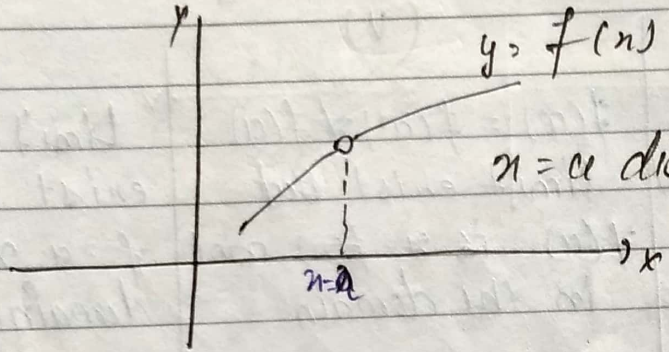
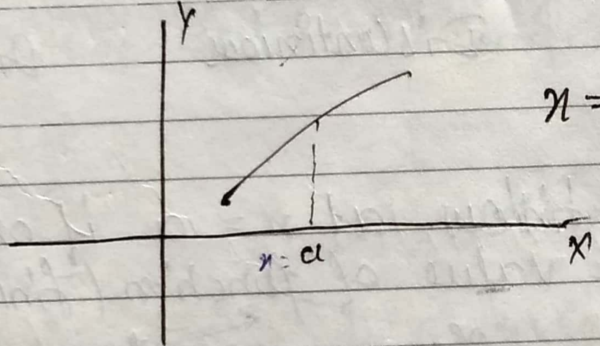
01/06/17

Continuity

A function is said to be a Continuous if y traveled informed L To Right or R to L one does not left to Right.

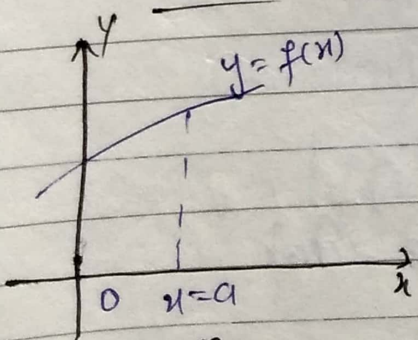


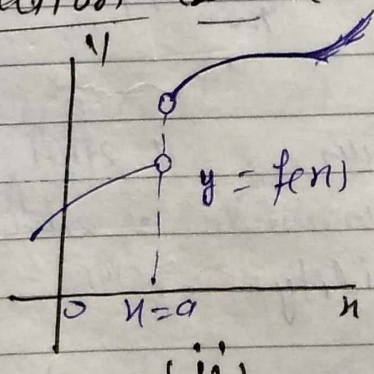
x = a discontinous

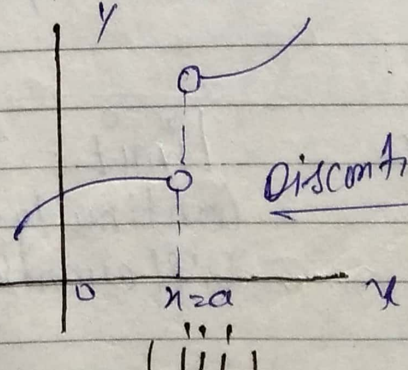


x = a Continuous.

* **Possible Cases of function y = f(x) at point of consideration at x = a**







Discontinous

(i)

$f(a^-) = f(a^+) = f(a)$

Continuous f

(ii)

L.H.L = $f(a^-) = l_1$

R.H.L = $f(a^+) = l_2$

Value of $f^n = f(a) = l$

$\because$ LHL $\neq$ RHL

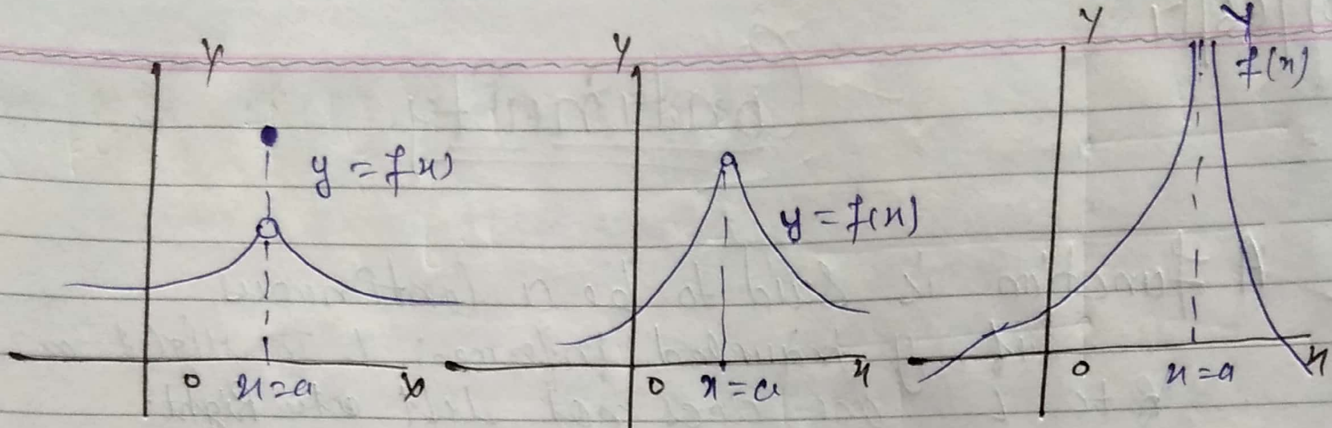
$\therefore$ Dis. at x.

(iii)

L.H.L = $f(a^-) = l_1$

$f(a^+) = l_2 (\neq l_1)$

at x = a is not in the domain



(iv) | (v) | (vi)

(iv)
$f(a^-) = f(a^+) \neq f(a)$
Limit exist but not equal to value of fn. therefore discontinous
Discontinous

(v)
$f(a^-) = f(a^+) \neq f(a)$
Limit exist But $f(a)$ ~~is x = f(a)~~ not in the domain
Discontinious

(vi)
Limit does not exist as well as $x = a$ not in the domain.
Discontinious

If f^n is continious at $x = a$ if and only if
L.H.L = R.H.L = value of function (finite quantity) ← $f(a)$.

OR.

$$L.H.L = f(a^-) = \lim_{h \to 0} f(a-h)$$

$$R.H.L = f(a^+) = \lim_{h \to 0} f(a+h)$$

* Limit - बचपन 1 सीढी
Continuity - जवानी 2 सीढी
Differentiability - बुढापा - 3 सीढी

* Continuity is $x = a$ ⇒ limit exist at $x = a$

* Limit exist at $x = a$ $\Rightarrow$ may or may not be Continuity at $x = a$

* Continuity is always top in the domain of function.

