivatives but satisfy the given DE

$$\frac{dy}{dx} = \frac{y}{x}$$

$$\frac{dy}{y} = \frac{dx}{x}$$

$$\int \frac{dy}{y} = \int \frac{dx}{x} + \text{Int. C}$$

$$\ln y = \ln x + \ln c$$

$$= \ln cx$$

$$y = cx$$

It is called General Solution.

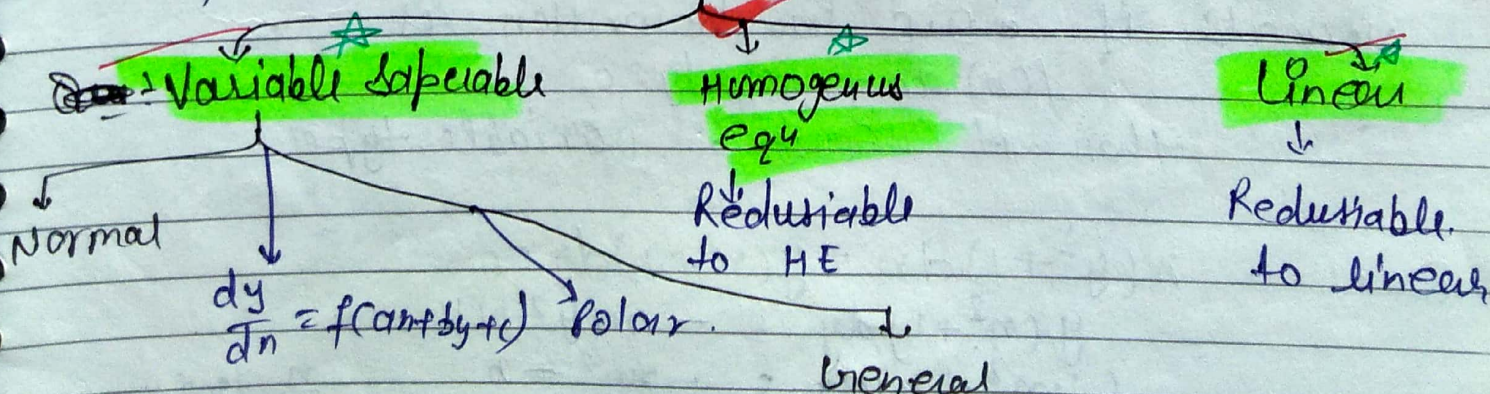
$$y = (2)x$$

Particular Solution.

When Particular value is attached to c called Particular Solution

$$\left(\frac{dy}{dx} \right) = f(x, y)$$

first order first degree DE.



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* Ques: And no. of Independent const.

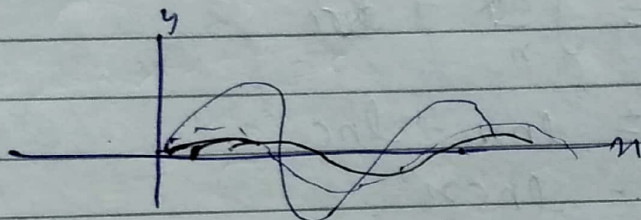
4) $y = Ae^{n+B}$
 $= e$

$= (A \cdot e^n \cdot e^B)$

Independent const.
 $= Ae^n = 1$

(2) $y = A \sin n + B \cos(n+C) \Rightarrow A \sin n + B \cos C - B \sin n \cos$
 $(A - B \sin C \cos n) \sin n + (B \cos C) \cos n$

(3) $y = A \sin(n+B)$



$A (\sin n \cos B + \cos n \sin B)$
 $A \cos B \sin n + (A \sin B) \cos n$

$\lambda \quad \lambda = 2 \text{ Const.}$

$\lambda \sin n + \lambda \cos n$

Ques: $y = (C_1 + C_2) \cos(n + C_3) - (C_4 e^{n+C_5})$
 $= 3$

Variable Separable:

Normal: If O.D.E. can be written as
 $f(n) dn + g(y) dy = 0$
 then it is ~~can be~~ variable type.

Ques: $n(y^2+1)dn + y(n^2+1)dy = 0$
 $\frac{y(n^2+1)dy}{(y^2+1)} = -\frac{n(y^2+1)dn}{(y^2+1)}$
 $= \frac{y dy}{(y^2+1)} = -\frac{n}{n^2+1}$

$\frac{n}{n^2+1} dn + \frac{y}{y^2+1} dy = 0$

$$\int \frac{n}{n^2+1} dn + \int \frac{y dy}{y^2+1} = 1$$

$$n^2+1 = k$$

$$n dn = \frac{1}{2} dk$$

$$\frac{1}{2} \int \frac{dk}{k} + \frac{1}{2} \int \frac{dp}{p} = 1$$

$$\frac{1}{2} \ln k + \frac{1}{2} \ln p = 1$$

$$\ln kp = 2$$

$$kp = e^2$$

$$(n^2+1)(n^2+1) = \text{constant.}$$

Type : 2 $\frac{dy}{dn} = f(an+by+c)$

$$an+by+c = k$$

diff w.r.t n

$$a+b \frac{dy}{dn} = \frac{dk}{dn}$$

$$\frac{dy}{dn} = \left(\frac{dk}{dn} - a \right) \cdot \frac{1}{b}$$

$$\frac{1}{b} \left(\frac{dk}{dn} - a \right) = f(k)$$

$$\frac{dk}{dn} - a = b f(k)$$

$$\frac{dk}{a+b(k)} = dn$$

$$\frac{dk}{dn} = b f(k) + a$$

$$\frac{dk}{b f(k) + a} = dn$$

Ques 1 $\sin^{-1} \frac{dy}{dn} = n + 2y$

$$\sin^{-1} \frac{dy}{dn} = n + 2y \quad \Rightarrow \quad \sin^{-1} \frac{dy}{dn} = 2y = n$$

$$\frac{dy}{dn}$$

$$\frac{dy}{dn} = 2y = \frac{n}{\sin^{-1}}$$

$$\frac{dy}{dn} = 2y = \sin^{-1} n$$

$$\frac{dy}{dn} = \sin(n + 2y) \quad \text{--- (1)}$$

$$n + 2y = k$$

$$1 + 2 \frac{dy}{dn} = \frac{dk}{dn}$$

$$\frac{dy}{dn} = \left(\frac{dk}{dn} - 1 \right) \frac{1}{2}$$

$$\frac{1}{2} \left(\frac{dk}{dn} - 1 \right) = \sin k$$

$$\frac{dk}{dn} - 1 = 2 \sin k$$

$$\frac{dk}{1 + 2 \sin k} = dn$$

Type-3 $\frac{dy}{dn} = \frac{a_1 n + b_1 y + c_1}{a_2 n + b_2 y + c_2}$

where $b_1 + a_2 = 0$

In this case simply cross multiply,
Note the Perfect diff. of (x, y) i.e. $d(x, y) = x dy + y dx$, and integrate term by term.

$$\left. \begin{aligned} y &= n^2 \\ \frac{dy}{dn} &= 2n \end{aligned} \right\}$$

cross

= y =

But tan

$$y = n^2 \quad \rightarrow \quad \frac{dy}{dn} = 2n \frac{dn}{dn}$$

$$\downarrow \quad dy = 2n dn$$

$$\boxed{d(ny) = n dy + y dn}$$

$$d\left(\frac{n}{y}\right) = \frac{y \cdot dn - n dy}{y^2}$$

Q. $y^3 = n^2 + y^2 \sin n$

$$3y^2 dy = 2n dn + 2y dy \sin n + y^2 \cos n dn$$

Q. $d(\tan^{-1} \frac{y}{n})$

$$\tan^{-1} \frac{y}{n} = \frac{1}{1 + \frac{y^2}{n^2}} \cdot d\left(\frac{y}{n}\right) = \frac{1}{1 + \frac{y^2}{n^2}} \left(\frac{n dy - y dn}{n^2} \right)$$

Ques! $\frac{dy}{dn} = \frac{n - 2y + 1}{2n - y}$ $-2 + 2 = 0$

$$= dy(2n - y) = dn(n - 2y + 1) \quad \cdot dy = \frac{dn(n - 2y + 1)}{(2n - y)}$$

$$2n dy - y dy - n dn + 2y dn - dn = 0$$

$$d(ny) - y dy - n dn - dn = 0$$

integrate w.r.t n.

$$\boxed{2ny - \frac{y^2}{2} - \frac{n^2}{2} - n = C}$$

$$= \frac{dn(n - 2y + 1)}{(2n - y)}$$

$$= d\left(\frac{y}{n}\right) = \frac{(n - 2y + 1)}{(2n - y)}$$

$$= \frac{1 - 2 \frac{dy}{dn}}{2n - y}$$

$$\begin{aligned} xdn - ydy &= rdx \\ xdn + ydy &= rdr \end{aligned}$$

$$\frac{xdy - ydx}{r^2} = \sec^2 \theta d\theta$$

$$\begin{aligned} xdy - ydx &= r^2 d\theta \\ xdy - ydx &= r^2 \sec^2 \theta d\theta \end{aligned}$$

type 4: polar

Some times Cartesian transform into polar form is help ful to solve dE

So put $x = r \cos \theta$

$y = r \sin \theta$

Here r and θ both are variable

$$x^2 + y^2 = r^2$$

$$xdx + ydy = rdr$$

$$\tan \theta = \frac{y}{x}$$

$$xdy - ydx = r^2 d\theta$$

$$\sec^2 \theta d\theta = \frac{xdy - ydx}{r^2}$$

Note!

Some times

New System

$$x = r \sec \theta$$

$$y = r \tan \theta$$

$$xdx - ydy = rdr$$

$$xdy - ydx = r^2 \sec^2 \theta d\theta$$

Que:

$$xdx + ydy = x^2 dy - xy dx$$

$$r \sec^2 \theta dr + r \tan \theta dy$$

$$= r^2 dy - r \sec \theta \tan \theta dr$$

$$r \tan \theta dy - r \sec^2 \theta dy = -r \sec \theta \tan \theta dr - r \sec^2 \theta dr$$

$$n dn + y dy = n(n dy - y dn)$$

$$r dr = r \cos \theta \cdot r^2 d\theta$$

$$\frac{dr}{r^2} = \cos \theta d\theta$$

Q. 5 $\frac{dy}{dn} = \sin(n+y) - \cos(n+y)$

$$n+y = p$$

$$1 + \frac{dy}{dn} = \frac{dp}{dn}$$

$$\frac{dp}{dn} - 1 = \sin p - \cos p$$

$$\frac{dp}{1 + \sin p - \cos p} = dn$$

$$\frac{dy}{dn} = \sin(n+y) - \cos(n+y)$$

$$= \frac{dy}{dn} = \cos(n+y)$$

$$= \frac{dy}{dn} = \cos(n+y) \cdot \frac{dp}{dn} = \cos(n+y) \cdot (1 + \frac{dy}{dn})$$

Q. 6 $\ln\left(\frac{dy}{dn}\right) = 3n + 4y$ with $y(0) = 0$ [for Constant]

$$= \ln\left(\frac{dy}{dn}\right) = 3n + 4y$$

$$= \frac{dy}{dn} = e^{3n+4y}$$

$$= \frac{dy}{dn} = e^{3n} \cdot e^{4y}$$

$$= \frac{dy}{e^{4y}} = e^{3n} dn$$

$$e^{-4y} dy = e^{3n} dn$$

Ans

$$\int e^{-4y} dy = \int e^{3n} dn + 1$$

$$\frac{e^{-4y}}{-4} = \frac{e^{3n}}{3} + 1$$

Put $n=0$ then

In that case $y=0$

Ques: $\frac{dy}{dn} = \frac{2n+3y-1}{4n+6y-5}$

$$= \frac{2n+3y-1}{2(2n+3y)-5}$$

$$= f(2n+3y)$$

$$2n+3y = t$$

$$= \frac{dy}{dn} = \frac{(2n+3y-1) + (1+y)t}{4n+6y-5}$$

Q. $\frac{dy}{dn} = \frac{n-y}{n+y}$

$$n dy + y dy - n dn + y dn = 0$$

$$d(ny) + y dy - n dn = 0$$

$$ny + \frac{y^2}{2} - \frac{n^2}{2} = C$$

$$= \frac{dy}{dn} = \frac{n-y}{n+y}$$

$$= \frac{dy}{dn} = \frac{n+y}{n+y} \cdot \frac{n-y}{1+\frac{y}{n}}$$

$$\frac{dy}{dn} = \frac{n-y}{1+\frac{y}{n}}$$

$$\frac{dy}{dn} = \frac{n-y}{1+\frac{y}{n}}$$

Ques: $\frac{dy}{dn} = e^{n-y} + n^2 e^{-y}$

$$= \frac{dy}{dn} = e^n \cdot e^{-y} + n^2 \cdot e^{-y}$$

$$\frac{dy}{dn} = e^{-y} (e^n + n^2)$$

Ans $\Rightarrow \frac{dy}{dn} = e^{-y} (e^n + n^2)$

$$e^y dy = (e^n + n^2) dn$$

Ques: $\frac{ndn + ydn}{\sqrt{n^2 + y^2}} = \frac{ydn - ndy}{n^2}$

$$= n^2 (-ndn + ydn)$$

$$n^2 (ndn + ydn) = ydn - ndy \cdot (n^2 + y^2)^{1/2}$$

$$n^2 (ndn + ydn + ndy (n^2 + y^2)^{1/2}) = ydn - ndy \cdot \sqrt{n^2 + y^2}$$

$$n^2 (ndn + ydn + ndy (n^2 + y^2)^{1/2} + ndy) = ydn$$

H.W: 0-1 $\Rightarrow 1, 2, 3, 6, 8,$
 3-1 $\Rightarrow$ P.N: 29 $\Rightarrow 1, 2(a), 3$ to 9