fn Natur
$\tan x$ is Continuous

$f(x) = \frac{1}{x-1}$ is a Continious

Ques whether $\tan x$ is Continious is at $\frac{\pi}{2}$

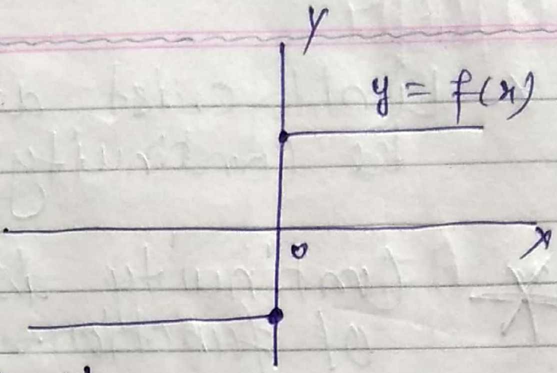
Ans: No

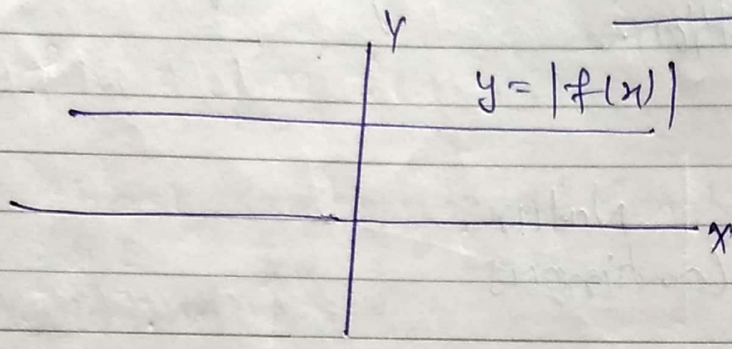
2) whether $f(x) = \frac{1}{x-1}$ is continious at $x = 1$
Ans: No
$f(1) = ND.$

* $y = f(x)$ is Cont. at $x = a \Rightarrow y = |f(x)|$ is Cont.
$\Rightarrow y = |f(x)|$ is continuous

* If $y = |f(x)|$ is Cont. then $y = f(x)$ need not be a Cont. function.

Example: $f(x) = \begin{cases} 1, & x \ge 0 \\ -1, & x < 0 \end{cases}$





* Polyonomial fn, trigonametric f^n modulus, f^n inverse fn, log f^n, exponcial fn, are continious.

* Continuity in a interval:

(i) Continuity in ~~close~~ open interval [a,b]

A f^n is continuous at each & every point of interval, then it is continous.

(ii) Continuity in close int. [a,b]:

i) Continuous at left boundry

ii) $f(a) = f(a^+) \Rightarrow$ cont. at a $x = a$

iii) & $f(b) = f(b^-) =$ cont. at $x = b$.

* Point f^n are Continuous

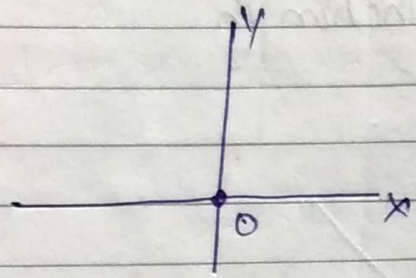
$$f(x) = \sqrt{x} + \sqrt{-x}$$

for Domain $x \geq 0$, $-x \geq 0$

$x \leq 0$

Domain $= \{0\}$

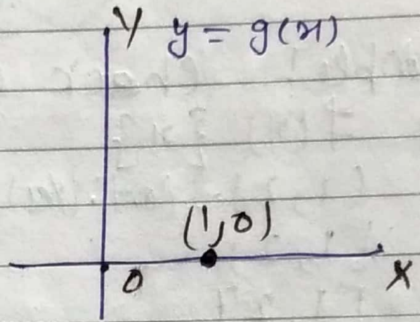
Range $= \{0\}$



$$g(x) = \sqrt{x-1} + \sqrt{1-x}$$

Domain $= \{1\}$

$f(1) = 0 =$ range.



* Reason of discontinuity !

(i) $x = a$ is not in the domain in the function $\rightarrow$ Fig: (iii), (v), (vi)

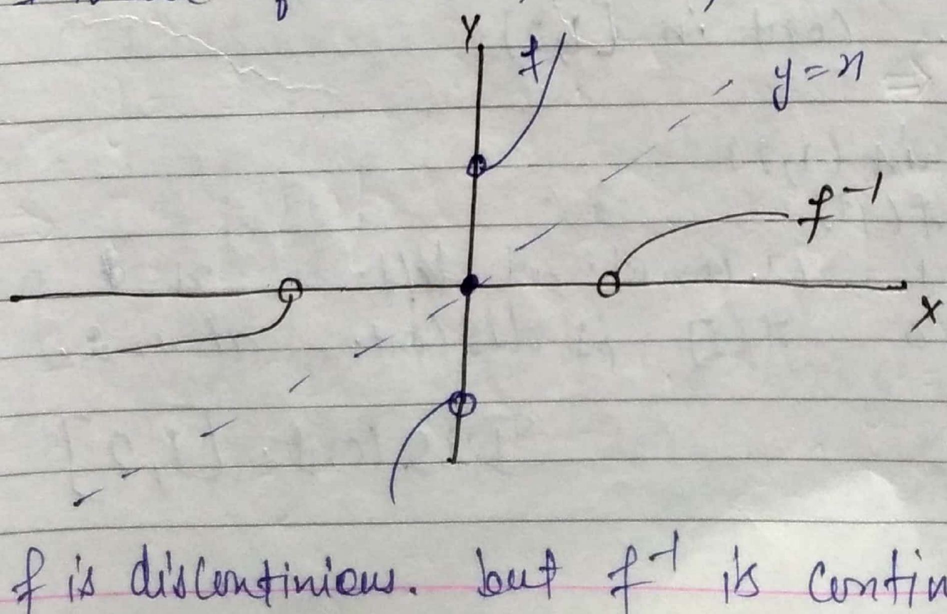
(ii) Limit does not exist [(II) (iii), (vi)]

(iii) Limit exit but not equal to value of f^n.

$$f(a^-) = f(a^+) \neq f(x).$$

(Fig. $\rightarrow$ (iv))

* Inverse of discontinious f^n Can be Continious.



f is discontinious. but f^{-1} is continous at $x = a$

Date :- 02/06/17

If f^n is Cont. at $x = a$

* $f(a^-) = f(a^+) = f(a)$

or

$$f(a) = \lim_{x \to a} f(x)$$

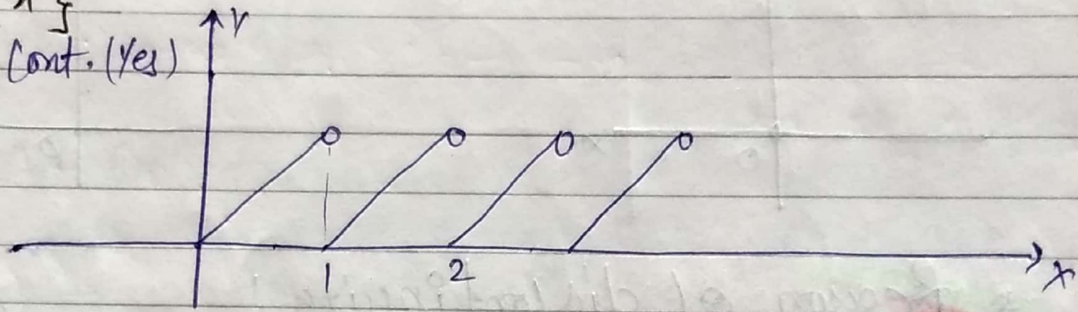
example! Check Continuity of function in

3) $f(x)$ $\{x\}$

(i) $(1, 2) \Rightarrow$ Cont. (Yes)

(ii) $[1, 2)$

(iii) $[1, 2]$



Ans: (i) Yes, Continious.

2) (ii) i) Cont. in $(1, 2)$

ii) Check at $x = 1$

$f(1)$ $f(1^+)$

$\downarrow$ $\downarrow$

$0 = 0$

$\therefore f(1) = f(1^+) \Rightarrow$ cont. at $x = 1$

$\therefore$ Cont in $[1, 2)$

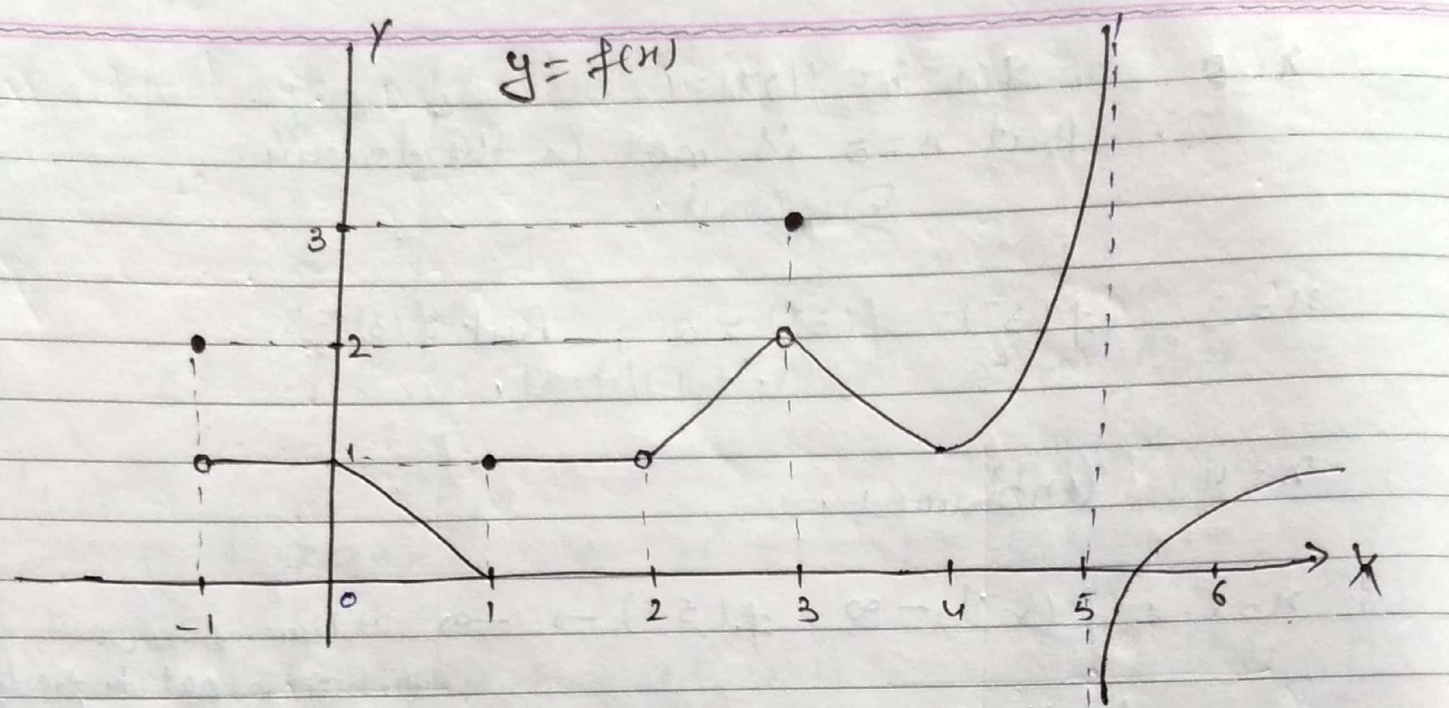
(3) (i) Cont. in $(1, 2)$

(ii) $f(1) = f(1^+)$

(iii) $f(2^-) = 1$ ($\because$ Conut. at left. $x = 2$

$f(2) = 0$ $f(2)$ is dis Cont. at $x = 2$.)

Dis. Cont. $[1, 2]$



Que! Consider $f(x)$ in interval $[-1, 6]$ and explain disconti. , if exist, belongs to $[-1, 6]$

Ans:-

i) Limit does not exist at $x = -1$ (discontinuous)

ii) Limit does ~~does~~ not exist at $x = 1$ (discontinuous)

iii) Limit exist but $f = x = 2$ not in domain (discontinuous)

(iv) Limit exist but $\lim \neq f(x)$ at $x = 3$ (discontinuous)

(v) $f(5^-) \neq f(5^+)$. (discontinuous).

Limit not exist and $x = 5$ not in the domain (discontinuous)

(vi) $x = 2$ not in the domain (discontinuous)

(vii) Cont. at $x = 0$

f $x = -1 \Rightarrow$ $(-1) = 2$, $f(-1) = 1$ $\because f(-1) \neq f(-1)^+)$ D.

$x = 0$ Cont. at $x = 0$

$x = 1$ $f(1^-) = 0$, $f(1) = 1$, $f(1^+) = 1$

$f(1^-) \neq f(1^+) = f(1)$

$0 \neq 1$ $\therefore$ Discont.

$(f(x-1)) x (g(x))$

$x = 2$ $\quad f(2^-) = f(2^+) = 1$

But $x = 2$ is not in the domain.

Discont.

$x = 3$ $\quad f(3^-) = f(3^+) = 2$, But $f(3) = 3$

$\therefore$ Discont.

$x = 4$ $\quad$ Continuous.

$x = 5$ $\quad f(5^-) \to \infty$, $f(5^+) \to -\infty$ (i) lim. D.N.E

(ii) $x = 5$, not in the domain

$x = 6$ $\quad$ Continuous.

Que:

$$f(x) = \begin{cases} \dfrac{(e^x - 1)^3 \cdot \operatorname{cosec} ax}{\ln(1+x^2)} & , x \neq 0 \\ b. & x = 0 \end{cases}$$

$\Rightarrow$ if f^n is cont. at $(x=0)$ then find Condition.

Ans:

$$f(0) = \lim_{x \to 0} f(x)$$

$$b = \lim_{x \to 0} \frac{(e^x - 1)^3 \operatorname{cosec} ax}{\ln(1+x^2)}$$

$$b = \lim_{x \to 0} \frac{\left(\frac{e^x - 1}{x}\right)^3}{\frac{\ln(1+x^2)}{x^2}} \times \frac{x^3}{x^2} \left(\frac{ax}{\sin ax}\right) \cdot \frac{1}{ax}$$

$$b = \lim_{x \to 0} = \frac{1}{a}$$

Que: $f(x) \begin{cases} (\cos x)^{\cot^2 x} & x \neq 0 \\ e^{\lambda} & x = 0 \end{cases}$

If f is cont. at $x=0$ then find λ.

Ans:

$(\cos x)^{\cot^2 x} \times e^{\lambda} = \times = \frac{\cos x^{\cot 3x}}{\cot^2 x} =$

$e^{\lambda} = \lambda + \frac{\lambda}{\lfloor 1} + \frac{\lambda^2}{\lfloor 2} + \frac{\lambda^3}{\lfloor 3} - \cdots = \cos x$

Ans:

$f(0) = e^{\lambda} = \lim_{x \to 0} (\cos x)^{\cot^2 x}$

$= e^{\lim_{x \to \infty} (\cos x - 1) \cot^2 x}$

$= e^{\lim_{x \to 0} -\frac{(1 - \cos x)}{x^2} \cdot x^2 \cdot \frac{\cos^2 x}{\sin^2 x}}$

$= e^{-1/2 \times 1}$

$= e^{-1/2}$

$\lambda = -\frac{1}{2}$ Ans

$f(\frac{\pi}{2}-x)$

Que: $f(x) \begin{cases} x + a\sqrt{2}\sin x & 0 \le x < \frac{\pi}{4} \\ 2x\cot x + b & \frac{\pi}{4} \le x \le \frac{\pi}{2} \\ a\cos 2x - b\sin x & \frac{\pi}{2} < x \le \pi \end{cases}$ check

if function is cont. in $[0, \pi)$ then find a, b.