Ans: $\frac{xdx}{x} = -d\left(\frac{y}{n}\right)$

$\frac{ydn}{n^2} = -$

$= -d(\tan \theta)$

$\int dx = - \int d(\tan \theta) + C$

$x = -\tan \theta + C$

$\int n^2 + y^2 = -\frac{y}{n} + C$

Homogeneous eqn:

DE of the form $\frac{dy}{dn} = \frac{f(n,y)}{g(n,y)}$

Here $f(n,y)$ and $g(n,y)$ are Homogeneous fn of n and y and of same degree is called Homogeneous eqn. It is always reducible in the form of $h\left(\frac{y}{n}\right)$. Homogeneous eqn is solved by putting $y = mn$ $\rightarrow$ variable.

where $\frac{dy}{dn} = m + n \frac{dm}{dn}$

$\Rightarrow \frac{dy}{dn} = \frac{(n-2y)}{(3n+4y)}$ Homogeneous eq. Deg. 1

$\Rightarrow \frac{dy}{dn} = \frac{n^2 - y^2}{n^2 + 3ny}$ Hom. eq.

$\frac{dy}{dn} = \frac{n-y-1}{n+y}$

X

$$* \frac{dy}{dn} = \frac{n-2y}{3n+4y} = \frac{1-2 \cdot \frac{y}{n}}{3+4 \frac{y}{n}} = h\left(\frac{y}{n}\right). \text{ Homo. eq.}$$

Que: $\frac{dy}{dn} = \frac{n+y}{n-y}$ $dy =$

Put $y = mn$. $= \frac{1+m}{1-m}$

$$m + \frac{dm}{dn} = \frac{1+m}{1-m}$$

$$n \cdot \frac{dm}{dn} = \frac{1+m}{1-m} - m$$

$$= \frac{1+m-m+m^2}{1-m}$$

$$\frac{1-m}{1+m^2} dn = \frac{dm}{dn}$$

Que: Solve $\frac{dy}{dn} = \frac{y}{n} + \tan \frac{y}{n}$

$$n + n \frac{dm}{dn} = m + \tan m$$

$$\cot m \, dm = \frac{dn}{n}$$

(2) $n \cos m \cdot \frac{dy}{dn} = y \cos m \frac{y}{n} + n$

$$(\cos m) \cdot \left[m + n \frac{dm}{dn} \right] = m \cos m + 1$$

$$n \cos m \cdot \frac{dm}{dn} = 1$$

$$\cos m \, dm = \frac{dn}{n}$$

$$dy(n-y) = dn$$

$$dy(n-y) = n+4y(dn)$$

$$n dy - y dy = n dn + 4y dn$$

$$-y dy = n dy + 4y dn$$

$$= \text{Put } n+y = t$$

$$+ 2dn = \frac{dt}{dn}$$

$$dn = \frac{dt}{2}$$

$$y = mn + t$$

$$\frac{dy}{dn} = m + \frac{dn}{dn}$$

$$\frac{dy}{dn} = \frac{y}{n} + \tan \frac{y}{n}$$

$$\frac{dy}{dn} = \frac{y + \tan y}{n}$$

$$\frac{y}{n} =$$

$$\frac{dy}{y + \tan y} = \frac{dn}{n}$$

$$\cot m \, dm = \frac{dn}{n}$$

$$n \cos y \frac{dy}{dn} = y \cos \frac{y}{n} + n$$

$$= \frac{dy}{dn} =$$

$$\frac{dy}{y \cos \frac{y}{n}} = \frac{n \cdot n}{n \cos y}$$

$$y = mn + c$$

Ques: $(n dy - y dx) \cdot \frac{y \sin y}{n} = (y dx + n dy) \cdot \frac{n \cos y}{n}$

$$\frac{n dy - y dx}{n^2} \cdot \frac{y \tan y}{n} = \frac{d(ny)}{n^2}$$

$$d\left(\frac{y}{n}\right) \tan \frac{y}{n} = \frac{d(ny)}{ny}$$

$$\int \tan\left(\frac{y}{n}\right) d\left(\frac{y}{n}\right) = \int \frac{d(ny)}{ny} + \ln 1$$

$$\ln\left(\sec\left(\frac{y}{n}\right)\right) = \ln(ny) + \ln 1$$

$$\sec \frac{y}{n} = ny \cdot 1$$

* equation reducible to

Homogeneous eqⁿ

$$\frac{dy}{dx} = \frac{a_1 x + b_1 y + c_1}{a_2 x + b_2 y + c_2}$$

name const -

eq is not hom eq using proper substitution will make it hom. eqⁿ.

$$b_1 + a_2 = 0$$

earlier written

step-

1. Cross multiply

2. = 0

3. Integrate term by term.

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

written earlier

$$\frac{dy}{dx} = \frac{x - y + 3}{2x - 2y}$$

$$= \frac{x - y + 3}{2(x - y)}$$

$$= \int (a + b y + c)$$

$$= \frac{1}{2} = -\frac{1}{2} \neq \frac{3}{0}$$

not a hom. eqⁿ

$$a_1 x + b_1 y + c_1 = 0 \quad (i)$$

$$a_2 x + b_2 y + c_2 = 0 \quad (ii)$$

after solving the over.

the value of

$$x = \alpha$$

$$y = \beta$$

$$\alpha \text{ and } \beta = \text{const}$$

$$x - y = t$$

$$1 - \frac{dy}{dn} = \frac{dt}{dn}$$

$$1 - \frac{dt}{dn} = \frac{t+3}{dt}$$

$$1 - \frac{t+3}{dt} = \frac{dt}{dn}$$

$$dn = \frac{2t}{t-3} dt$$

$$x = h + \alpha \rightarrow \text{constant.}$$

$$y = k + \beta$$

Variable Variable

$$dn = dh, dy = dk$$

so eqⁿ becomes

$$\frac{dk}{dn} = \frac{a_1 h + b_1 k}{a_2 h + b_2 k}$$

$$a_1 m + b_1 \neq 0 \quad \text{--- (i)}$$

$$\text{Put } k = mh$$

$$\text{then } \frac{dk}{dh} = m + h \frac{dm}{dh}$$

Ques: $\frac{dy}{dn} = \frac{x+y}{2x-y+3}$

$$= \frac{0+y}{2 \cdot 0 + 3}$$

$$\boxed{\begin{matrix} x = h+1 \\ y = k+1 \end{matrix}}$$

$$\frac{dk}{dh} = \frac{h+k}{2h-k}$$

$$= \frac{dn}{dh}$$

$$\frac{dy}{dn} = \frac{dn}{dn} = 1$$

$$= \frac{dy}{dn} = \frac{x+y}{2x-y+3}$$

$$x+y=0 \quad \text{--- (i) } x=0$$

$$2x-y+3=0 \quad \text{--- (ii)}$$

$$= x+y+2x-y=0+3$$

$$x+2x=3$$

$$3x=3$$

$$x=1$$

$$x+y=0$$

$$y=-1$$

$$x = h+1$$

$$y = k+1$$

$$\frac{dk}{dh} = \frac{x(h+1) + y(k+1)}{2x(h+1) - y(k+1)}$$

$$k = mh$$

$$\frac{dk}{dh} = m + h \frac{dm}{dh}$$

$$m + h \frac{dm}{dh} = \frac{1+m}{2-m}$$

$$h \frac{dm}{dh} = \frac{1+m}{2-m} - m = \frac{1+m - m(2-m)}{2-m}$$

$$\frac{2-m}{m^2-m+1} dm = \frac{dh}{h}$$

Put $m = \frac{k}{h}$ $k = \frac{y-1}{x+1}$

Ques

write length of T, S, T, S, N, N,
write Leibniz formula.

Linear DE :-

1st order Linear DE :-

$$\frac{dy}{dx} + P(x) \cdot y = Q(x)$$

Here $P(x)$ and $Q(x)$ are fⁿ of x or const.

$$I. F = e^{\int P(x) dx}$$

Soln:

$$y(I.F) = \int Q(x) \cdot (I.F) dx + A$$

$$= e^{\int P(x) dx} \cdot \frac{dy}{dx} + P(x) \cdot y e^{\int P(x) dx} = Q(x) \cdot e^{\int P(x) dx}$$

$$\frac{d}{dx} (y \cdot e^{\int P(x) dx}) = Q(x) \cdot e^{\int P(x) dx}$$

$$d(y e^{\int P(x) dx}) = Q(x) \cdot e^{\int P(x) dx} \quad \therefore \text{Integrate}$$

Que: $\cos^2 x \cdot \frac{dy}{dx} + y = \tan x$

$$\frac{dy}{dx} + (\sec^2 x) y = \sec^2 x \tan x$$

$$I.F = e^{\int \sec^2 x dx} = e^{\tan x}$$

$$\frac{dy}{dx} = \frac{\tan x}{\cos^2 x} = \sec^2 x \tan x$$

$$\frac{dy}{dx} + \sec^2 x y = \sec^2 x \tan x$$

$$y e^{\tan x} = \int \sec^2 x \tan x e^{\tan x} dx + A$$

1st order

Other form of Linear DE:

$$(1) \quad \frac{dy}{dx} + P(x)y = Q(x), \quad If = e^{\int P(x) dx}$$

$$\text{sol}^n: \quad y(If) = \int Q(x) \cdot (If) dx + C$$

$$(2) \quad \frac{dx}{dy} + P(y)x = Q(y), \quad If = e^{\int P(y) dy}$$

$$x(If) = \int Q(y) (If) dy + C$$

$$(3) \quad f'(y) \frac{dy}{dx} + P(x)f(y) = Q(x)$$

$$f(y) = k$$

$$f'(y) \frac{dy}{dx} = \frac{dk}{dx}$$

$$\frac{dk}{dx} + P(x)k = Q(x), \quad k(If) = \int Q(x) (If) dx + C$$

* Bernoulli's form:

Plurite

$$(4) \quad \frac{dy}{dx} + P(x)y = Q(x) \cdot y^n$$

$$\frac{1}{y^n} \cdot \frac{dy}{dx} + P(x) \frac{1}{y^{n-1}} = Q(x) \quad \frac{1}{y^{n-1}} = k$$

$$\frac{1}{1-n} \frac{dk}{dn} + P(n) \cdot k = Q(n)$$

$$\frac{dk}{dn} + \underbrace{(P(n)(1-n))}_{P(n)} k = \underbrace{(1-n)Q(n)}_{Q(n)}$$

$$\begin{aligned} \frac{1}{y^{n+1}} &= k \\ y^{-(n+1)} &= k \\ y^{-(n+1)-1} &= k \\ y^{-n-2} &= k \\ -n-2 &= \frac{dk}{dn} \end{aligned}$$

$$\frac{1}{y^n} \frac{dy}{dn} = \frac{1}{1-n} \frac{dk}{dn}$$

Que: $n \ln n \frac{dy}{dn} + y = 2 \ln n$

$$\frac{dy}{dn} + y \cdot \underbrace{\left(\frac{1}{n \ln n}\right)}_{P} = \underbrace{\left(\frac{2}{n}\right)}_{Q}$$

$$I f = \int \frac{1}{n \ln n} dn$$

$$\begin{aligned} \ln n &= t \\ \frac{1}{n} dn &= dt \end{aligned}$$

$$e^{\int \frac{dt}{t}} = e^{\ln t} = t = \ln n$$

$$y \ln n = \int \frac{2}{n} (\ln n) dn + I$$

$$\int n \ln n = \frac{n^2}{2} \ln n - \frac{1}{2} n^2$$

$$\begin{aligned} \frac{dy}{dn} + \frac{1}{2} y &= 2 \ln n \\ I f &= e^{\int \frac{1}{2} dn} \end{aligned}$$

Que: $(n+2y^3) \frac{dy}{dn} = y$

$$\frac{dy}{dn} = \frac{y}{n+2y^3}$$

$$\frac{dn}{dy} = \frac{n+2y^3}{y}$$

$$\frac{dn}{dy} + n \left(-\frac{1}{y} \right) = \underbrace{2y^2}_Q$$

$$\begin{aligned} &= (n+2y^3)^2 \\ &= \frac{n^2}{2} + 2 \frac{y^4}{4} = \frac{n^2}{2} + \frac{y^4}{2} \\ &= \frac{dy}{dn} + \frac{n^2}{2} + \frac{y^4}{2} + y = \end{aligned}$$

Labuize.
sign diff. formulae
L + P.S. + diff.

trick!

#3 #4 #5
Q-1: E-1: 16, 14, 15
E-2: 1, 5, 6, 7, 8, 9
Linear: E-3: 2, 3, 5, 6, 7, 8, 9

$$Pf = e^{\int \frac{1}{y} dy} = e^{\ln \frac{1}{y}} = \frac{1}{y}$$

$$x \cdot \frac{1}{y} = \int 2y^2 \cdot \frac{1}{y} dy$$

$$\boxed{\frac{x}{y} = 2 \int y dy + \lambda}$$

$$\tan \theta = \frac{dy}{dx}$$

* Length of Tangent $(T) = y_1 \operatorname{cosec} \theta$

Length of Normal $= y_1 \sec \theta$

length of Sub-tangent $= y_1 \cot \theta$

Length of Sub-normal $= y_1 \tan \theta$

Radius vector of point P = dist. of point P from origin

$$= \sqrt{x_1^2 + y_1^2}$$

* Leibnitz's formulae:

$$* \int_0^{nt} f(n) dn = n \int_0^T f(n) dn \cdot x$$

Not Leibnitz's formula

$$* \frac{d}{dn} \left(\int_{g(n)}^{h(n)} f(t) dt \right) = f(h(n)) \cdot \frac{d}{dn} h(n) - f(g(n)) \cdot \frac{d}{dn} g(n)$$

Property - 5:

$$\int_0^a f(n) dn = \int_0^a f(a-n) dn.$$

$$\int_a^b f(n) dn = \int_a^b f(a+b-n) dn.$$

Que! $\frac{dy}{dn} = \frac{1}{n \cos y + \sin y}$

Ans! $\frac{dn}{dy} = n \cos y + \sin y$

$$\frac{dn}{dy} + n \frac{(-\cos y)}{1} = \frac{\sin y}{1}$$

$$I f = e^{-\int \cos y dy}$$

$$= e^{-\sin y}$$

$$\Rightarrow n e^{-\sin y} = \int \sin y \cdot e^{-\sin y} dy + 1$$

Que! $\frac{dy}{dn} + ny = ny^2$

$$\frac{1}{y^2} \frac{dy}{dn} + \frac{1}{y} \cdot n = n$$

$$\frac{1}{y} = k$$

$$-\frac{1}{y^2} \frac{dy}{dn} = \frac{dk}{dn}$$

$$\frac{dy}{dn} = \frac{1}{n \cos y + \sin y}$$

$$\frac{dy}{dn} - \frac{1}{\sin y} = \frac{1}{n \cos y}$$

$$\frac{dy}{dn} = \frac{n \cos y + \sin y}{n \cos y + \sin y}$$

$$\frac{dy}{dn} + ny = ny^2$$

$$\frac{dy}{dn} + ny - ny^2$$

$$dy + n(y - y^2) = 0$$

$$\frac{dn}{dn} + n(y - y^2) = 0$$

$$\frac{dn}{dy} \cdot \frac{1}{n(1-y^2)} = 0$$

$$\frac{dn}{dy} \cdot \frac{1}{n} = -y - y^2$$

$$\frac{dn}{dy} = -y - y^2$$

$$- \frac{dk}{dn} + k = n$$

$$\frac{dk}{dn} + k(-n) = -n$$

$$k \cdot e^{-n} = \int -n e^{-n^2/2} dn + \lambda$$

$$If = e^{\int -n dn} = e^{-n^2/2}$$

Ques!

$$n(n^2+1) \frac{dy}{dn} = y(1-n^2) + n^2 \ln n$$

$$\frac{n(n^2+1)}{n(n^2+1)} \frac{dy}{dn} = \frac{y(1-n^2)}{n(n^2+1)} + \frac{n^2 \ln n}{n(n^2+1)}$$

$$\frac{dy}{dn} + \left(- \frac{(1-n^2)}{n(n^2+1)} \right) y = \frac{n \ln n}{n^2+1}$$

$$If = e^{\int - \frac{(1-n^2)}{n(n^2+1)} dn} = e^{\int \frac{1-\frac{1}{n^2}}{n+\frac{1}{n}} dn}$$

$$If = e^{\int \frac{n^2-1}{n(n^2+1)} dn} = e^{\int \frac{(n^2-1)(n+1)}{n(n+1)(n^2+1)} dn}$$

$$= e^{\int \frac{n^2-1}{n(n^2+1)} dn}$$

$$= e^{\int \frac{n^2-1}{n(n^2+1)} dn} = \int n \frac{\ln n}{n^2+1} \cdot \frac{n^2-1}{n(n^2+1)} dn + \lambda$$

$$If = e^{\int \frac{1-\frac{1}{n^2}}{n+\frac{1}{n}} dn}$$

$$= e^{\int \frac{dp}{p}} = e^{\ln p} = p = n + \frac{1}{n}$$

$$y \left(n + \frac{1}{n} \right) = \int \frac{n^2 \ln n}{n(n^2+1)} \cdot \left(n + \frac{1}{n} \right) dy + \lambda$$

Ques! $\frac{dy}{dn} = \frac{y}{2y \ln y + y - n}$

$$\frac{dn}{dy} = \frac{2y \ln y + y - n}{y} \quad \text{I.P.C.}$$

$$= 2 \ln y + 1 - \frac{n}{y}$$

Let $n =$

$$\frac{dy}{dn} = \frac{y}{(2y \ln y + y - n)}$$

$$\frac{dn}{dy} = \frac{2y \ln y + y - n}{y}$$

$$= \frac{dn}{dy} + \frac{n}{y} = \frac{2y \ln y + y}{y}$$

$$= \frac{dn}{dy} + \left(\frac{1}{y}\right)n = \frac{2y \ln y + y}{y}$$

$$\text{I.F.} = e^{\int \frac{1}{y} dy} = e^{\ln y} = y$$

$$ny = \int (2 \ln y + 1) dy =$$

Ques! $\frac{dy}{dn} + \frac{1}{n} \sin 2y = n^3 \cos^2 y$

$$\left(\frac{1}{\cos^2 y} \frac{dy}{dn} + \frac{1}{n} (2 \tan y) \right) = n^3$$

$$\tan y = k$$

$$\sec^2 y \frac{dy}{dn} = \frac{dk}{dn}$$

$$\frac{dk}{dn} + k \cdot \frac{2}{n} = \frac{n^3}{n}$$

$$k \cdot n^2 = \int n^3 e^{n^2} dn + A$$

$$\text{I.F.} = e^{\int \frac{2}{n} dn}$$

$$= e^{2 \ln n}$$

$$= e^{\ln n^2}$$

$$= n^2$$

$$\frac{dy}{dn} + n^3 \cos^3 y = -\frac{1}{n} \sin 2y$$

$$= \frac{dy}{dn} + (-n^3 \cos^3) y = -\frac{1}{n} \sin 2y$$

$$\text{I.F.} = \int -n^3 \cos^3$$

$$= + \frac{n^4}{4} \cos^3 n \cdot n^3 \sin^3 n$$

$$= \frac{n^4}{4} \cos^3 n \cdot n^3 \sin^3 n$$

$$n = \sin 2y$$

$$\frac{dn}{dy} = \cos^2 y = k$$

$$\sin^2 y dy = \frac{dk}{dn}$$

$$= 2 \sin y dy = \frac{dk}{dn}$$

$$\frac{dy}{dx} + (m)y^n = \frac{p}{x}$$

Ques: $\frac{dy}{dn} - \frac{\tan y}{n+1} = (1+n)e^n \sec y$

$$\frac{dy}{dn} - \frac{\sin y}{(n+1)\cos y} = (1+n)e^n \frac{1}{\cos y}$$

$$\cos y \frac{dy}{dn} - \frac{\sin y}{(n+1)} = (1+n)e^n$$

$$\sin y = k$$

$$\cos y \frac{dy}{dn} = \frac{dk}{dn}$$

$$\frac{dk}{dn} + k \cdot \left(-\frac{1}{n+1}\right) = (n+1)e^n$$

$$IF = e^{-\int \frac{1}{n+1} dn} = e^{-\ln(n+1)} = e^{\ln \frac{1}{n+1}} = \frac{1}{n+1}$$

$$\frac{k}{n+1} = \int \frac{(n+1)e^n}{n+1} dn + C$$

* Equation ~~to~~ Reducible to Linear DE by suitable Substitution:

Ques: Solve: $(n-y^2)dn + 2ny dy = 0$

Ans:

$$(n-y^2)dn + 2ny \frac{dy}{dn} = 0$$

$$y^2 = k$$

$$2y \frac{dy}{dn} = \frac{dk}{dn}$$

$$(n-k) + n \frac{dk}{dn} = 0$$

$$n \frac{dk}{dn} + (-k) = -n$$

$$\frac{dk}{dn} + k \left(-\frac{1}{n} \right) = \frac{-1}{1}$$

Ques: $n \frac{dy}{dn} + y \ln y = n y e^n$

~~$$\frac{n}{n} \frac{dy}{dn} + \frac{y \ln y}{n} = \frac{n y e^n}{n}$$~~

~~$$\frac{n}{n} \frac{dy}{dn} + \frac{y \ln y}{n} = \frac{n y e^n}{n}$$~~

$$\frac{n}{y} \frac{dy}{dn} + \frac{y \ln y}{y} = \frac{n y e^n}{y}$$

$$\ln y = k$$

$$\frac{1}{y} \frac{dy}{dn} = \frac{dk}{dn}$$

$$n \cdot \frac{dk}{dn} + k = n e^n$$

~~$$\frac{dk}{dn} + k \cdot \frac{1}{n} = e^n$$~~

Ques: $\frac{dy}{dn} = \frac{e^y}{n^2} - \frac{1}{n}$

$$e^{-y} \frac{dy}{dn} = \frac{1}{n^2} - \frac{e^{-y}}{n}$$

$$e^{-y} = k$$

$$e^{-y} \frac{dy}{dn} = \frac{dk}{dn}$$

$$-\frac{dk}{dn} = \frac{1}{n^2} - \frac{k}{n}$$

$$\frac{dk}{dn} + k \left(-\frac{1}{n} \right) = -\frac{1}{n^2}$$

~~$$\frac{dy}{dn} = \frac{e^y}{n^2} - \frac{1}{n}$$~~

~~$$\frac{dy}{dn} = \frac{e^y}{n^2} - \frac{1}{n}$$~~

~~$$\frac{dy}{dn} + \frac{1}{n} = \frac{e^y}{n^2}$$~~

~~$$\frac{dy}{dn} + \left(-\frac{e^y}{n^2} \right) = -\frac{1}{n}$$~~

~~$$\frac{e^y}{n^2} = k$$~~

~~$$e^{n^2 e^y} - 2 n e^y = k$$~~

~~$$= \frac{e^y (n^2 - 2n)}{n^4} \frac{dk}{dn}$$~~

Q. $\frac{dy}{dn} = \frac{y^2 - n}{2y(n+1)}$

$2y \frac{dy}{dn} = \frac{y^2 - n}{(n+1)}$

$y^2 = k$

$2y \frac{dy}{dn} = \frac{dk}{dn}$

$\frac{dk}{dn} = \frac{k - n}{n+1}$

$\frac{dy}{dn} = \frac{y^2 - n}{2y(n+1)}$

$= \frac{y^2 (1 - \frac{n}{y^2})}{2y(n+1)}$

$= \frac{\frac{y^2}{2y(n+1)} - \frac{n}{2y(n+1)}}{2y(n+1)}$

$n+1 = k$

$k \frac{dn}{dy} = \frac{dk}{dn}$

$y(y - \frac{n}{y})$

X

$y - \frac{n}{y} = k$

Que: $(n^2 + y) \frac{dy}{dn} = 6n$

$\frac{dy}{dn} = \frac{6n}{n^2 + y}$

$(n^2 + y) = 6n \frac{dn}{dy}$

$n^2 = p$

Leibner + P5 + 1st order DE = Individual Combination

$(n^2 + y) \frac{dy}{dn} = 6n - 0$

$\frac{(n^2 + y)}{(n^2 + y)} \frac{dy}{dn} = \frac{(-6)n}{(n^2 + y)}$

$\frac{dy}{dn} + \frac{n^2 + y}{n^2 + y} = 0$

$2n + \frac{dy}{dn} = \frac{dk}{dn}$

$dn + dy$

* Leibner + P5 + 1st order Linear DE = Individual Combination

* Que: Find DE fcn with satisfy

$f(n) = \int_0^n f(t) \cos t - \cos(t-n) dt$

$= f(n) = f(n) \cos n - \cos(n/n) dt$

$f(n) = f(n) \cos n$

{E3 and E4 Complete!}

P.No: 18. Exact DE.