Ans $x = \frac{\pi}{4}$ or $x = \frac{\pi}{2}$ function change our nature. Thus both are ~~not~~ check.

A check at $\frac{\pi}{4} \Rightarrow f(\frac{\pi}{4}^-) = f(\frac{\pi}{4}) = f(\frac{\pi}{4}^+)$

$=$ ~~$x + a\sqrt{2}\sin x = f(x + a\sqrt{2}\sin x) = f(x + a\sqrt{2}\sin x^+)$~~

$\frac{\pi}{4} + a\sqrt{2}\cdot\frac{1}{\sqrt{2}} = 2\frac{\pi}{4}\cot\frac{\pi}{4} + b$

$\frac{\pi}{4} + a = 2\cdot\frac{\pi}{4} + b \Rightarrow \boxed{a = b + \frac{\pi}{4}}$ — (i)

$\frac{\pi}{2} \Rightarrow f(\frac{\pi}{2}^-) = f(\frac{\pi}{2}) = f(\frac{\pi}{2}^+)$

$2\frac{\pi}{2}\cot\frac{\pi}{2} + b = a\cos\pi - b\sin\frac{\pi}{2}$

$\boxed{b = -a - b}$ — (ii)

Que:

Que: $f(x) \begin{cases} \dfrac{1-\cos 4x}{x^2}, & x<0 \\ a, & x=0 \\ \dfrac{\sqrt{x}}{\sqrt{16+\sqrt{x}}}, & x>0 \end{cases}$

If f^n is cont. at $x=0$ then find a.

→ L.H.L R.H.L Value of function

$\lim_{x\to 0^-} \dfrac{1-\cos 4x}{x^2}$ $\lim_{x\to 0^+} \dfrac{\sqrt{x}}{\sqrt{16}+\sqrt{x}}$ a

$\cancel{\dfrac{1-\cos 4x}{4x}}\ x^2$

$\lim_{x\to 0^+} \cancel{\dfrac{\sqrt{x}}{\sqrt{16}+\sqrt{x}}} =$

$\lim_{0^-} = \dfrac{1}{x^2}$

$\cancel{\dfrac{(\sqrt{x})^2}{(\sqrt{16}+\sqrt{x})(\sqrt{x})}} = \dfrac{x^2}{4\sqrt{x}+\sqrt{x^2}}$

$= \cancel{\dfrac{x^2}{4\sqrt{x}+x}}$

L.H.L

L.H.L $= f(0^-) =$

$\lim_{h\to 0} f(0-h) = \lim_{h\to 0}$

$\dfrac{1-\cos 4(0-h)}{(0-h)^2} = \lim_{h\to 0} \dfrac{1-\cos 4h}{4h^2} \cdot \dfrac{(4h)^2}{h^2}$

$\cos(-\theta) = \cos\theta$

$= \dfrac{1}{2} \cdot 16 = 8$ A

R.H.L

$f(0^+) = \lim_{x\to 0} f(0+h)$

$\lim_{x\to 0^+} \dfrac{1-\cos 4(0+h)}{(0+h)^2}$

$\lim_{x\to 0} = \dfrac{1-\cos 4h}{(0+h)^2} =$

$\lim_{h\to 0} \dfrac{1-\cos 4x}{h^2} =$

R.H.L $= f(0^+) = \lim_{h\to 0} f(0+h) = \lim_{x\to 0} \frac{\sqrt{h}}{\sqrt{16+\sqrt{x}}-4}$ $\left(\frac{0}{0}\right)$

$\lim_{h\to 0} \frac{\sqrt{h}}{(\sqrt{16+\sqrt{x}}-4)} \frac{(\sqrt{16+\sqrt{x}}+4)}{(\sqrt{16+\sqrt{x}}+4)}$

$\lim_{h\to 0} \frac{\cancel{\sqrt{h}}(\sqrt{16+\sqrt{h}}+4)}{\cancel{16}+\cancel{\sqrt{h}}-\cancel{16}} = 8$

$\Rightarrow$ Cot du $= 0$

$f(0^-) = f(0) = f(0^+)$

$8 = a = 8 \quad \therefore a = 8$ Au

Que: If $f(x) = \frac{\sin 2x + A\sin x + B\cos x}{x^3}$ is cont. at $x=0$, then find A and B.

Ans: $f(0) = \lim_{x\to 0} f(x)$

$\lim_{x\to 0} \frac{\sin 2x + A\sin x + 3\cos x}{x^3}$

$\lim_{x\to 0} \Rightarrow \left(2x - \frac{(2x)^3}{\lfloor 3} + \cdots\right) + A\left(x - \frac{x^3}{\lfloor 3} + \cdots\right) + B\left(1 - \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3}\right)$

$\lim_{x\to 0} \frac{x(2+A+\overset{0}{B}) + x^3\left(-\frac{8}{\lfloor 3} - \frac{A}{\lfloor 3}\right) + x^2\left(-\frac{B}{\lfloor 2}\right) + x^4 \cdots}{x^3}$

~~f(0)=~~ $2 + A + B = 0$

$\cancel{-\frac{B}{\lfloor 2} = 0}$

H.W : Q-1 : 1, 2, 3, 4, 5, 6, 7, 11,
S-1 : 1, 2, 3, 5, 8, 9, 10

$$\frac{B + x(2+A) + x^2\cdot\left(-\frac{B}{l^2}\right) + x^3\left(-\frac{8}{l^3} - \frac{A}{l^3}\right) + x^4 \text{———}}{x^3}$$

$\Rightarrow \quad f(0) = -\frac{8}{l^3} - \frac{A}{l^3}$

$B = 0$

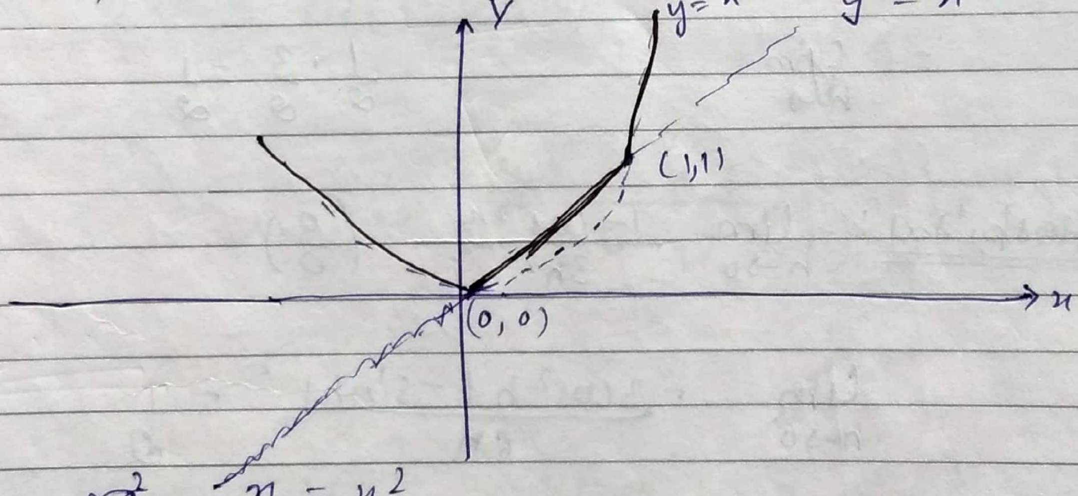
$2 + A = 0$

$-\frac{B}{l^2} = 0$

Que: Check Continuity and dif.