1, 2, 4, 6, 7, 8, 9, 11 write down.
same only same

Ans $f(0) = 0$

$$= \int_0^n f(t) \cos t dt - \int_0^n \cos(t-n) dt$$

$$t \Rightarrow 0+n-t$$

$$\int \cancel{\cos t} dt - \int_0^n \cos t dt$$

$$f(n) = f(n) \cos n$$

$$f(n) = f(n) \cos n - \cos(0) dn$$

$$f(n) = f(n) \cos n$$

$$f(n) = f(n) \cos n - \cos n - dn$$

$$0 = \cos n - \cos(n+1)$$

$$f(n) = f(n) \cos n$$

$$f(n) =$$

$$f'(n) = f(n) \cos n - \cos n$$

$$\frac{df}{dn} + f(n) (-\cos n) = -\cos n$$

$$f(n) \cdot e^{-\sin n} = \int -\cos n \cdot e^{-\sin n} dn + A$$

$$If = e^{-\int \cos n dn}$$

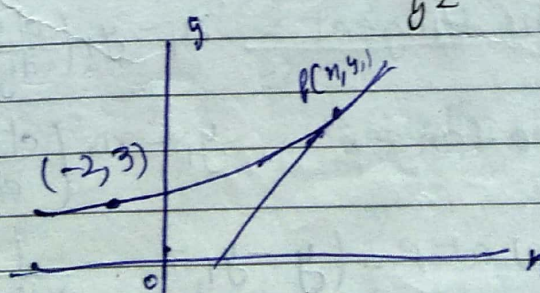
$$= e^{-\sin n}$$

$$= \int e^p dp + A$$

$$= e^p + A = e^{-\sin n} + A$$

$$f(0) = 0$$

Que: Find eqn of curve passing through $(-2, 3)$ given that slope of tangent at any point on it is $\frac{2n}{y^2}$



$$\frac{dy}{dn} = \frac{2n}{y^2}$$

$$y^2 dy = 2n dn$$

$$\Rightarrow \frac{y^3}{3} = n^2 + A$$

$$\frac{27}{3} = 4 + A \Rightarrow A = 5$$

* Exact differentials:-

$$* \quad x dy + y dx = d(xy)$$

$$* \quad \frac{x dy - y dx}{n^2} = d\left(\frac{y}{n}\right)$$

$$* \quad \frac{x dy + y dx}{xy} = d(\ln xy)$$

$$* \quad \frac{dx + dy}{x+y} = d(\ln(x+y))$$

$$* \quad \frac{x dy - y dx}{xy} = d\left(\ln \frac{y}{x}\right)$$

$$* \quad \frac{x dy - y dx}{x^2 + y^2} = d\left(\tan^{-1} \frac{y}{x}\right)$$

$$* \quad d\left(\frac{-1}{xy}\right) = \frac{x dy + y dx}{x^2 y^2}$$

$y = f(x)$

Point $P = \left(\frac{dy}{dn}, n_1, y_1\right)$

* i) The eq of the tangent at P is - $y - y_1 = \frac{dy}{dn} (n - n_1)$

x -intercept of the tangent: $n_1 - y_1 \left(\frac{dy}{dn}\right)$

y -intercept of the tangent: $y_1 - n_1 \left(\frac{dy}{dn}\right)$

ii) The eqn of normal at P = $(y - y_1 = - \frac{1}{(dy/dn)} (n - n_1)$

x and y -intercept of Normal; $n_1 + y_1 \frac{dy}{dn}$ and

$y_1 + n_1 \frac{dy}{dn}$

$$(iii) \text{ L. of Tangent: } PT = |y_1| \sqrt{1 + \left(\frac{dn}{dy}\right)^2}_{(n_1, y_1)}$$

$$(iv) \text{ L. of Normal: } PN = |y_1| \sqrt{1 + \left(\frac{dy}{dn}\right)^2}_{(n_1, y_1)}$$

$$(v) \text{ L. of } \phi \text{ tangent: } ST = \left| y_1 \left[\frac{dn}{dy} \right]_{(n_1, y_1)} \right|$$

$$(vi) \text{ L. of } \phi \text{ Normal: } \phi N = \left| y_1 \left[\frac{dy}{dn} \right]_{(n_1, y_1)} \right|$$

$$(vii) \text{ L. of Radius Vector: } \sqrt{n_1^2 + y_1^2}.$$

Note: If equation of the form

$$y \cdot f(ny) dn + n g(ny) dy = 0$$

then variable separable can be achieved by substitution.

$$ny = t \quad \text{--- variable}$$

$$\boxed{y = \frac{t}{n}}$$

$$dy = \frac{ndt - tdn}{n^2}$$

Ex: Solve $(n^3y^3 + n^2y^2 + ny + 1)ydn + (n^3y^3 - n^2y^2 - ny + 1)ndy = 0$
 Put $y = \frac{t}{n}$

$$(t^3 + t^2 + t + 1) \cdot \frac{t}{n} dn + (t^3 - t^2 - t + 1) n \cdot \frac{ndt - tdn}{n^2} = 0$$

$$(t^3 + t^2 + t + 1) \frac{t}{n} dn + (t^3 - t^2 - t + 1) n \cdot \frac{ndt - tdn}{n^2} = 0$$

$$t \ln [(t^3 + t^2 + t) - (t^3 - t^2 - t + 1)] + n dt [t^3 - t^2 - t + 1]$$

$$2(t^2 + t) t dn + n dt \cdot (t-1)^2 (t+1) = 0$$

$$2t^2 (t+1) dn + n dt \cdot (t-1)^2 (t+1) = 0$$

$$\frac{dn}{n} + \frac{(t-1)^2}{2t^2} dt = 0$$

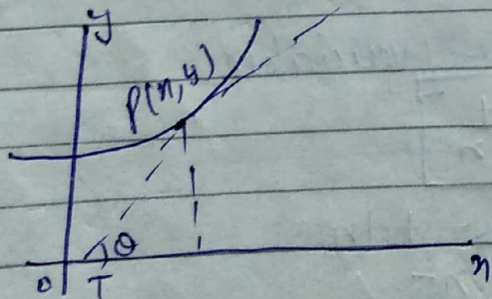
Ques 1 $(1 - ny + n^2 y^2) dn = n^2 dy$

$$(1 - t + t^2) dn = n^2 \frac{ndt - tdn}{n^2}$$

$$dn [1 - t + t^2 + t] = n dt$$

$$\frac{dn}{dn} = \frac{dt}{1+t^2}$$

Ques 2 find eqn of Curve not passing through origin and for which the length of tangent at any point P is equal to radius vector of that point.



$$PT = PO$$

$$y \operatorname{cosec} \theta = \sqrt{n^2 + y^2}$$

$$y^2 \operatorname{cosec}^2 \theta = y^2 + n^2$$

$$y^2 \cos^2 \theta = n^2$$

$$\tan^2 \theta = \frac{y}{n}$$

$$\ln y = \ln n + \ln m$$

$$\frac{dy}{y} = \frac{dn}{n}$$

$$\frac{dy}{y} = \frac{dn}{n}$$

$$\ln y = \ln n + \ln m$$

$$y = mn$$

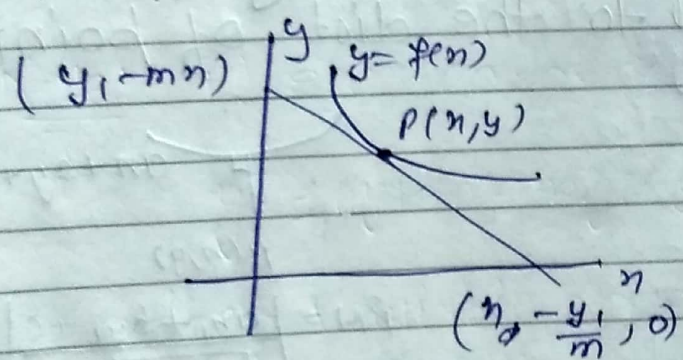
$$\frac{dy}{dn} = -\frac{y}{n}$$

$$\frac{dy}{y} + \frac{dn}{n} = 0$$

$$\ln y + \ln n = \ln m$$

$$ny = m \quad \text{rectangular}$$

Ques: find curve for which segment of the tangent contained b/w the coordination axis is bisected by the point. given that curve passes to point (2,3)



$$\frac{y_1 - mn_1 + 0}{2} = y_1$$

Ans: $\frac{n_1 - \frac{y_1}{m} + 0}{2} = n_1$

$$n - y \frac{dn}{dy} = 2n$$

$$-y \frac{dn}{dy} = n$$

$$-\frac{dn}{n} = \frac{dy}{y}$$

$$y_1 = mx_1$$

$$y = \frac{dy}{dn} \cdot n$$

$$\frac{dy}{dn} = \frac{y}{n}$$

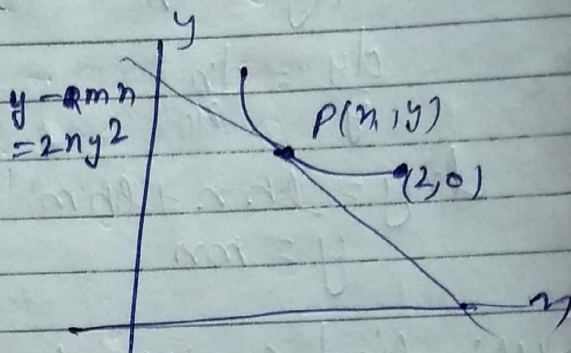
Ques! find eqⁿ of Curve such that intercept of tangent on the y axis is equal to $2ny^2$. given that Curve passes through $(2,0)$

$$y - n \left(\frac{dy}{dn} \right) = 2ny^2$$

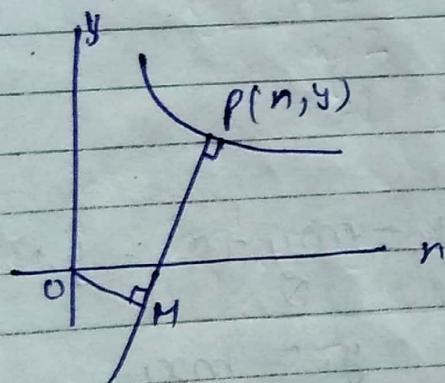
$$y - mx = 2ny^2$$

$$y - n \frac{dy}{dn} = 2ny^2$$

$$y - 2ny^2 = n \frac{dy}{dn}$$



Ques! find Curve $y = f(n)$ such that $\perp$ dist. from origin to the normal at any pt of the Curve is equal to the dist. of point P from the n axis.



$$PM = \frac{|an_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$PM = \frac{|an_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$m(y - y_1) + (n - n_1) = 0$$

$$n + my - (n_1 + my_1) = 0$$

$$OM = \frac{|0 + 0 - (n_1 + y_1 m)|}{\sqrt{1 + m^2}} = y_1$$

$$(n_1 + my_1)^2 = y_1^2 (1 + m^2)$$

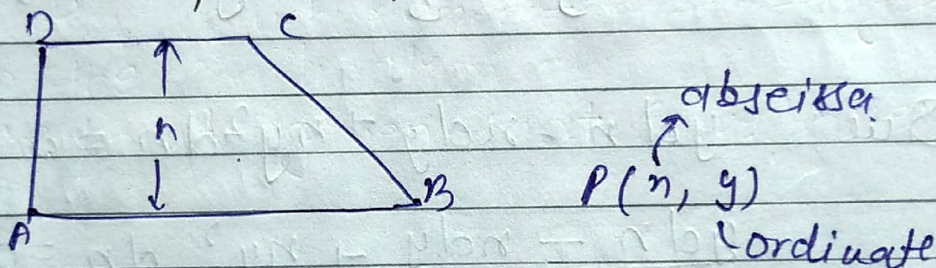
$$n_1^2 + m^2 y_1^2 + 2mn_1 y_1 = y_1^2 + m^2 y_1^2$$

$$m = \frac{y_1^2 - n_1^2}{2n_1 y_1}$$

$$\frac{dy}{dn} = \frac{y^2 - n^2}{2ny}$$

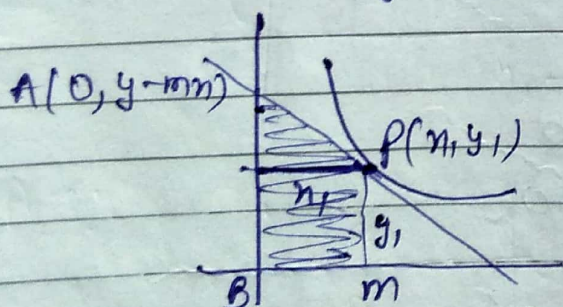
$$y - y_1 = \frac{-1}{m}(n - n_1)$$

Ques: find Curve such that the area of trapezium formed by coordinate axes ordinate of an arbitrary point and tangent at this point equal half the square of its abscissa.