$f(x) = \max\{x, x^2\}$ $\quad y = x$

$y = x^2$



$x^2 \quad x = x^2$

$x = 0, 1$

$$\max\{x, x^2\} = \begin{cases} x^2, & 1 \le x < \infty \\ x, & 0 \le x < 1 \\ x^2, & -\infty < x < 0 \end{cases}$$

Date: 03/06/17 Q-1

(7) $f(0) = \lim_{x \to 0} \frac{x - e^x + \cos 2x}{x^2} \quad \left(\frac{0}{0}\right)$

$= \lim_{x \to 0} x - (1 + x + \frac{x^2}{\lfloor 2} + (1 - \frac{4x^2}{\lfloor 2} + - -)$

$= \lim_{x \to 0} \frac{x^2 (-\frac{1}{\lfloor 2} - \frac{4}{\lfloor 2}) + - -}{x^2}$

$f(0) = -\frac{5}{2}$

(9) L.H.L $= f(\frac{\pi^-}{2}) = \lim_{h \to 0} f(\frac{\pi}{2} - h) = \lim_{h \to 0} \frac{1 - \sin^3(\frac{\pi}{2} - h)}{3\cos^2(\frac{\pi}{2} - h)}$

$\lim_{h \to 0} \frac{1 - \cos^3 x}{3 \frac{\sin^2 h}{h^2} \cdot h^2} = \lim_{x \to 0} \left[\frac{(1 - \cos h)(1 + \cos^2 h + (h).}{h^2 \quad 3} \right.$

$= \lim_{h \to} \qquad \frac{1}{2} \cdot \frac{3}{2} = \frac{1}{2}$

L Hospital: $\lim_{h \to 0} \frac{1 - \cos^2 x}{3h^2} \quad \left(\frac{0}{0}\right)$

$\lim_{h \to 0} \frac{-3\cos^2 h(-\sin h)}{6h} = \frac{1}{2}$

(3) L.H.L: $f(\frac{\pi^-}{2}) = \lim_{h \to 0} f(\frac{\pi}{2} - h)$

$\lim_{h \to 0} \left(\frac{6}{5}\right)^{\frac{\tan 6(\frac{\pi}{2} - h)}{\tan 5(\frac{\pi}{2} - h)}}$

$= \lim_{h \to 0} \left(\frac{6}{5}\right)^{\frac{-\tan 6h}{\cot 5h}} = \lim_{h \to 0} \left(\frac{6}{5}\right)^{\frac{0 \cdot 0}{1}}$

$= \left(\frac{6}{5}\right)^0$

$= 1$ Ans

(2)

$f(-1)$, $f(-1^-)$ $\rightarrow$ $|-a+3|$ = Cont. at $x = -1$

$f(-1^+) = |-3+a|$

$$|-a+3| = |-3+a|$$

$$|-1|\,|a-3| = |a-3| \quad \left[\,|a-b| = |b-a|\,\right]$$

$$|a-3| = |a-3|$$

(8) L.H.L

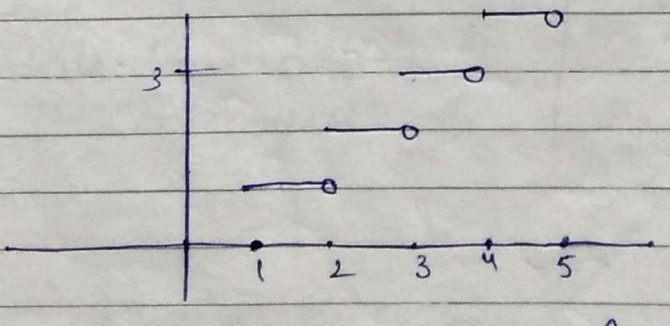
$$f(0^-) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} \frac{e^{\sin 4(0-h)} - 1}{\ln(1+\tan 2(0-h))}$$

$$\lim_{h\to 0} \frac{e^{-\sin 4x} - 1}{\ln(1-\tan 2x)} = \lim_{h\to 0} \frac{e^{-\sin 4x} - 1}{-\sin 4x}$$

$$\lim_{h\to 0} \frac{e^{-\sin 4x} - 1}{-\sin 4x} \times \frac{\frac{-\sin 4x}{4x}\cdot 4x}{\frac{\ln(1-\tan 2h)}{-\tan 2h} \times \frac{-\tan 2h}{2h}\cdot 2h}$$

$$= 2$$

Que: [1,4] find no. of Continuity of $f(x) = [x]$ in interval [1,4].



$f(4^-) = 3$

$f(4) = 4$

Ans = 2, 3, 4

Discontinuous at 3 Points

Que: find no. of point of discont. in [1,4]

$$f(x) = [x^2]$$

$1 \le x \le 4$

$1 \le x^2 \le 16$ $\qquad = 16 - 1 = \underline{15}$

$x = \sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5} \; --- \; \sqrt{16}.$ Ans

Que: $[1, 4]$

$f(x) = [x^2 - 1]$

$= [x^2] - 1 \qquad = [x + I] = [x] + I$

$= 15$

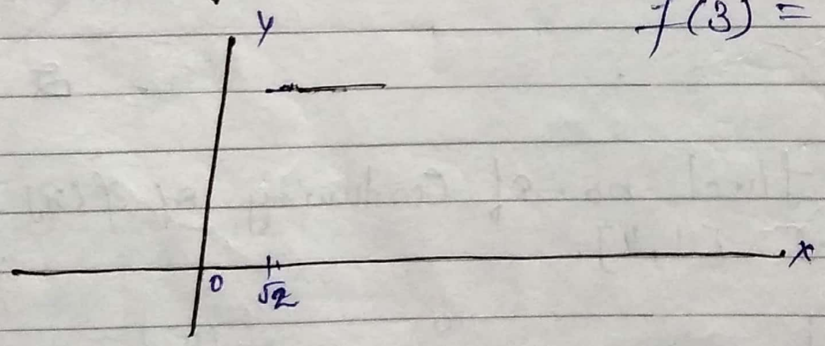
Q. Let $f : R \to R$ be the continuous fn

$f(x) = 5 \;\; \forall$ is t irrational value.

Ans: find $f(3) = ?$

$f(x) = 5$

$f(3) = 5$ Ans



$y = f(x)$	$g = g(x)$	$f(x)+g(x)$	$f-g$	$f \cdot g$	f/g
C	C	C	C	C	C
C	D	D	D	CNS	CNS
D	C	D	D	CNS C	CNS
D	D	CNS	CNS	CNS	CNS

C → Continuous
D → Discontinuous
CNS → Can't say.

* Type of discontinuity:

Type 1 or Removable D.

- In this case limit exist at $x=a$ i.e $\lim_{x \to a} f(x) =$ Exist but either it is not equal to $f(a)$ or $f(A)$ is not defined
- It is possible to redefine fn to make it continuous

Type 2 or Non Removable D.

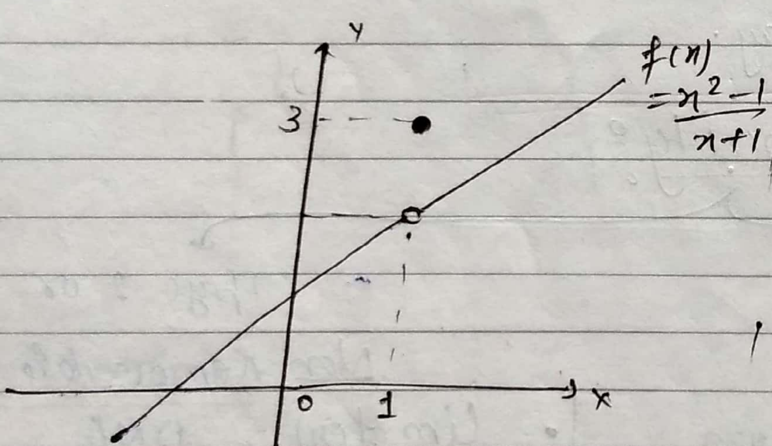
- $\lim_{x \to a} f(x) =$ DNE
- Not possible to define to make such fn Continuous.

Removable D.

Isolated P.t D | **missing pt. D.**

$$f(x) = \begin{cases} \dfrac{x^2-1}{x-1}, & x = 1 \\ 3, & x = 1 \end{cases}$$

Here value of fn is not equal to its limit



$$f(x) = \frac{x^2-1}{x-1}$$

$$= \frac{(x-1)(x+1)}{(x-1)}$$

$$= x+1$$

$$\lim_{x \to 1} \frac{x^2-1}{x-1} = 2 \neq f(1)$$

* $f(x) = \dfrac{x^2-1}{x-1}$ at $x = 1$
* $x = a$ is missing from domain.

* $f(x) = \dfrac{x^2-1}{x-1}$ at $x = 1$
* $\lim_{x \to 1} f(x) = 2$
* f but $x = 1$ is not in the domain
* Herefore DisCont.

~~$f(x) = x$~~

$f(x) \neq f(1) = 2$

It is possible to ~~reco~~ redefine to make it continuous.

H.W : Q-2 : 7, 8, 9, 10, 11
8-2 : 9(A),
J.M = full complete.

Non-Removable or Type 2

1) finite time

* Here L.H.L and R.H.L both are unequal and finite.
* and their Jump

$\text{Jump} = |L.H.L - R.H.L|$

* $f(x) = \left|\frac{\sin x}{x}\right| \quad x \neq 0$
* Cheack its C/D at $x=0$
* $R.H.L = f(0^+) = \lim_{x \to 0} \frac{\sin x}{x} = 1$
* $L.H.L = f(0^-) \to \lim_{x \to 0} -\frac{\sin x}{x} = -1$, therefore
* Non Renuable finite type
* Jump $= 1-(-1)$ $= 2$

2) infinite Time.

Either one and both limit aproches to infine

$f(x) = 2^{\tan x}$ at $x = \frac{\pi}{2}$

$f\left(\frac{\pi}{2}^+\right) = 2^{-\infty} = \frac{1}{2^\infty}$

$= \frac{1}{\infty} = \infty$

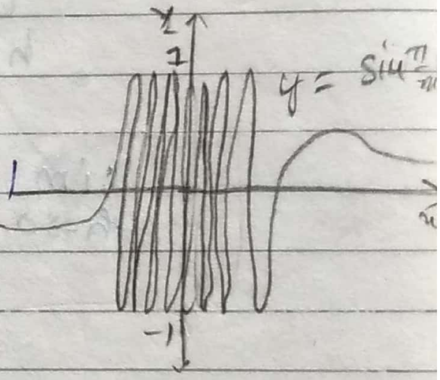
$f\left(\frac{\pi}{2}^-\right) = 2^\infty = \infty$

Oscillatory

Here limit is oscillatory b/w two fixed point.

$y = \sin\frac{1}{x}$

$y = \cos\frac{1}{x}$



Here f^n oscillate b/w -1 to 1 when x is zero therefore Dis continuous, 05/06/17 Ex: O-2

Que: 9 (D) $f(x) = \frac{1}{\ln|x|}$ = limit exist, non Removable

J.M

(5) $\lim_{x \to \pi} f(\pi) = \lim_{x \to \pi} f(x)$

$x - \pi = h$

$x = \pi + h$

$\cos x = \cos(\pi + h)$

$= -\cosh$

$K = \lim_{x \to \pi} \frac{\sqrt{2 + \cos x} - 1}{(x - \pi)^2}$

$\lim_{h \to 0} \frac{\sqrt{2 - \cos h} - 1}{h^2} \quad \left(\frac{0}{0}\right)$

$\lim_{h \to 0} \frac{1}{2\sqrt{2 - \cos h}} \times \frac{(-x - \sin h)}{2h}$

$= \frac{1}{2 \cdot 2 \cdot 1} = \frac{1}{4}$ Am

(1) $f(x) = \begin{cases} \frac{1}{x} - \frac{2}{e^{2x} - 1} & R - \{0\} \\ \lambda & 0 \end{cases}$

$f(0) = \lambda = \lim_{x \to 0} \frac{1}{x} - \frac{2}{e^{2x} - 1}$

$= \lim_{x \to 0} \frac{e^{2x} - 1 - 2x}{\frac{(e^{2x} - 1)}{2x} \cdot x \cdot 2x}$

$= \frac{\left(1 + 2x + \frac{(2x)^2}{\underline{2}}\right) \cancel{+} - 1 - 2x}{2x^2}$

$= \frac{\frac{4}{\underline{2}}}{2} = 1$ Am

H.W: J.A and complete - 0.1

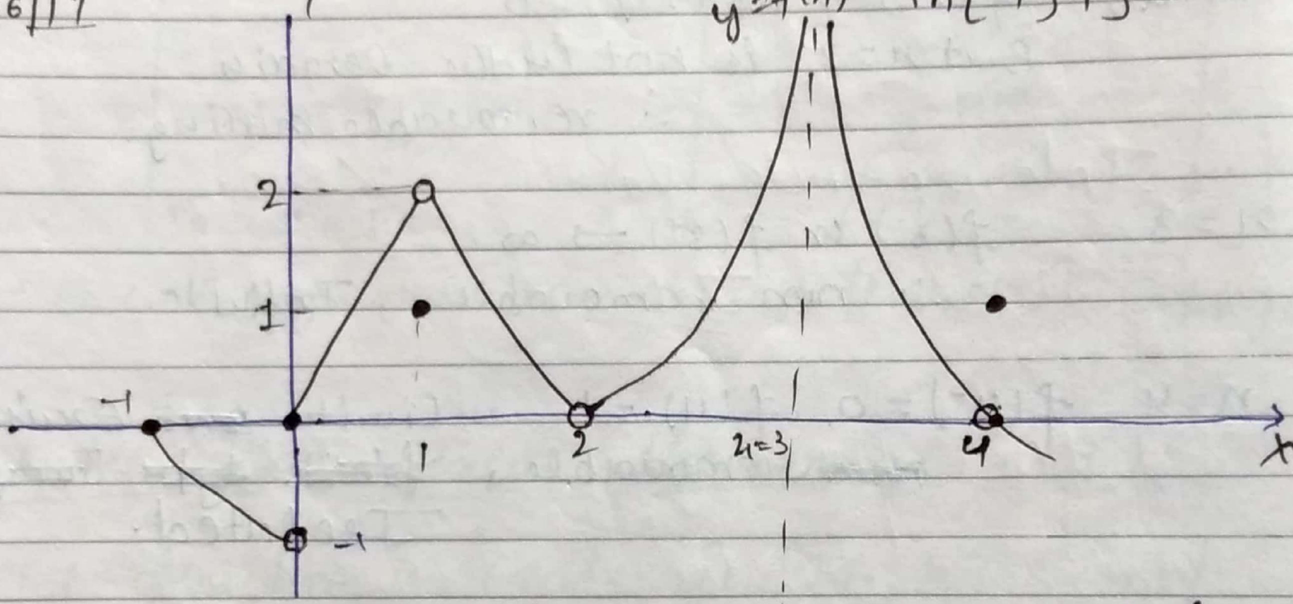
(b) $f(x) = x \sin \frac{1}{x}$

$f(0) = \lim_{x \to 0} f(x)$

$= \lim_{x \to 0} x \sin \frac{1}{x} = 0$

05/06/17

$y = f(x)$ in $[-1, 4]$



Discuss continuity / discontinuity at all integral points in the given interval. If discontinuous then conside nature of discontinuous