$$A = \frac{1}{2} (AB + CD) \times h$$

$$y - y_1 = m(n - n_1)$$



$$\frac{1}{2} \times (OA + BM) \times OM = \frac{1}{2} n^2$$

$$= \frac{1}{2} (y, -mn, + y_1) \times m, = \frac{1}{2} n^2$$

$$y_1 - mn_1 = n_1$$

$$\frac{dy}{dn} = \frac{ny}{dn} + n$$

$$\frac{ndy}{dn} = 2y - n$$

* Exact DE:

Ques: Solve $(x^2 - ay) dx + (y^2 - ax) dy = 0$

$$x^2 dx - ay dx + y^2 dy - ax dy = 0$$

$$x^2 dx + y^2 dy - a(y dx + x dy) = 0$$

$$x^2 dx + y^2 dy - ad(xy)$$

$$\frac{x^3}{3} + \frac{y^3}{3} - axy = 1$$

Ques: $y dx - x dy + xy^2 dx = 0$

$$\frac{y dx - x dy}{y^2} + \frac{xy^2 dx}{y^2} = 0$$

$$\frac{y dx - x dy}{y^2} + x dx = 0$$

$$d\left(\frac{x}{y}\right) + \frac{x^2}{2} = 0$$

Ques: $xy dx + y dx + 2x^3 dx = 0$

$$\frac{xy dx + y dx}{x^2} + 2x dx = 0$$

$$d\left(\frac{y}{x}\right) + 2x dx = 0$$

$$d\left(\frac{y}{x}\right) + x^2 = 0$$

Ques! $y \, dn - n \, dy + (1+n^2) \, dn + n^2 \sin y \, dy = 0$

$$\frac{y \, dn - n \, dy}{y^2} + \frac{dn}{y^2} + \frac{n^2 \, dn}{y^2} + \frac{n^2 \sin y \, dy}{y^2} = 0$$

$$- d\left(\frac{n}{y}\right) + \left(\frac{1}{n^2} + 1\right) \, dn + \sin y \, dy = 0$$

Sol. $n \, dn + y \, dy = (n \, dy - y \, dn)$

$$= n \, dn + y \, dy = \frac{(n \, dy - y \, dn)}{n^2} \cdot n^2$$

$$\frac{1}{2} \, dn^2 + \frac{1}{2} \, dy^2 = n^2 \, d\left(\frac{y}{n}\right)$$

$$\frac{1}{2} \frac{d(n^2 + y^2)}{(n^2 + y^2)} = \frac{n^2 \, d\left(\frac{y}{n}\right)}{n^2 y^2}$$

$$\frac{1}{2} \int \frac{d(n^2 + y^2)}{(n^2 + y^2)} = \int \frac{d\left(\frac{y}{n}\right)}{1 + \left(\frac{y}{n}\right)^2} + C$$

$$\frac{1}{2} \ln(n^2 + y^2) = \tan^{-1} \frac{y}{n} + C$$

Ques! $y \ln n - n dy + (1 + n^2) dn + n^2 \sin y dy = 0$

$$\frac{y \ln n - n dy}{y^2} + \frac{dn}{y^2} + \frac{n^2 dn}{y^2} + \frac{n^2 \sin y dy}{y^2} = 0$$

$$- d\left(\frac{n}{y}\right) + \left(\frac{1}{n^2} + 1\right) dn + \sin y dy = 0$$

Ans, $n dn + y dy = (n dy - y \ln n)$

$$= n dn + y dy = \left(\frac{n dy - y \ln n}{n^2} \right) n^2$$

$$\frac{1}{2} dn^2 + \frac{1}{2} dy^2 = n^2 d\left(\frac{y}{n}\right)$$

$$\frac{1}{2} \frac{d(n^2 + y^2)}{(n^2 + y^2)} = \frac{n^2 d\left(\frac{y}{n}\right)}{n^2 + y^2}$$

$$\frac{1}{2} \int \frac{d(n^2 + y^2)}{(n^2 + y^2)} = \int \frac{d\left(\frac{y}{n}\right)}{1 + \left(\frac{y}{n}\right)^2} + 1$$

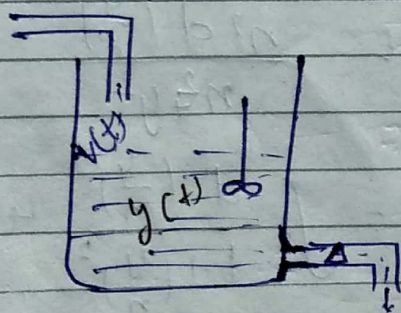
$$\frac{1}{2} \ln(n^2 + y^2) = \tan^{-1} \frac{y}{n} + 1$$

* Mixture Problem!

Q. $\Rightarrow$ A chemical in a liquid solⁿ (or dissolved in a gas) runs into a container holding the liquid (or the gas) with, possibly, a specify amount of chemical dissolved as well. the mixture escape uniform by streaming and flows out of the container at uniform rate.

In this process it is imp. to know the concⁿ of chemical of container at any given time. The diff. eq of process is given by (Rate of change of Amount in container)

$$= \left(\text{rate of which chemical in} \right) - \left(\text{rate of which chemical out} \right)$$



$y(t)$ - concⁿ

If $y(t)$ is Amount Chemical in the container at time t and $V(t)$ is the total of the liquid of the container

then

flow rate (known) = $\frac{m^3}{\text{sec}}$

Calculated

$$\text{Conc}^n = \frac{y(t)}{V(t)} = \frac{gm}{m^3}$$

$$\frac{y(t)}{V(t)} = \frac{gm}{m^3}$$

$\therefore$ rate of departure of Chemical = (Concⁿ) $\times$ out flow rate

$$= \frac{y(t)}{V(t)} \times \text{out flow rate}$$

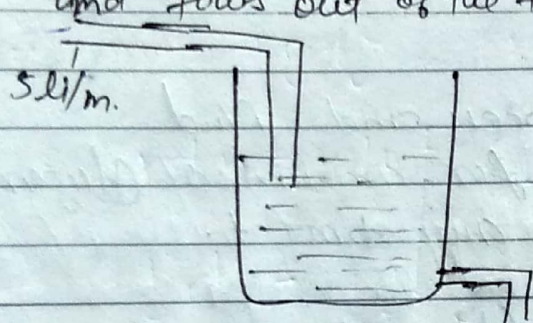
$$\frac{gm}{m^3} \times \frac{m^3}{\text{sec}} =$$

$$\frac{dy(t)}{dt} = \text{In rate} - \frac{y(t)}{V(t)} \text{ out flow rate}$$

1st order DE.

Ques: A tank initially contained 100 litres of water in which 50 g of salt dissolved. A water containing 2 g/L of salt runs into the tank at the rate of 5 l/m. the mix. stirring and flows out of the tank at the rate of 4 l/m

2g/L



$$t(0) = 100 \text{ l. water}$$

50 g salt Sugar

$$1 - 2g \text{ su.}$$

$$\frac{V}{t} = 5 \text{ l/m.}$$

(a) Rate at which does salt enter the tank at time t = $\frac{g}{\text{sec.}}$

1 min = 5 lit and
In each of water = 2 g salt

$$5 \text{ litre water contains} = 5 \times 2 \text{ gm}$$

$$\therefore \text{rate at which} = \frac{10 \text{ gm}}{1 \text{ min}} = 10 \text{ gm/min}$$

Chemical

$$\begin{aligned} & \frac{V(t)}{t} = \frac{g}{\text{sec}} \\ & \frac{dV}{dt} = 5 \text{ l/m.} \\ & = \frac{5 \times 60}{60} \\ & = \frac{500}{6} = 83. \end{aligned}$$

(b) what is the Volume of liquid in the tank. at time t .

$$t(0) = 100$$

$$= 5 \text{ l/m} = 5 = 5t$$

Net in volume after time $t = 5t$

Net out Volume time $t = 4t$

$$\text{Net vol. of liquid} = 100 + 5t - 4t$$

$$= 100 + t \text{ Ans}$$

(iii) at what rate does salt leave the tank at time t .
 Super $v(t) = y$
 Amount of Sugar in the container = $y(t)$

$$t = y(t)$$

$$1 = 2g$$

$$t = 2t$$

Deposited $\left(\frac{y(t)}{100+t}\right) \times y$. An
 Rate.

$$\text{Total} = 80t - 2t = 48t$$

$$= 48t$$

(iv) form diff DE of the process and find
 and write general expression for amount of Sugar
 present in the container at any time t .

$$\frac{dy(t)}{dt} = \text{In rate} - \frac{y(t)}{v(t)} \quad (\text{at flow rate})$$

$$\frac{dy}{dt} = 10 - \frac{4y}{100+t}$$

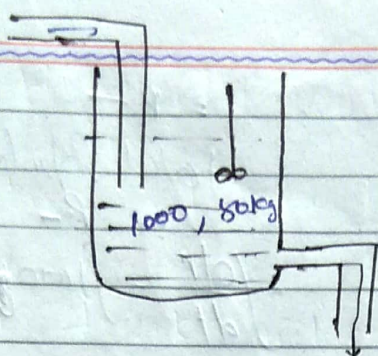
$$\frac{dy}{dt} + \frac{y}{100+t} = 10$$

$$I.F. = e^{\int \frac{1}{100+t} dt} = e^{\ln(100+t)} = 100+t$$

$$y(t+100)^4 = \int 10(t+100)^4 dt + C$$

$$t=0, y=80$$

Ques 1



Q. A tank contain 1000 l of brine with 80 kg salt dissolved. Pure water enter the tank at the rate 10 l/min. Salt can settle and grains from the same rate. Find how much salt is there in tank after time t .

Suppose:

after time t

amount of salt in the container = y

$$V(t) = 1000 + 10t = 10t$$

$$= 1000$$

$$\frac{dy(t)}{dt} = \text{In rate} - \frac{y(t)}{Vt}$$

$$\frac{dy}{dt} = 0 - \frac{y}{1000} \times 10$$

$$\frac{dy}{dt} = -\frac{y}{100}$$

$$\int_{50}^y \frac{dy}{y} = -\frac{1}{100} \int_0^t dt$$

$$[\ln y]_{50}^y = -\frac{t}{100}$$

$$\ln y - \ln 80 = -\frac{t}{100}$$

$$\ln \frac{y}{80} = -\frac{t}{100}$$

$$y = 80 e^{-t/100}$$

$$\left. \begin{array}{l} 1000 \text{ l} \text{ --- } 150 \text{ g salt} \\ 100 \text{ l} \text{ --- } \frac{15}{1000} \\ 1000 \text{ --- } \frac{15}{100} = 0.15 \\ = 0.18y. \\ y = 15 \times \frac{100}{y} \\ (t) = \frac{y - 150}{y} \end{array} \right\}$$

How? SM complete.

Q.10

$$y(t) = P$$

Time $t =$

$$\frac{dP}{dt} = (P - 1000)k$$

$$\int_{2800}^P \frac{dP}{P - 1000} = -k \int_0^t dt$$

$$\begin{aligned} & \ln(P - 1000) = -kt + \ln(P - 1000)_0 \\ & \ln \frac{P - 1000}{P_0 - 1000} = -kt \\ & \frac{P - 1000}{P_0 - 1000} = e^{-kt} \\ & P = 1000 + (P_0 - 1000)e^{-kt} \end{aligned}$$

* Orthogonal trajectory!

Suppose $\phi(x, y, a) = 0$ are given family of curves which depends on a single parameter (constant) a . If a curve meeting at each new

of its points a fixed angle α with the curve of the family passing through that point is called an isogonal trajectory of that family. If $\alpha = \frac{\pi}{2}$ then it is called

Orthogonal trajectory.

If we have to set up dE of such Orthogonal trajectory then follow the following step.

(i) Form dE corresponding to the given curve.

$$F(x, y, \frac{dy}{dx}) = 0 \quad (\text{Curve})$$

(ii) for orthogonal trajectory Replaced by $\frac{dy}{dx}$

H.W $y = n, |n|, n^2, n^3, \frac{1}{|n|}, \frac{1}{n^2}, \ln n, \ln |n|$
 $y = e^n, (n-1), (n-1), (|n|-1), \sin |n|, |\sin n|, \cos |n|, \cos n$

DE of curve

O.T of DE

$$f\left(n, y, -\frac{1}{\frac{dy}{dn}}\right) = 0 \quad \xrightarrow{\text{soln.}}$$

$$\frac{dy}{dn} \rightarrow -\frac{dn}{dy}$$

(3) solⁿ of this eqⁿ will be the orthogonal trajectory of given curve.

Q. Derive O.T of $n^2 + y^2 = a^2$

solⁿ

$$n^2 + y^2 = a^2 \quad \text{--- (i) Step 1}$$

$$2n + 2y \frac{dy}{dn} = 0 \quad \text{--- D.E of family of Curve. (2) Step.}$$

Now we have to write O.T of family of give eqⁿ (i)

So, replaced

$$\frac{dy}{dn} \rightarrow -\frac{dn}{dy}$$

$$2n + 2y \left(-\frac{dn}{dy}\right) = 0$$

$$\frac{dy}{y} = \frac{dn}{n}$$

$$\ln y = \ln n + \ln c$$

$$y = mn \quad \text{--- it is O.T equation.}$$

$$|y| = \sin n.$$

$$y = n^2 - 3|n| + 2$$

$$|y| = n^2 - 3n + 2$$

$$y = \max\{n, n^2, 1\}$$

$$y \geq n$$

$$y \leq n$$

$$|n| + |y| \leq 1$$

Figure and Graph Related to Area under the Curve

Chapter

*

$$y = n(n-1)$$

$$y = n(n-1)(n+2)$$

$$n^2 + y^2 \leq 4$$

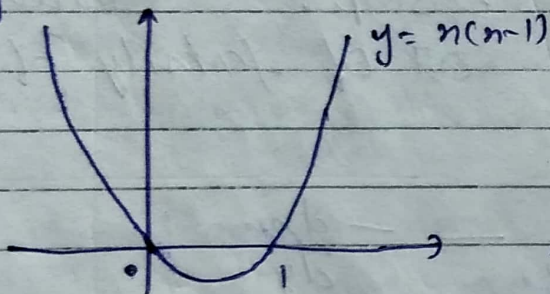
$$n^2 + y^2 \geq 4$$

$$y = e^n - 1$$

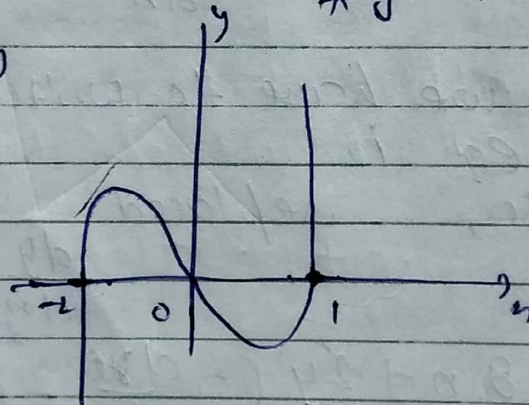
$$y = e^{|n|}$$

$$y = \frac{n}{1+n^2}, \frac{1}{1+n^2}, \frac{e^n + e^{-n}}{2}, \dots$$

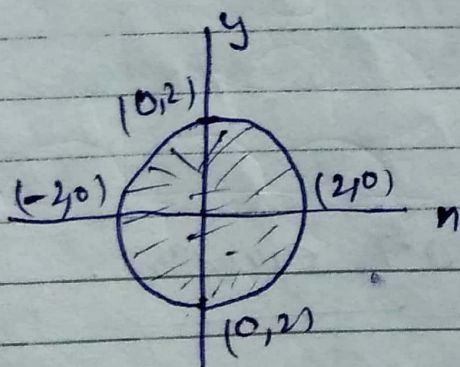
* $y = n(n-1)$



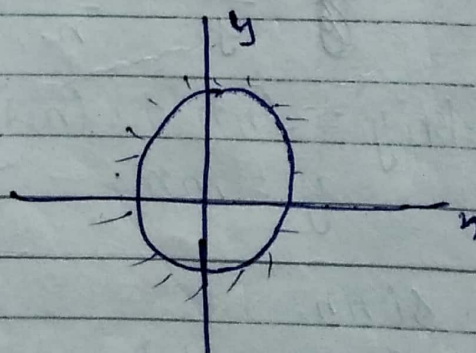
* $y = n(n-1)(n+2)$



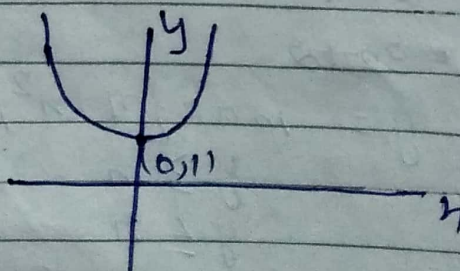
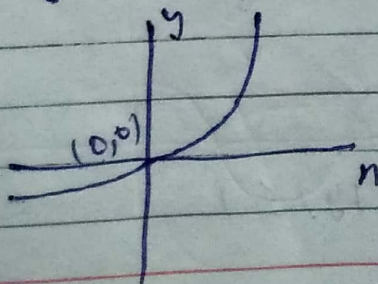
* $n^2 + y^2 \leq 4$