Ans: $\lim_{x \to 0} f(0^-) \neq f(0^+)$

$f(-1) = f(1^-) = 0$, $f(1^+) = -1$, $= f(1^-) \neq f(1^+) \neq f(1)$

continuous.

$f(0) = f(0^-) = -1$, $f(0^+) = 0 = f(0^-) \neq f(0^+) \neq f(0)$

Discontinuous

$f(1) = f(1^-) = 2$, $f(1^+) = 2$, $f(1) = 1$, $f(1^-) \neq f(1^+)$

Discontinuous

$f(2) = f(2^-) = 2$, $f(2^+) = \infty$,

Ans:
$x = -1$ continuous.

$x = 0$ $f(0^+) = 0$, $f(0^-) = -1$ D
non-removable, finite, Jump = 1

$x = 1$, $f(1^-) = f(1^+) \neq f(1) \Rightarrow$ removable.
Isolated.

$x = 2$, $f(2^-) = f(2^+) = 0$
But $x = 2$ is not in the Domain
$\therefore$ removable missing.

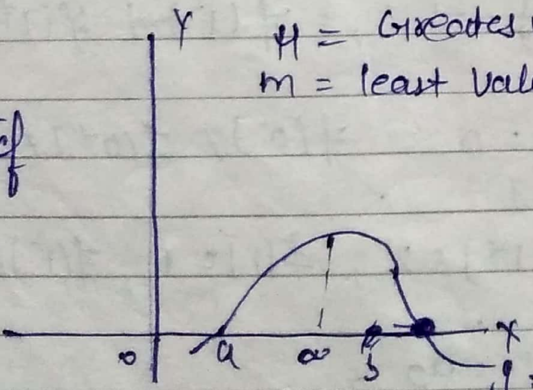
$x = 3$ $f(3^-)$ or $f(3^+) \to \infty$
$\therefore$ non-removable, Infinite.

$x = 4$ $f(4^-) = 0$, $f(4) = 1$ $\Rightarrow$ Limit ~~DNE~~ Exist
$\therefore$ ~~non~~ removable, ~~finite type Jump =1~~
Isolated.

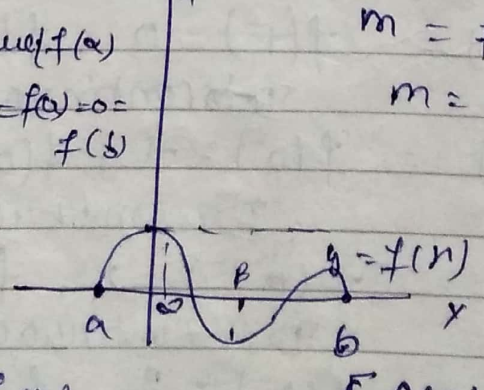
Note: If function $y = f(x)$ is continuous in $[a, b]$ then it will be bounded. that is function must have greatest ~~&~~ least somewhere in their interval

Ief

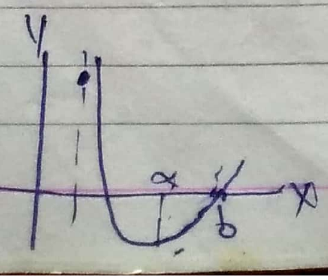
M = Greatest value $f(\alpha)$
m = least value $= f(a) = 0 = f(b)$



$m = f(\alpha)$
$m = f(\beta)$

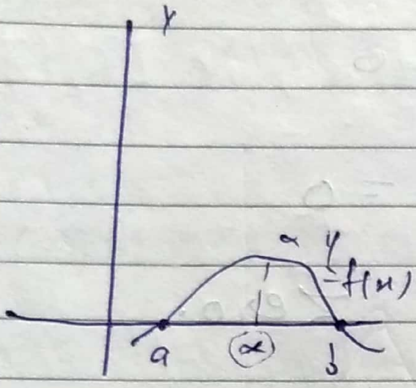


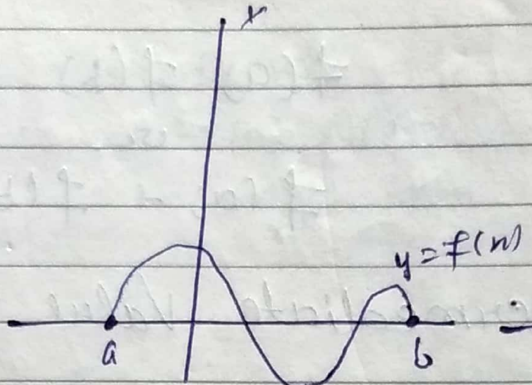
range $[f(\beta), f(a)]$

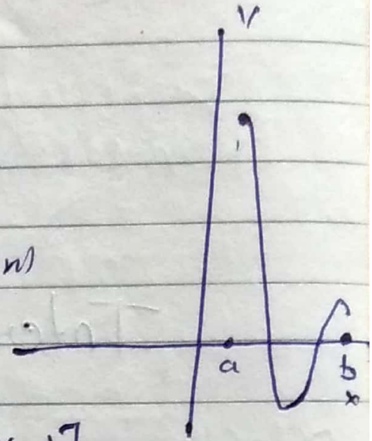


range $(f(\alpha), \infty)$
$m = f(a)$
$m = f(\alpha)$

If f is cont. in $[a,b]$ then f takes a least value and a greatest value on this interval this is called an extreame value theorem.