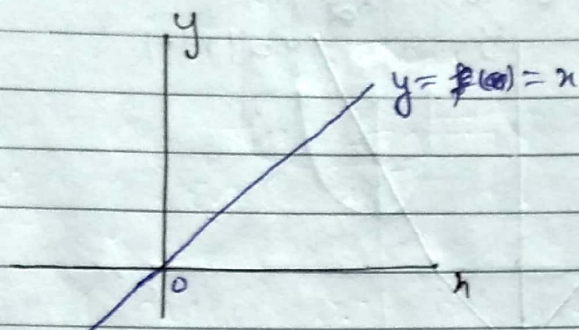
* $n^2 + y^2 \geq 4$



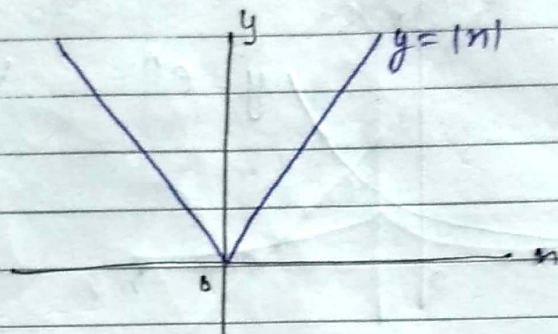
* $y = e^n - 1$



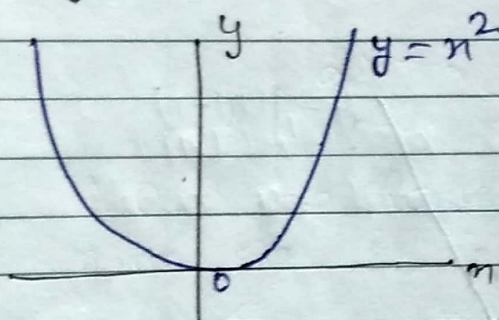
(1) $y = x$



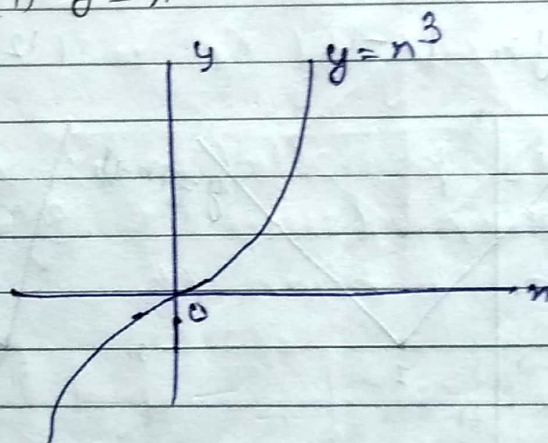
(2) $y = |x|$



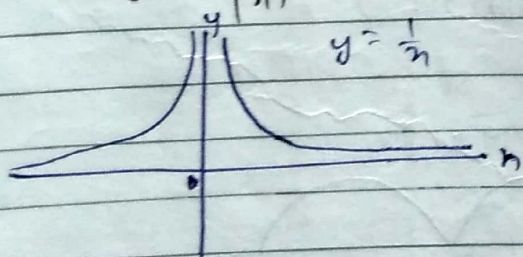
(3) $y = x^2$



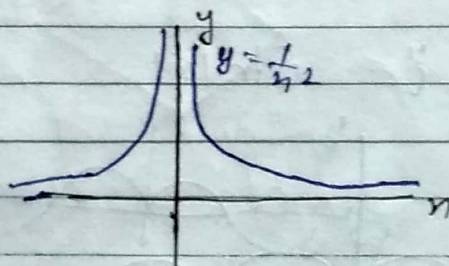
(4) $y = x^3$



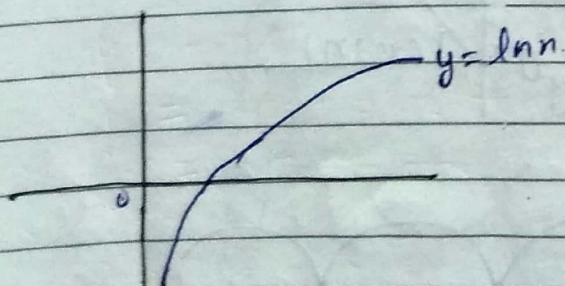
(5) $y = \frac{1}{|x|}$



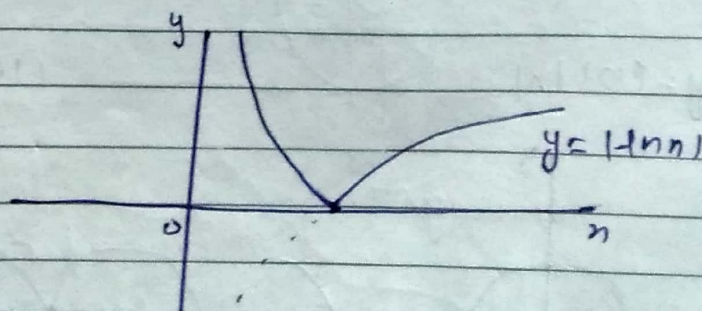
(6) $y = \frac{1}{x^2}$



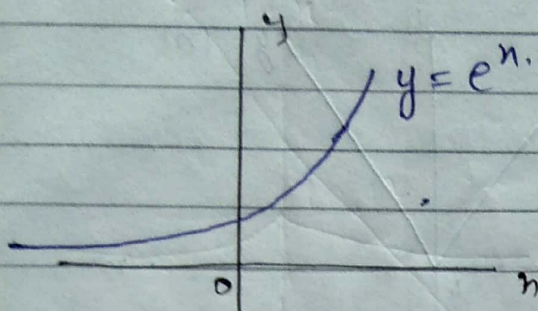
(7) $y = \ln x$



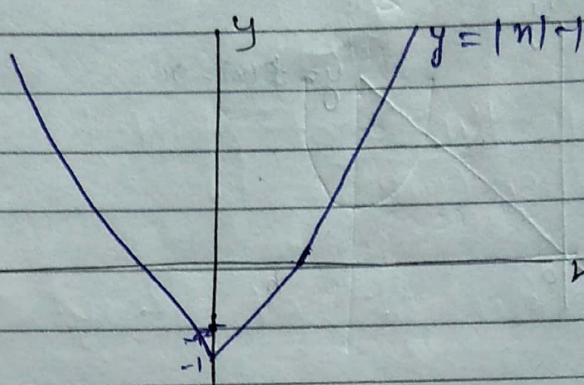
(8) $y = |\ln x|$



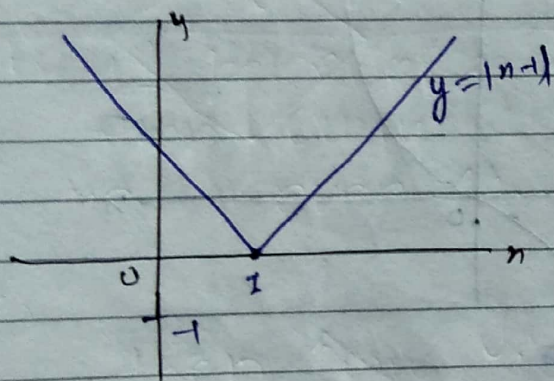
(9) $y = e^n$



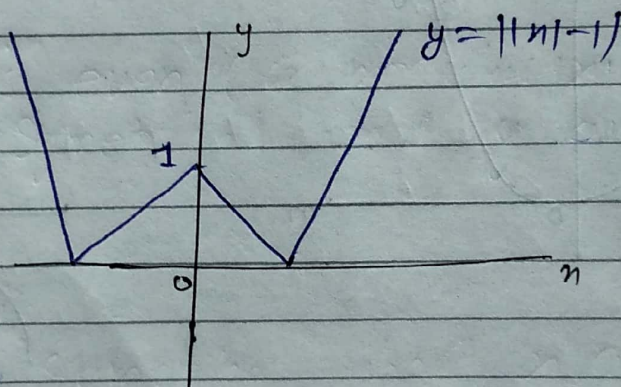
(10) $y = |n| - 1$



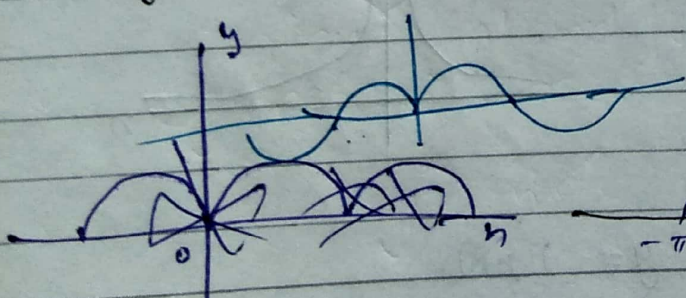
(11) $y = |n-1|$



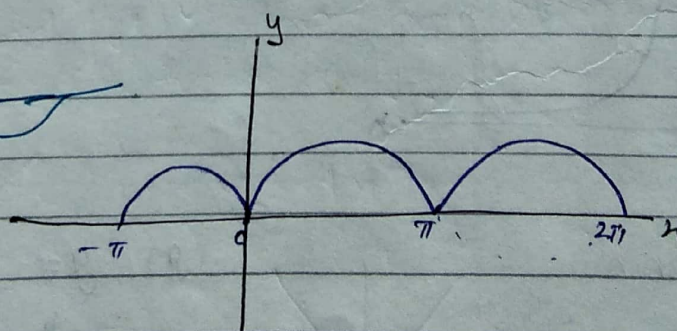
(12) $y = ||n|-1|$



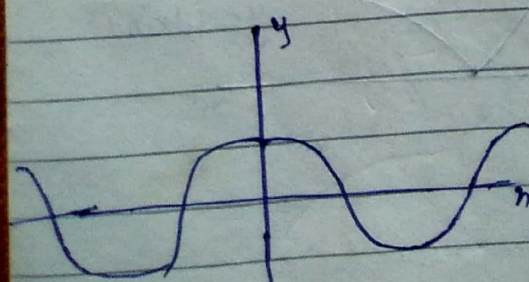
(13) $y = \sin |n|$



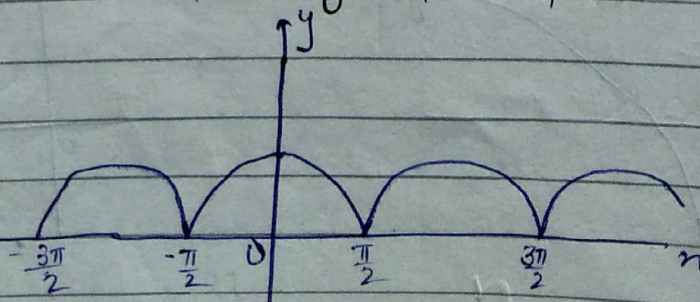
(14) $y = |\sin n|$



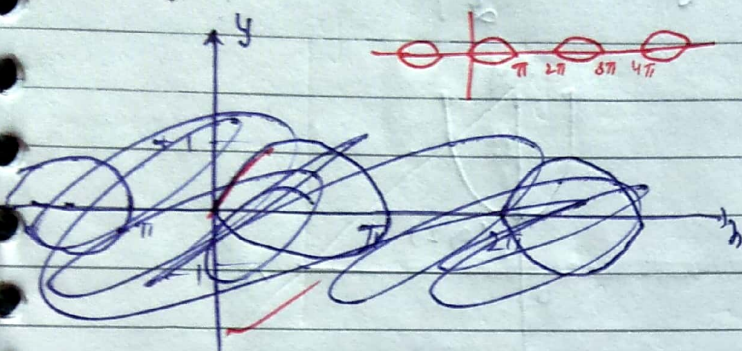
(15) $y = \cos |n|$



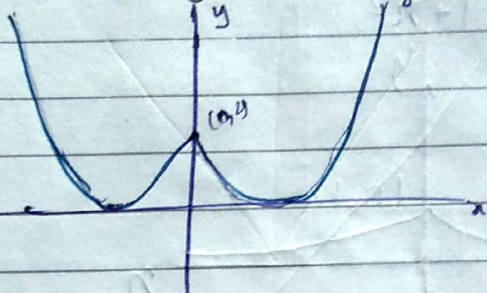
(16) $y = |\cos n|$



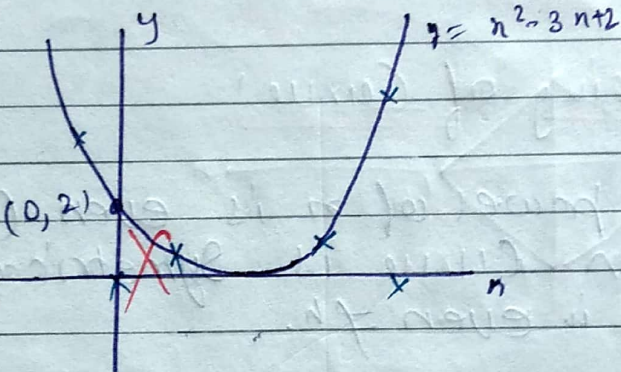
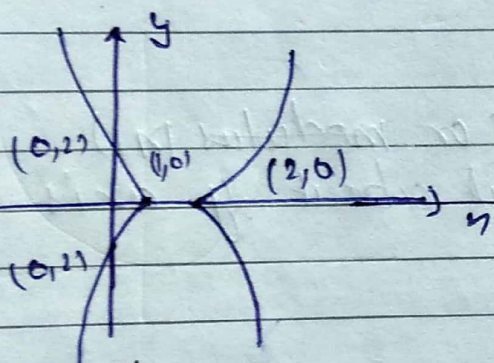
(17) $y = |x| = \delta \ln x$



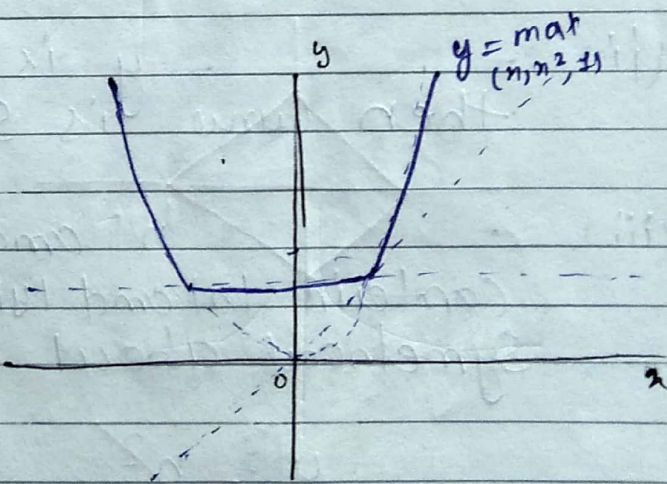
(18) $y = n^2 - 3/n + 2$



(19) $|y| = n^2 - 3n + 2$

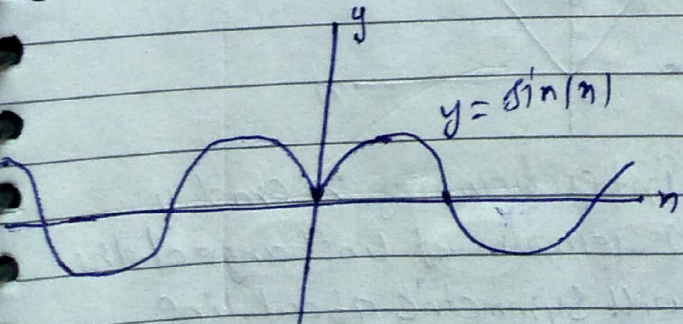


(20) $y = \max \{n, n^2, 1\}$

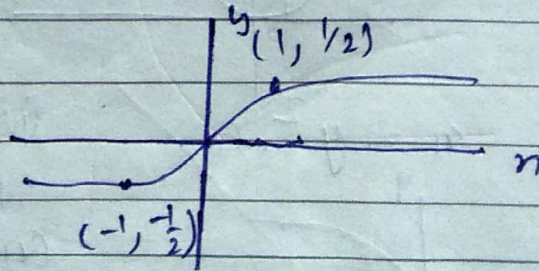


(13) $y = \delta \ln |n|$

$y = \delta \ln(n)$



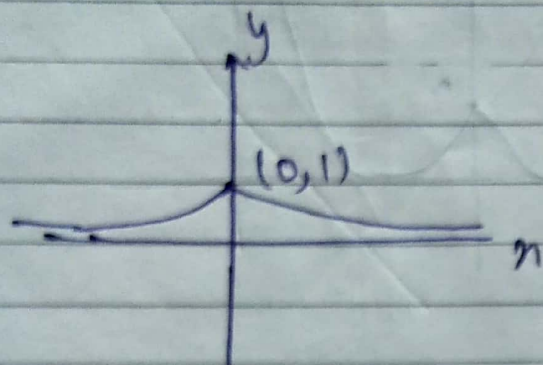
* $y = \frac{n}{1+n^2}$



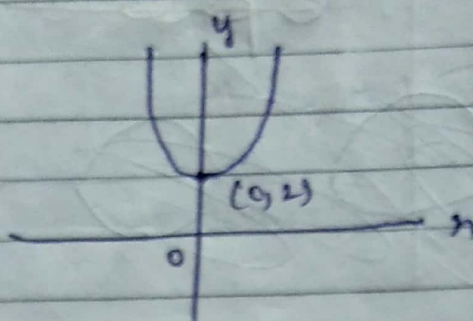
Area Under the Curve



* $y = \frac{1}{1+x^2}$



* $y = \frac{e^x + e^{-x}}{2}$

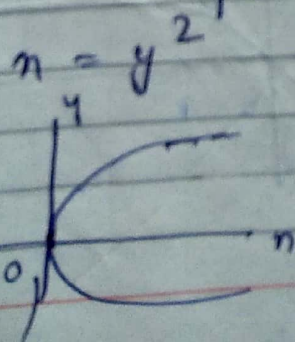
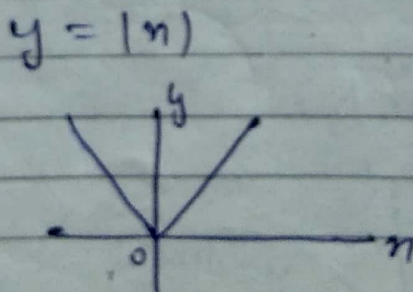
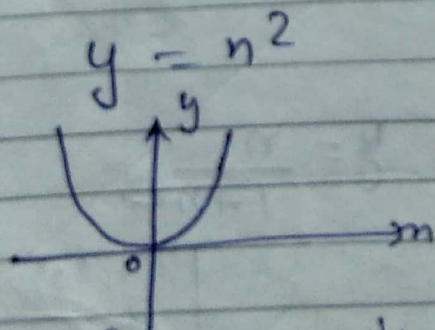


* Tracing of Curve

(i) If power of x is even (or modulus $|x|$) then curve is symmetrical about y axis. i.e. is even $f(x)$.

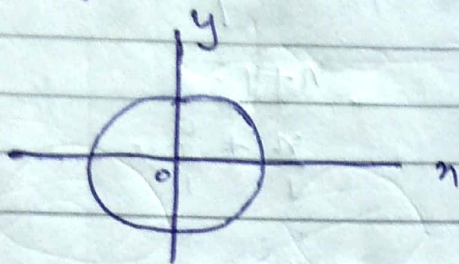
(ii) - y is - then curve is symmetrical about x axis.

(iii) - x and y both even (or eqn contain $|x|$ and $|y|$) the eqn of curve symmetrical about both axis.

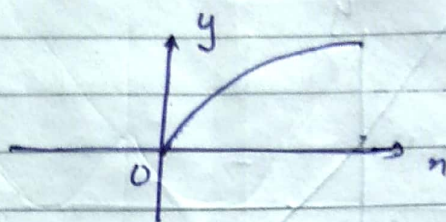


(iv) by Interchanging x and y if eqn remain unchanged the curve will be symmetric about line $y = x$.

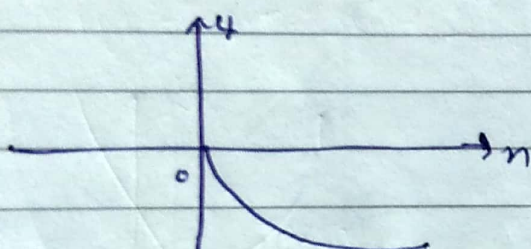
$$x^2 + y^2 = 4$$



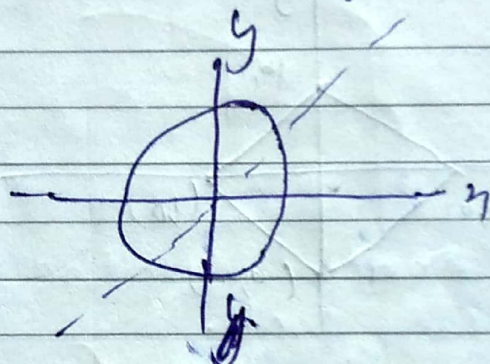
$$y = \sqrt{x}$$



$$y = -\sqrt{x}$$

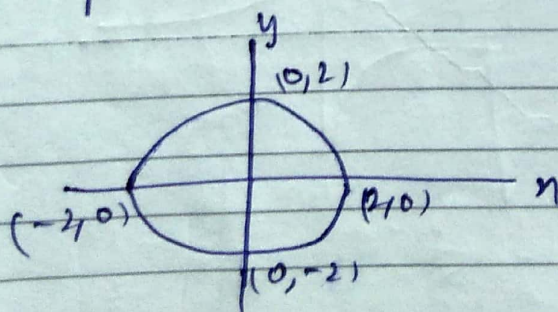
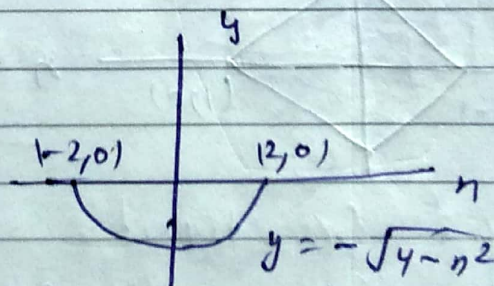
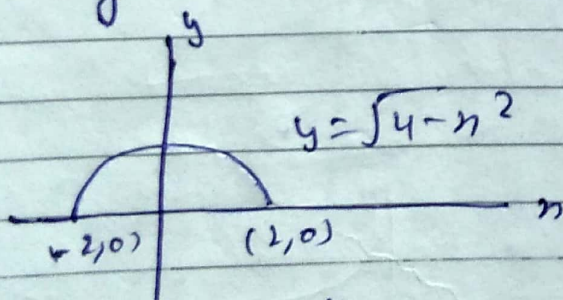


$$x \leftrightarrow y$$



$$y^2 = 4 - x^2$$

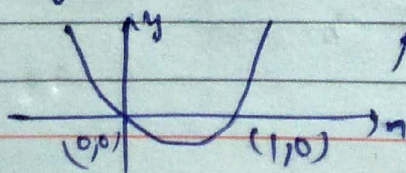
$$y = \pm \sqrt{4 - x^2}$$



$$x \text{ Plot } |x| + |y| = 1$$

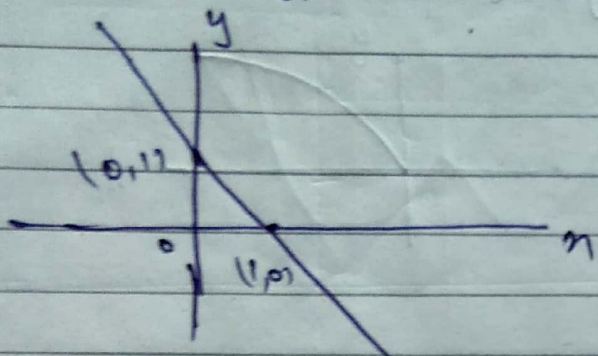
Hence Symmetrical about both axes.

$$x y = x^2 - x$$



No Symmetry

$$|x| + |y| = 1$$



in first Quad.

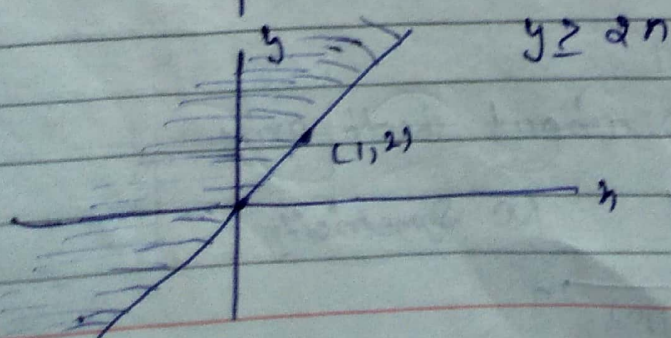
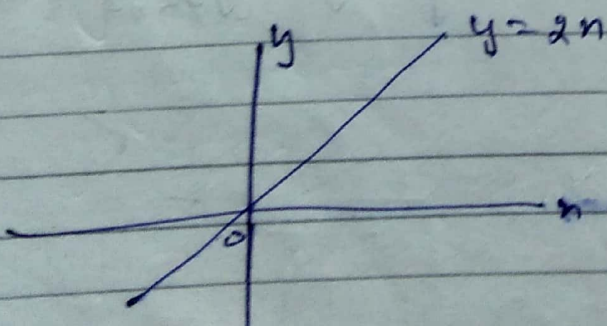
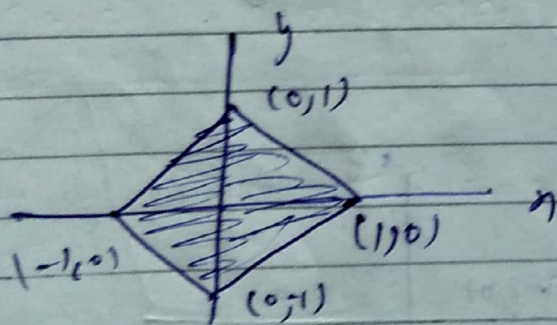
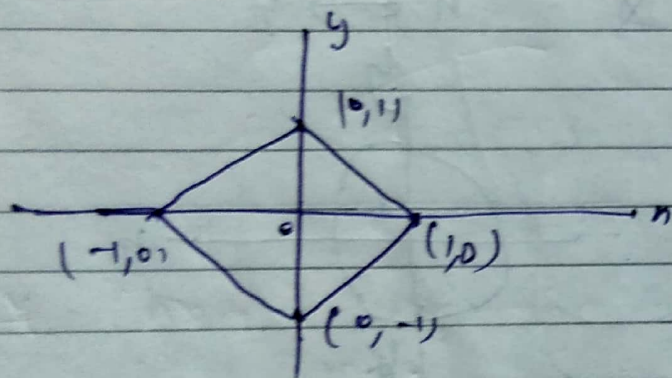
$$x + y = 1$$

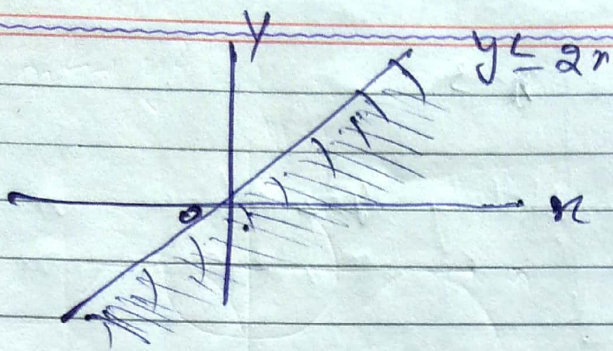
$$\frac{x}{1} + \frac{y}{1} = 1$$

$$-x + y = 1$$

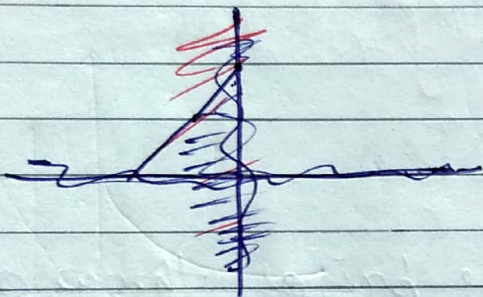
$$y = x + 1$$

$$y = -x + 1$$

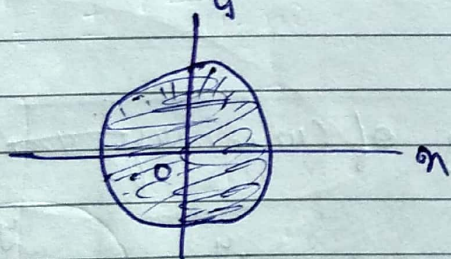




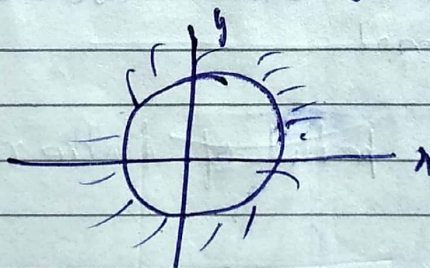
* $|x| + |y| \leq 1$



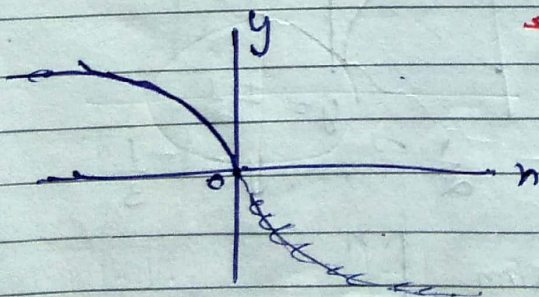
* $x^2 + y^2 \leq 4$



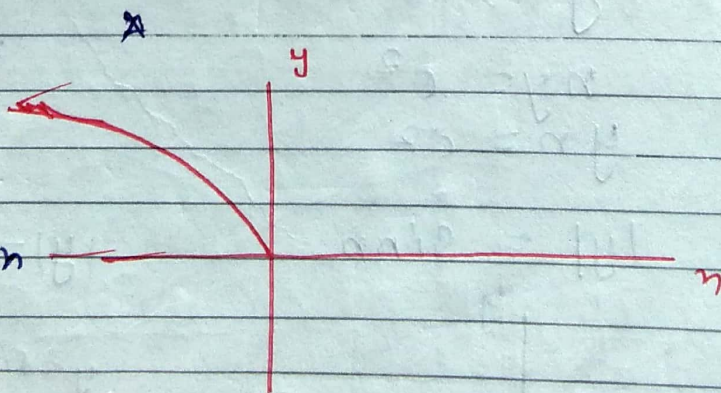
* $x^2 + y^2 \geq 4$



* $y = \sqrt{-x}$

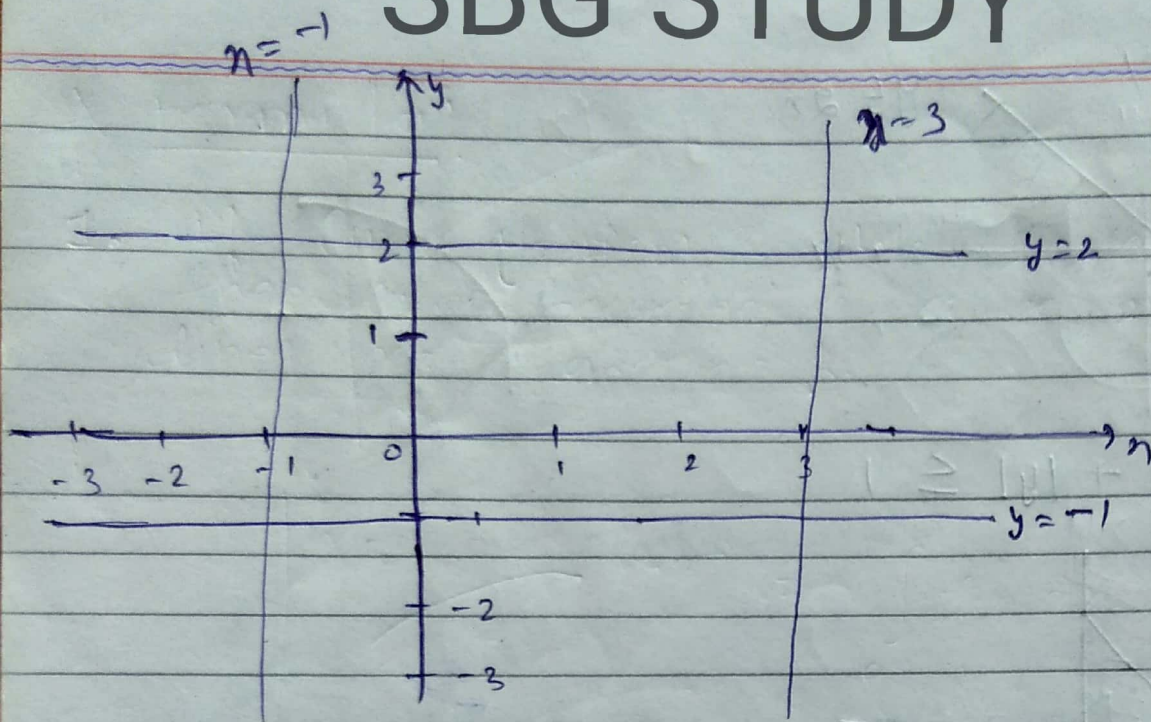


$y^2 = -x$



SBG STUDY

Con



* Check whether curve passes through origin or not.

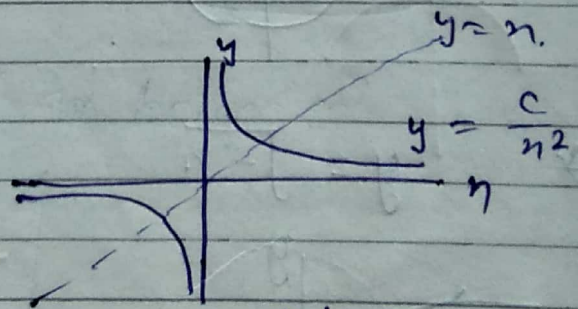
* Constant term present will pass through origin or not.

* Find point of intersection of curve with y-axis coordinate.

* $ny = \text{Constant}$

$$ny = c^2$$

$$yn = c^2$$



* $|y| = \sin n$

$$|y| = \frac{1}{x} \Rightarrow y = \pm \frac{1}{x}$$

$$y = -\frac{1}{3}$$