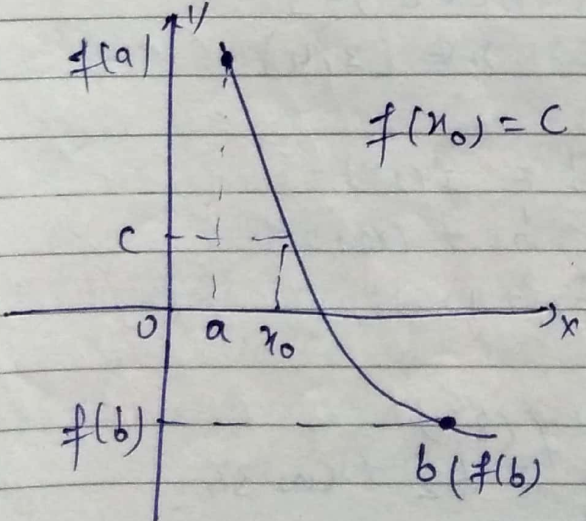
M = Go Greatest value

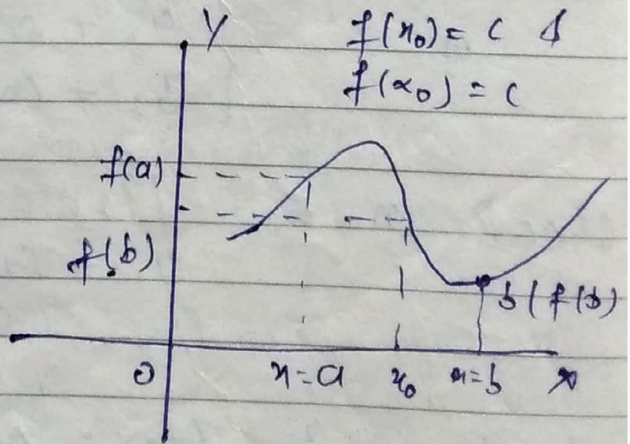
range $= [f(\beta), f(\alpha)]$

$m = f(a)$

* Intermediate Value theorem : (IVT):

If function is cont. in $[a,b]$ and $f(a) \neq f(b)$ then for any $c \in (f(a), f(b))$ their exist atleast one no. $x = x_0 \in (a,b)$ for which $f(x_0) = c$





Note: If f^n is cont. in $[a, b]$ and if $f(a)$ and $f(b)$ have opposite sign then their must atleast one (a, b)

$$f(a)\, f(b) < 0$$
or
$$f(a) + f(b) = 0$$

Intermediate Value = Zero.

06/06/17

J.A

Que: 2

$$f(x) = \begin{cases} a_n + \sin \pi x & x \in [2n, 2n+1] \\ b_n + \cos \pi x & [2n-1, 2n] \end{cases}$$

$$n = 1 = \begin{cases} a_1 + \sin \pi x & x \in [2, 3] \\ b_1 + \cos \pi x & x \in (1, 2) \end{cases}$$

$$n = 2 \quad \begin{cases} a_2 + \sin x & x \in [4, 5] \\ b_2 + \cos \pi x & x \in (3, 4) \end{cases}$$

check $x = 2$ $\quad f(2) = f(2^+) = f(2^-)$

$$a_1 + \sin \pi . 2 = b_1 + \cos 2\pi$$
$$a_1 = b_1 + 1 \quad -(i)$$

check $x = 3$ $\quad f(3) = f(3^-) = f(3^+)$

$$a_1 + \sin 3\pi = b_2 + \cos 3\pi$$
$$a_1 = b_2 - 1$$

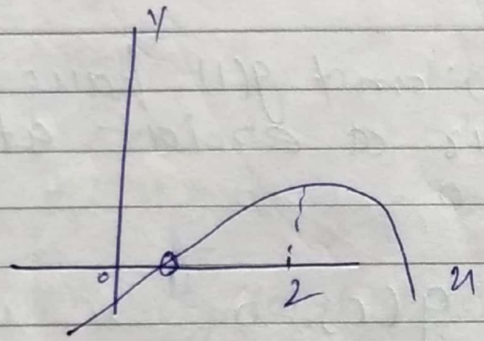
06/06/17

Que) If $f(x) = ax^2 + bx + c$ such that
$f(0) + f(2) = 0$
then roots of equation $f(x) = 0$ will lie in the interval

i) $(-1, 0)$
ii) $(0, 1)$
iii) $(1, 2)$
~~iv)~~ $(0, 2)$

$f(0) = -f(2)$
$f(0)$ & $f(2)$ must have opp sign.

Ans:



$a, b \in R$.

Que! If roots of eqn $bx^2 - 2ax + a = 0$ are real and distinct then Proff that above equation must have one real root lie in the Interval $(0, 1)$

Ans! $f(0)$ & $f(1)$ must have opposite sign.

$f(0) \cdot f(1) = a(b - 2a + a)$
$= a(b-a) < 0$

$f(0)$ & $f(1)$ must have opp sign.

$D > 0$
$4a^2 - 4ba > 0$
$4a(a-b) > 0$
$a(a-b) > 0$
$a(b-a) < 0$

बनाके L.H.S में लगाके
अपना फंक्शन बनाना है

31

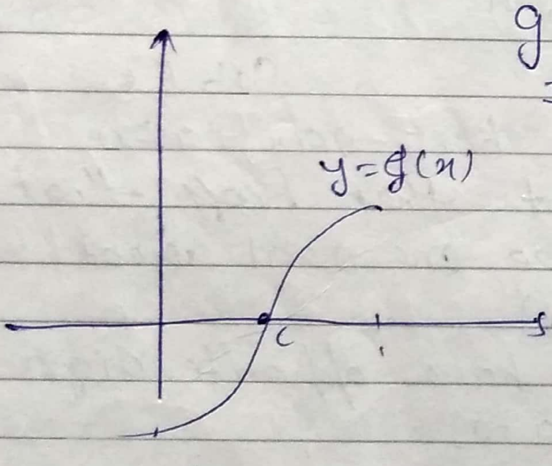
Que: Suppose $f(x)$ is cont. at x $[0,1]$
$f(0) = 0$
$f(1) = 0$
P.T $f(c) = 1 - 2c^2$ for some $c \in (0,1)$

Define a new function $g(x)$ in $[0,1]$
where $g(x) = f(x) + 2x^2 - 1$

$g(x)$ is cont. in $[0,1]$

because $g(0)$ and $g(1)$ have opposite sign then their a exist at c where $g(x) = 0$

$g(c) = 0$
$f(c) + 2c^2 - 1 = 0$
$f(c) = 1 - 2c^2$



* Continuity of function is defined by some functional roots:

If fn $f(x)$ satisfy $f(x+y) = f(x) f(y)$ following property and f is cont. at $x = 1$ then proof that it is cont. at. ~~x=1~~ for all x.

$f(x+y) = f(x)\, f(y)$

Continuity at $x = 1$ L.H.L. = R.H.L = value of f.

$f(1^-) = f(1^+) = f(1)$

$$\lim_{h \to 0} f(1-h) = \lim_{h \to 0} f(1+h) = f(1)$$

$$\boxed{\lim_{h \to 0} \cancel{f(1)}\, f(-h) = \lim_{h \to 0} \cancel{f(1)}\, f(h) = f(1)}$$

check ~~at~~ cont. at $x = a$

$$L.H.L = f(a^-) = \lim_{x \to 0} (a-h) = \lim_{h \to a} f(a)\, f(-h) = f(a)$$

$$R.H.L = f(a^+) = \lim_{x \to 0} f(a+h) = \lim_{h \to 0} f(a) + f(h) = f(a)$$

Value of f^n.

$\therefore$ Continuous

* Some functions are continuous ~~at~~ only at one point however define every where.

Que: Prove that $f(x) = \begin{cases} x, & x \in Q \\ x, & x \in Q^c \end{cases}$

f^n is cont. only at $x = 0$

$$f(x) = \begin{cases} x & x \in \mathbb{Q} \\ 0 & x \in \mathbb{Q}^c \end{cases}$$

$f(0) = 0$

LHL or R.H.L $= f(0^+)$ or $f(0^-)$

If rational $= 0^+ = 0$

If irrational $\to 0$

$\therefore$ cot. at $x = 0$

* $x = 1$

$f(1) = 1$

R.H.L $= f(1^+) =$

rational $= 1$

Irrational $= 0$

Discontinuous at $x = 1$